Announcements

• TNvoice evaluations
  - 5 points EC on final if we get 100% participation
  - 67%, currently; deadline Monday 4/27
Final Exam

• One-week, take-home exam
  – Same format/restrictions as Midterm 2
    ▪ Open-book, open-course-website, no other online resources
    ▪ No collaboration
    ▪ MATLAB, LTSpice, or other computer tools can be used but will receive no credit unless otherwise stated

• Cumulative, covering entire course content
• Exam password e-mailed out April 27th before midnight
• Exam due to canvas before midnight, May 4th

Course Content

• Magnetically Coupled Circuits (Ch 13)
• Sinusoidal Steady-State Analysis (Ch 10)
• AC Circuit Power Analysis (Ch 11)
• Circuit An Analysis in the s-Domain (Ch 14)
• Frequency Response (Ch 15)
• Fourier Circuit Analysis (Ch 17)
• Polyphase Circuits (Ch 12)
• Two-Port Networks (Ch 16)
Transform Domains

- **Phasor Transform**
  \[ u(t) = A \cos(\omega t + \varphi) \]

- **Fourier Transform**
  \[ u(t) = \sum_{n} c_n \cos(n \omega t + \varphi_n) \]

- **Laplace Transform**
  \[ u(t) = \sum_{n} k_n e^{s_n t} \]

---

Ch 13 – Magnetically Coupled Circuits

**Defining Equations**

- Circuit 1:
  \[ v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + v_1 \]
- Circuit 2:
  \[ v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + v_2 \]

**Dot convention**
- Current into the dot on one terminal produces a positive open circuit voltage w.r.t. the dot on the other.

**Coupling Coefficient**

\[ k = \frac{M}{\sqrt{L_p L_s}} \]

**Recall**
- Equivalent circuits

**Defining Equations**

- Ideal Transformer
  \[ \frac{V_1}{N_1} = \frac{V_2}{N_2} = \ldots \]
  \[ 0 = N_1 I_1 + N_2 I_2 + \ldots \]

**Coupled inductors with:**
- no energy storage \((L \to \infty)\)
- Perfect coupling \((k=1)\)

**Recall**
- \(Z/V/I\) reflection

---

Slido: #Q711
**Ch 10 – Sinusoidal Steady State**

<table>
<thead>
<tr>
<th>Time Domain</th>
<th>Phasor Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(t) = i(t)R$</td>
<td>$V = IR$</td>
</tr>
<tr>
<td>$v(t) = L \frac{di}{dt}$</td>
<td>$V = j\omega LI$</td>
</tr>
<tr>
<td>$v(t) = C \frac{di}{dt}$</td>
<td>$V = \frac{j}{\omega C}I$</td>
</tr>
</tbody>
</table>

$V_1 = j\omega L_1I_1 + j\omega M_1I_2$

$V_2 = j\omega M_1I_1 + j\omega L_2I_2$

**Phasor Notation**

$A\cos(\omega t + \varphi) \Rightarrow A\varphi$

$= \text{Re}\{Ae^{j(\omega t + \varphi)}\}$

$= \text{Re}\{A\cos(\omega t + \varphi) + jA\sin(\omega t + \varphi)\}$

**Impedance and Admittance**

$Z = R + jX$

$Y = \frac{1}{Z} = G + jB$

**Impedance**  | **Resistance**  | **Reactance**
**Admittance**  | **Conductance**  | **Susceptance**

**Circuit Analysis**
- Real circuits always have all real signals in the time domain
- All 201 analysis techniques apply
- Gives only forced/steady-state/particular response, for single sinusoidal source
- Phasor superposition

---

**Ch 11 – AC Power Analysis**

**Average (DC) Power:**  
$P = \int p(t)dt$

For periodic signals:  
$P = \int_{t_0}^{t_0+T} p(t)dt$

**Average power in a resistor:**  
$P_R = \left[ \int [i(t)]^2 dt \right] R$

For sinusoids:  
$I_{\text{rms}} = \frac{I_A}{\sqrt{2}}$

**Sinusoidal Power**

$p(t) = [V_A \cos(\omega t + \varphi_V)][I_A \cos(\omega t + \varphi_I)]$

$= \frac{V_A I_A}{2} \cos(2\omega t + \varphi_V + \varphi_I) + \frac{V_A I_A}{2} \cos(\varphi_V - \varphi_I)$

