Complex Numbers

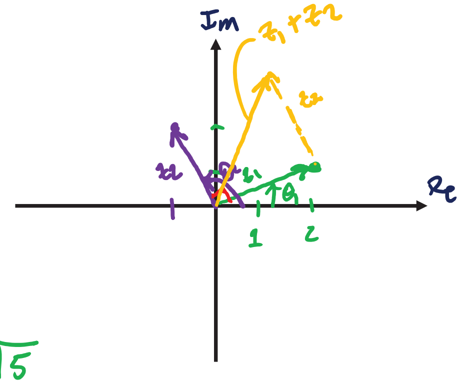
$$z = x + jy = re^{j\theta}$$

$x = Re\{z\}$ "real part"

$y = Im\{z\}$ "imaginary part"

$r = |z|$ magnitude

$\theta = \angle z$ phase



Complex numbers can represent vectors in 2D

Rect: (x, y)

Polar: (r, θ)

$z_1 = 2 + j1 = \sqrt{5}\, e^{j\tan^{-1}(\frac{1}{2})}$ $\quad \theta_1 = \tan^{-1}(\frac{1}{2}) \quad r = \sqrt{5}$

$z_2 = jz_1 = 2j - 1 = -1 + j2$

$j = 0 + j1 = 1e^{j\tan^{-1}(\frac{1}{0})} = e^{j\frac{\pi}{2}}$

$$\left.\right\} z_2 = jz_1 = e^{j\frac{\pi}{2}} \cdot z_1 = e^{j\frac{\pi}{2}} \sqrt{5} e^{j\theta_1} = \sqrt{5} e^{j(\frac{\pi}{2} + \theta_1)}$$

→ multiplying a complex number by j is a $90°$ phase shift

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Multiply

$$z_1 \cdot z_2 = (r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

Sinusoids as Complex Numbers

$$v(t) = A\cos(\omega t + \varphi) = Re\{Ae^{j(\omega t + \varphi)}\} = \boxed{Re\{\underbrace{A}_{\text{amplitude}}\underbrace{e^{j\varphi}}_{\text{phase}}\underbrace{e^{j\omega t}}_{\text{sinusoid}}\}}$$

project onto Real axis = cos

Euler: $e^{j\theta} = \cos\theta + j\sin\theta$

$\cos\theta = Re\{e^{j\theta}\}$

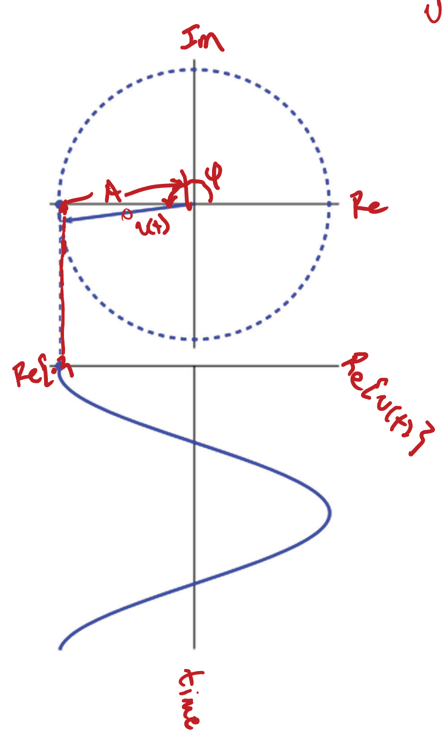
$$\frac{d}{dt}v(t) = -A\omega\sin(\omega t + \varphi) = \omega A\cos(\omega t + \varphi + 90°)$$

$$= Re\{\omega A e^{j(\omega t + \varphi + \frac{\pi}{2})}\}$$

$$= Re\{\omega A e^{j\frac{\pi}{2}} e^{j\varphi} e^{j\omega t}\}$$

$$= \boxed{Re\{j\omega A e^{j\varphi} e^{j\omega t}\}}$$

$$\begin{tikzcd} v(t) \arrow[r, "\text{differentiation}"] \arrow[d, "\text{transform into complex}"'] & v'(t) \\ {} \arrow[r, "\text{multiply by } j\omega"'] & {} \arrow[u, "\text{transform back}"'] \end{tikzcd}$$



$v(t) = Re\{Ae^{j\varphi}e^{j\omega t}\}$

at $t=0$ $e^{j\omega 0} = 1$

$e^{j\omega t}$ → has magnitude 1

phase is zero @ $t=0$

as t goes $0 \to \frac{2\pi}{\omega} = \frac{1}{f} = T$

phase of $e^{j\omega t}$: $0 \to 2\pi$

Phasor Transformation

Phasor: A complex vector that represents a sinusoid

$A\cos(\omega t + \varphi)$ $\left\{ \begin{array}{ll} \text{Amplitude} & A \\ \text{phase} & \varphi \\ \cancel{\text{frequency}} & \cancel{\omega} \\ \cancel{\text{function}} & \cancel{\cos/\sin} \end{array} \right.$

→ known for single-frequency problem

→ by convention, everything is a cos

$$v(t) = A\cos(\omega t + \varphi) = \cancel{Re}\{Ae^{j\varphi}\cancel{e^{j\omega t}}\}$$

$\Big\downarrow$ phasor transform

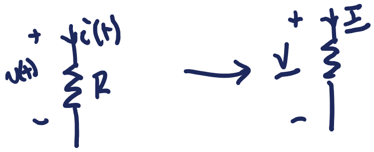
$$\underline{V} = Ae^{j\varphi} \xLeftrightarrow{\text{short hand}} A\angle\varphi$$

(Bold in book)

$$v_2(t) = B\sin(\omega t + \theta)$$
$$= B\cos(\omega t + \theta - 90^\circ)$$

$$\underline{V_2} = Be^{j(\theta - \frac{\pi}{2})} \Longleftrightarrow B\angle(\theta - \tfrac{\pi}{2})$$

Phasor Circuit Elements

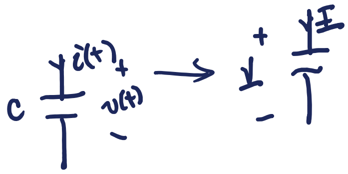


$v(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{V} = Ae^{j\varphi}$

$i(t) = \frac{A}{R}\cos(\omega t + \varphi) \longrightarrow \underline{I} = \frac{A}{R}e^{j\varphi}$

$v(t) = i(t)R$

$\boxed{\underline{V} = \underline{I}R}$



$v(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{V} = Ae^{j\varphi}$

$i(t) = -CA\omega\sin(\omega t + \varphi)$

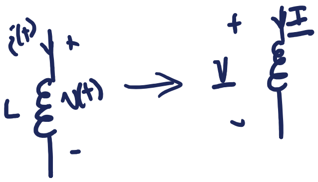
$= CA\omega\cos(\omega t + \varphi + 90°) \longrightarrow \underline{I} = CA\omega e^{j(\varphi + \frac{\pi}{2})}$

$= CA\omega e^{j\varphi}e^{j\frac{\pi}{2}}$

$= CA\omega j e^{j\varphi}$

$i(t) = C\frac{dv}{dt}$

$\boxed{\underline{V} = \underline{I}\,\frac{1}{j\omega C}}$



same process

$\boxed{\underline{V} = \underline{I}\,j\omega L}$