Form of the Solution

write out all KVL/KCL eqs & element diff EQs & hope to manipulate to

$$k_N \frac{d^N}{dt^N} v_{rx}(t) + \dots + k_2 \frac{d^2}{dt^2} v_{rx}(t) + k_1 \frac{d}{dt} v_{rx}(t) + k_0 v_{rx}(t) = V_{TX} \sin(\omega t)$$

$$\sum_{i=0}^{N} k_i \frac{d^i}{dt^i} v_{rx} = V_{TX} \sin(\omega t)$$

solution for $v_{rx}(t)$ will look like

$$v_{rx}(t) = v_{rx,h}(t) + v_{rx,p}(t)$$

Homogeneous / Natural / Transient response

$$\sum_{i=0}^{N} k_i \frac{d^i}{dt^i} v_{rx} = \emptyset \quad \rightarrow \quad v_{rx} = \sum_{i=1}^{N} A_i e^{s_i t}$$

assume no repeated roots

$s_i \rightarrow$ roots of characteristic polynomial

$A_i \rightarrow$ from ICs

$s_i \rightarrow$ some time constants / frequencies associated with the circuit independent of input

transient response $\rightarrow$ for any real circuit (damped) this response will die out over time

~~particular~~ / steady-state / Natural Response

$$\sum_{i=0}^{N} k_i \frac{d^i}{dt^i} v_{rx} = V_{TX} \sin(\omega t)$$

pick a guess, for this equation $v_{rx} = A \sin(\omega t + \varphi)$

$$\begin{aligned}
v_{rx}(t) &= A\sin(\omega t + \varphi) &&= A\cos(\omega t + \varphi - 90^\circ) \\
v'_{rx}(t) &= \omega A\cos(\omega t + \varphi) &&= \omega A\cos(\omega t + \varphi) \\
v''_{rx}(t) &= -\omega^2 A\sin(\omega t + \varphi) &&= \omega^2 A\cos(\omega t + \varphi + 90^\circ) \\
v'''_{rx}(t) &= -\omega^3 A\cos(\omega t + \varphi) &&= \omega^3 A\cos(\omega t + \varphi + 180^\circ) \\
v''''_{rx}(t) &= \omega^4 A\sin(\omega t + \varphi) &&= \omega^4 A\cos(\omega t + \varphi + 270^\circ)
\end{aligned}$$

every derivative multiply by ω & add 90° to the phase.

Any linear combination will be

$$v_{rx}(t) = C\cos(\omega t + \theta)$$

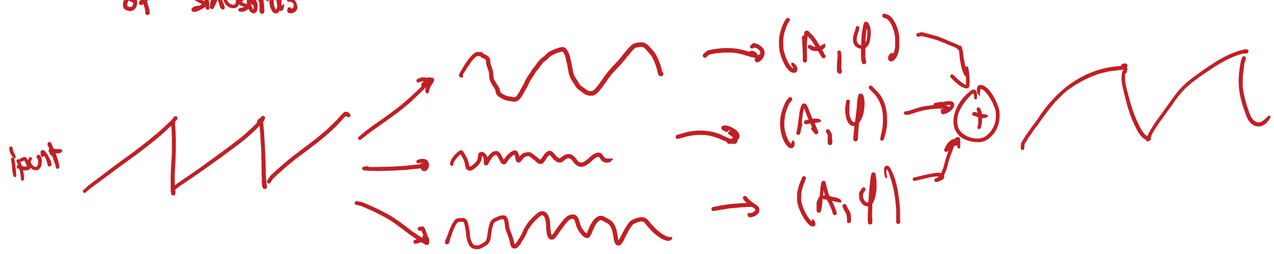
$\sin(\theta) = \cos(\theta - 90^\circ)$

Property of all LTI systems: if input is a sinusoid at ω, output is a sinusoid at ω. (only mag & phase change)

Frequency Domain: Preview (sneak peek)

LTI: sinusoids in → sinusoids out (same freq)

1. Make it really easy to solve circuits for sinusoidal inputs
2. Find a way to represent any signal as a linear combination of sinusoids



Fourier → Fourier Series → for periodic signal

Fourier → Fourier Transform → for everything else

Sinusoidal Steady State

$v(t) = A\cos(\omega t + \varphi)$ → cos

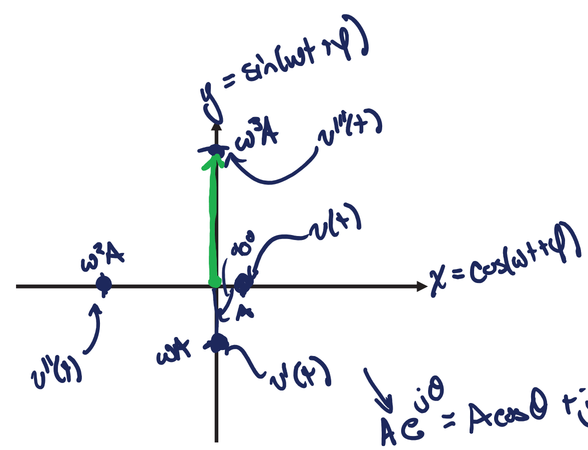
$v'(t) = \omega A\cos(\omega t + \varphi + 90°)$ → −sin

$v''(t) = \omega^2 A\cos(\omega t + \varphi + 180°)$ → −cos

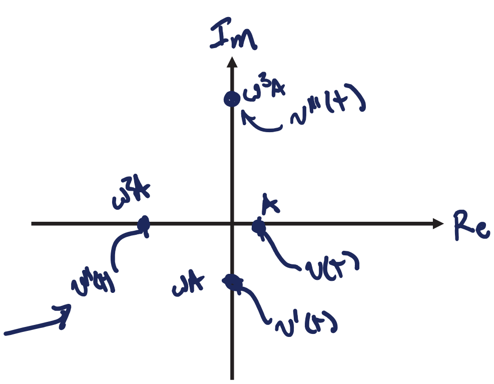
$\vdots$ sin

cos

Derivative → rotate 90° & scale length by ω



$Ae^{j\theta} = A\cos\theta + j\sin\theta$



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Complex Numbers (Review)

$z = x + jy = re^{j\theta}$

Rectangular form   Polar Form

$\begin{cases} r = \sqrt{x^2+y^2} \rightarrow \text{Magnitude} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow \text{phase} \end{cases}$

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Euler's Formula

$Ae^{j\theta} = A\cos\theta + Aj\sin\theta$

$z = x + jy = re^{j\theta}$

$jz = jx - y$

$\rightarrow$ $Re\{z\} = x$ $Im(z) = y$

$Re\{jz\} = -y$ $Im(jz) = x$