Transformers

special case of coupled inductors where $k \approx 1 \iff M \approx \sqrt{L_1 L_2}$

$$v_1 = L_1 \frac{di_1}{dt} \pm \sqrt{L_1 L_2}\,\frac{di_2}{dt}$$
$$v_2 = L_2 \frac{di_2}{dt} \pm \sqrt{L_1 L_2}\,\frac{di_1}{dt}$$

$$v_1 = v_2\left(\frac{\sqrt{L_1 L_2}}{L_2}\right)$$

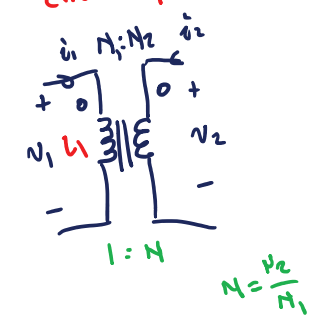
$$\boxed{v_1 = v_2\sqrt{\frac{L_1}{L_2}}}$$

$$v_1 = v_2\sqrt{\frac{\cancel{k_{11}} N_1^2}{\cancel{k_{22}} N_2^2}}$$

if $k = 1$, $k_{11} = k_{22}$

$$\boxed{v_1 = v_2\frac{N_1}{N_2}}$$

Transformer ($k=1$) circuit symbol



$N = \frac{v_2}{N_1}$

Ideal Transformer

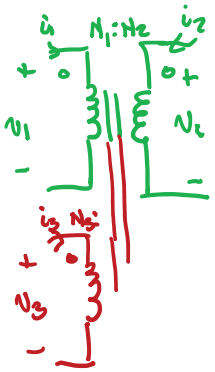
$k = 1$ (still) & inductances are "very large"

Recall: Inductors & Transformers cannot have DC voltage applied

- $v = L\frac{di}{dt} \rightarrow i = \frac{1}{L}\int_0^t v\,dt \rightarrow$ DC voltage causes current to ramp to ∞
- $v = \frac{d\Phi}{dt}$, $\Phi = kNi \rightarrow$ only get signal when $\frac{d\Phi}{dt} \neq \emptyset$
- Materials: if using core material, cores "saturate" $\rightarrow$ at high Φ permeability $\mu_r \rightarrow 1 \rightarrow L_x$ to drop & k drops drastically

with "large" inductances $E = \frac{1}{2}L_\infty i^2 = \frac{1}{2}L_\infty\left(\frac{1}{L_\infty}\int_0^t v\,dt\right)^2 = \frac{1}{2}\frac{1}{L_\infty}\left(\int_0^t v\,dt\right)^2$

↳ transformer has zero energy stored (as long as $v < \infty$)



$$i_1 v_1 + i_2 v_2 = \emptyset$$
$$i_1\left(\cancel{v_2}\frac{N_1}{N_2}\right) + i_2\cancel{v_2} = \emptyset$$

$$\boxed{\emptyset = i_1 N_1 + i_2 N_2 + i_3 N_3}$$

$$\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_3}}$$

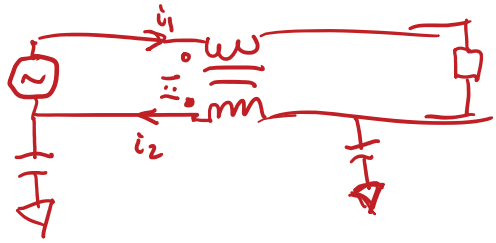
for > 2 windings

$$\frac{v_x}{N_x} = \frac{v_y}{N_y}$$

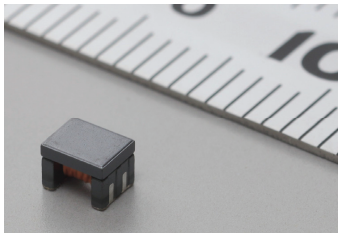
$$\sum_x i_x N_x = \emptyset$$

Transformer Uses

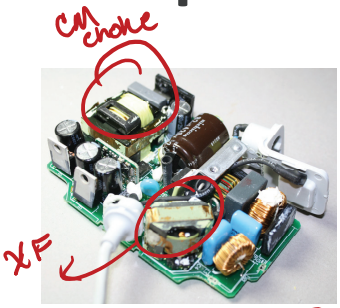
- ac voltage conversion
- impedance matching
- galvanic isolation
- common mode choke