$V_{\text{rms}}I_{\text{rms}} = \frac{V_A I_A}{2}$

**Complex Power**

$S = \frac{VI^*}{2} = V_{\text{rms}}I_{\text{rms}}$

**Apparent Power:**  
$|S| = \frac{V_A I_A}{2} = V_{\text{rms}}I_{\text{rms}}$

**Power Factor:**  
$PF = \frac{P}{|S|}$ (leading/lagging)

**Impedance match for max power transfer:**  
$Z_L = Z_{\text{th}}^*$
Ch 14 – Laplace Transform

Unilateral Laplace Transform:
\[ F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t) \, dt \]

Inverse Laplace Transform:
\[ f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{\sigma_0-j\infty}^{\sigma_0+j\infty} e^{st} F(s) \, ds \]

Complex frequency = Laplace variable = \( s = \sigma + j\omega \)

Laplace transform is a linear transformation. Other properties and transforms in tables

Inverse Transforms: Long Division \( \rightarrow \) Factor \( \rightarrow \) PFE \( \rightarrow \) Tables

PFE Special cases:

Repeated:
\[ \frac{N(s)}{(s + 5)^2} = \frac{k_1}{s + 5} + \frac{k_2}{(s + 5)^2} \]

(Differentiation or coefficient matching)

Complex:
\[ \frac{N(s)}{s^2 + 4} = \frac{k_1}{s - j2} + \frac{k_1^*}{s + j2} \]

Ch 14 – Laplace Circuit Analysis

Poles and Zeros:
\[ F(s) = \frac{N(s)}{D(s)} \]

Transfer Functions:
\[ H(s) = \frac{V_o(s)}{V_i(s)} \]

Poles of \( H(s) \) define “form” of terms in natural response of the circuit

Poles of \( V_i(s) \) define “form” of terms in forced response of the circuit

Convolution:
\[ v_o(t) = v_i(t) * h(t) = \int_{-\infty}^{\infty} v_i(t - \tau) h(\tau) \, d\tau \]
Ch 15 – Frequency Response

**Frequency Response:** $H(s \rightarrow j\omega)$
- Circuit response to sinusoidal inputs
- Valid if all poles in LHP

**Bode plot:** mag-phase plots on log-log axes
- $\|H(j\omega)\|_{dB} = 20 \log(\|H(j\omega)\|)$
- $4 \cdot H(j\omega)$

**Templates and Approximations:**
- Circuit response to sinusoidal inputs
- Valid if all poles in LHP

**Filter Design**
- Bandwidth, graphical analysis, Chebyshev and Butterworth, Sallen-Key Amplifier
- Resonant circuits

Ch 17 – Fourier Series and Transform

**Fourier Series:**
For periodic $f(t)$ with period $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \cos(k\omega_0 t) \, dt$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \sin(k\omega_0 t) \, dt$$

**Fourier Transform:**
For periodic or non-periodic $f(t)$

$$F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t} f(t) \, dt$$

$$f(t) = \mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} F(\omega) \, d\omega$$

*Frequency content of a signal*
*Gives mag/phase of sinusoids that add up to original signal*
A Return to Lecture 1

FUTURE TOPICS
Power Systems
• ECE 325 -- Electric Energy System Components

Nonlinear Circuits
• ECE 335 & 336 – Electronic Devices
Closed-Loop Control

- ECE 316 – Signals and Systems

**Classic Control**

\[
X(s) \xrightarrow{G(s)} Y(s) \xrightarrow{H(s)}
\]

**Root Locus**: How poles and zeros move as you change feedback

**Nyquist Plots**: Frequency response in the complex plane

**Modern Control**

\[
x(t) = \dot{A}(t)x(t) + B(t)u(t) \\
y(t) = C(t)x(t) + D(t)u(t)
\]

\[H(s) = C(sI - A)^{-1}B + D\]

---

Digital / Discrete Time Signals

- ECE 315 – Signals and Systems

**Cont. time**

\[\sum_n A_n \frac{d^n}{dt^n} v_o(t) = \sum_m B_m \frac{d^m}{dt^m} v_i(t)\]

\[\mathcal{L}\{\sum_n A_n v_o(s)\} = \sum_m B_m v_i(s)\]

**Discrete time**

\[\sum_n A_n v[k] = \sum_m B_m u[k - m]\]

\[\sum_n A_n z^{-n} v(z) = \sum_m B_m z^{-m} u(z)\]

**Sampling and aliasing:**

---
FINAL REMARKS

Thank you for all your hard work

Good luck

Slido: #Q711