Finding the Conversion Ratio M(D,K)

Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance
\[
\langle v_L \rangle = \frac{1}{T_s} \int_{0}^{T_s} v_L(t) \, dt = 0
\]

Capacitor charge balance
\[
\langle i_C \rangle = \frac{1}{T_s} \int_{0}^{T_s} i_C(t) \, dt = 0
\]

Small ripple approximation sometimes applies:
\[
v(t) = V \quad \text{because} \quad \Delta v \ll V
\]
\[
i(t) = I \quad \text{is a poor approximation when} \quad \Delta i > I
\]

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

Buck Converter in DCM

\[
\Delta i_L \ll I_L
\]
\[
\Delta v_c \ll V
\]

[Diagrams of the Buck Converter in DCM showing different subintervals and control states]
Subinterval Analysis

1. 
\[ v_c(t) = v_g - v(t) = v_g - V \]
2. 
\[ i_c(t) = i_L(t) - \frac{v(t)}{R} \approx i_L(t) - \frac{V}{R} \]
3. 
\[ v_c(t) = -V \]
\[ i_c(t) = i_L(t) - \frac{V}{R} \]

Waveforms in DCM

1. 
\[ \langle v_L \rangle_z \phi = D_1 (v_g - V) - D_2 V \]
\[ \phi = D_1 v_g - (D_1 + D_2) V \]
2. 
\[ \frac{V}{V_g} = \frac{D_1}{D_1 + D_2} \]
\[ \langle i_L \rangle = \phi = \frac{1}{T_S} \int_0^{T_S} i_L(t) \, dt \]
\[ \phi = \frac{1}{T_S} \int_0^{T_S} (i_L(t) - \frac{V}{R}) \, dt \]
\[ \phi = \frac{1}{T_S} \int_0^{T_S} i_L(t) \, dt - \frac{V}{R} \]
3. 
\[ \frac{V}{R} = \frac{1}{T_S} \left[ \frac{1}{2} (D_1 + D_2) T_S \int i_L(t) \, dt \right] \]
Solving $M(D,K)$

Two equations and two unknowns ($V$ and $D_z$):

\[ V = V_s \frac{D_1}{D_1 + D_z} \]  
\[ \frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_z) (V_s - V) \]  
\[ \text{(from inductor volt-second balance)} \]
\[ \text{(from capacitor charge balance)} \]

Eliminate $D_z$, solve for $V$:

\[ \frac{V}{V_s} = \frac{2}{1 + \sqrt{1 + 4K / D^2}} \]

where \[ K = \frac{2L}{RT_s} \]
valid for \[ K < K_{crit} \]

---

Buck Converter $M(D,K)$

\[ M(D,K) = \begin{cases} 
  D & \text{for } K > K_{crit} \\
  \frac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit}
\end{cases} \]

Buck in CCM \[ \frac{V_o}{V_i} = D \]

Design \# load

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Fundamentals of Power Electronics  18  Chapter 5: Discontinuous conduction mode

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Fundamentals of Power Electronics  19  Chapter 5: Discontinuous conduction mode
Boost Converter in DCM

Mode boundary:

\[ I > \Delta i \quad \text{for CCM} \]

\[ I < \Delta i \quad \text{for DCM} \]

Previous CCM soln:

\[ I = \frac{V_s}{D^2 R} \quad \Delta i = \frac{V_s}{2L} DT \]

Boost DCM Boundary

\[ \frac{V_s}{D^2 R} > \frac{DT V_s}{2L} \quad \text{for CCM} \]

\[ \frac{2L}{RT_s} > DD^2 \quad \text{for CCM} \]

\[ K > K_{crit}(D) \quad \text{for CCM} \]

\[ K < K_{crit}(D) \quad \text{for DCM} \]

where \[ K = \frac{2L}{RT_s} \quad \text{and} \quad K_{crit}(D) = DD^2 \]
Boost Converter Subintervals

Fundamentals of Power Electronics 23 Chapter 5: Discontinuous conduction mode

Boost Conversion Ratio in DCM

\[ V_L(+) = V_g \]

\[ i_C(+) = -\frac{V}{R} \]

\[ V_L(+) = V_g - V \]

\[ i_C(-) = i_C(+) - \frac{V}{R} \]

\[ V_L(-) = \emptyset \]

\[ i_C(-) = -\frac{V}{R} \]
Boost Waveforms in DCM

\[ \langle v_L \rangle = \phi = (D_1 + D_2) V_s - D_2 V \]
\[ \frac{v}{V_s} = D_1 + \frac{1}{D_1} \langle i_d(t) \rangle D_2 T_s \]
\[ \langle i_C \rangle = \phi = -\frac{V}{R} + \frac{1}{D_1} \langle i_d(t) \rangle D_2 T_s \]
\[ \langle i_L \rangle = \phi = -\frac{V}{R} + \langle i_d \rangle \]
\[ \frac{v}{R} = \frac{1}{D_1 T_s} \left[ \frac{1}{2} D_2 T_s \langle i_d \rangle \right] \]
\[ \frac{v}{R} = \frac{1}{D_1 T_s} \left[ \frac{1}{2} D_2 T_s \frac{V}{L} \frac{V}{L} D_1 T_s \right] \]

Boost DCM Conversion Ratio

\[ V^2 - VV_s - \frac{V^2 D^2_1}{K} = 0 \]

Use quadratic formula:

\[ \frac{V}{V_s} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2} \]

Note that one root leads to positive \( V \), while other leads to negative \( V \). Select positive root:

\[ \frac{V}{V_s} = M(D_1, K) = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2} \]

where

\[ K = \frac{2L}{RT_s} \]

valid for

\[ K < K_{\text{red}}(D) \]

Transistor duty cycle \( D = \text{interval} \) duty cycle \( D_1 \)

Fundamentals of Power Electronics

Chapter 5: Discontinuous conduction mode
Boost Conversation Ratio

Approximate $M$ in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

Summary of DCM Characteristics

<table>
<thead>
<tr>
<th>Converter</th>
<th>$K_{ccm}(D)$</th>
<th>DCM $M(D,K)$</th>
<th>DCM $D(D,K)$</th>
<th>CCM $M(D)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>$(1 - D)$</td>
<td>$\frac{2}{1 + \sqrt{1 + 4K / D^2}}$</td>
<td>$\frac{K}{D} M(D,K)$</td>
<td>$D$</td>
</tr>
<tr>
<td>Boost</td>
<td>$D(1 - D)^2$</td>
<td>$\frac{1}{2} + \sqrt{1 + 4D^2 / K}$</td>
<td>$\frac{K}{D} M(D,K)$</td>
<td>$\frac{1}{1 - D}$</td>
</tr>
<tr>
<td>Buck-boost</td>
<td>$(1 - D)^2$</td>
<td>$-\frac{D}{\sqrt{K}}$</td>
<td>$\sqrt{K}$</td>
<td>$- \frac{D}{1 - D}$</td>
</tr>
</tbody>
</table>

with $K = 2L / RT$, DCM occurs for $K < K_{ccm}$.
Chapter 5 Summary

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity.

2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.

3. The dc conversion ratio $M$ of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.

4. Extra care is required when applying the small-ripple approximation. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more inductor currents, may have large ripple that cannot be ignored.

5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load-dependent, resulting in an increase in the converter output impedance.