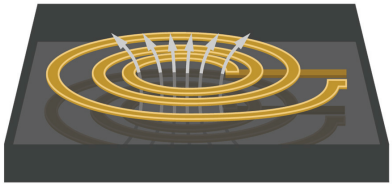
Applications of Coupled Inductors



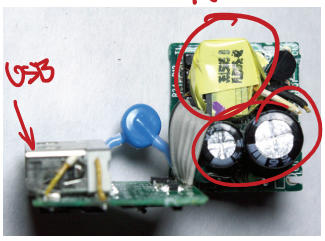
CM choke



laptop charger



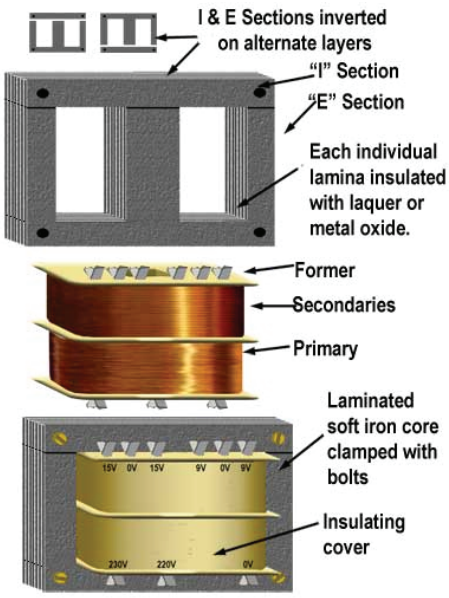
WPT



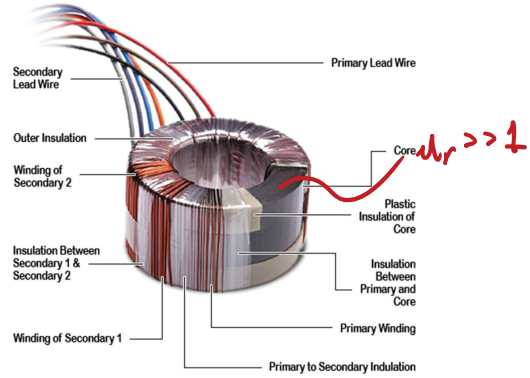
cell phone charger

Transmission scale

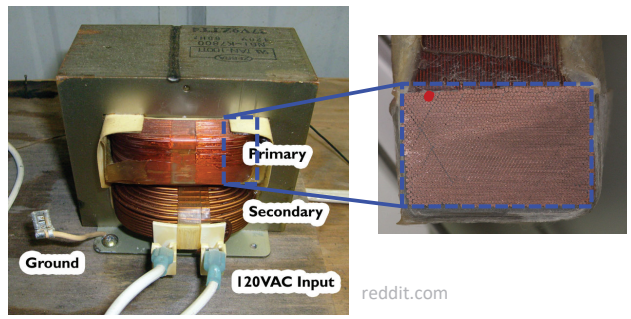
Transformer Design



http://www.learnabout-electronics.org/ac_theory/transformers03.php

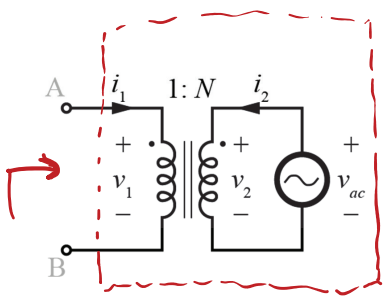


https://www.altrancorp.com/products/toroidal-power-transformers/

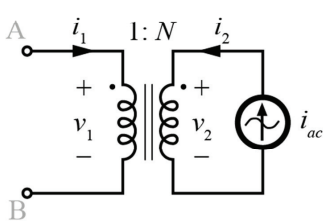


reddit.com

Transformer Reflection

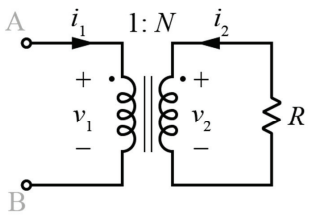


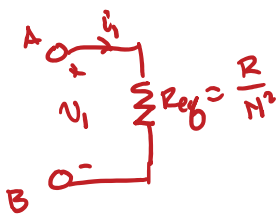
$$\frac{v_1}{1} = \frac{v_2}{N}, \quad v_2 = v_{ac}$$



$$1 \cdot i_1 + N i_2 = 0, \quad i_2 = i_{ac}$$

$$i_1 = -N i_{ac}$$

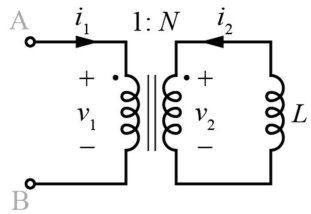


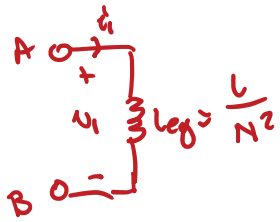


$$v_2 = -i_2 R$$

$$N v_1 = -\left(-\frac{i_1}{N}\right) R$$

$$\boxed{v_1 = i_1 \frac{R}{N^2}}$$

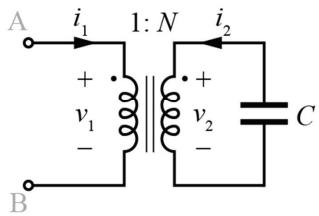


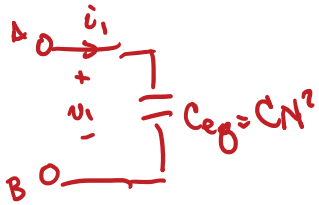


$$v_2 = -L \frac{di_2}{dt}$$

$$N v_1 = -L \frac{d}{dt}\left(-\frac{i_1}{N}\right)$$

$$\boxed{v_1 = \frac{L}{N^2} \frac{di_1}{dt}}$$





$$-i_2 = C \frac{dv_2}{dt}$$

$$-\left(-\frac{i_1}{N}\right) = C \frac{d}{dt} N v_1$$

$$i_1 = N^2 C \frac{dv_1}{dt}$$

Example Circuit Simplification