Finding the Conversion Ratio M(D,K)

Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance

$$\langle v_L \rangle = \frac{1}{T_s}\int_0^{T_s} v_L(t)\, dt = 0$$

Capacitor charge balance

$$\langle i_C \rangle = \frac{1}{T_s}\int_0^{T_s} i_C(t)\, dt = 0$$

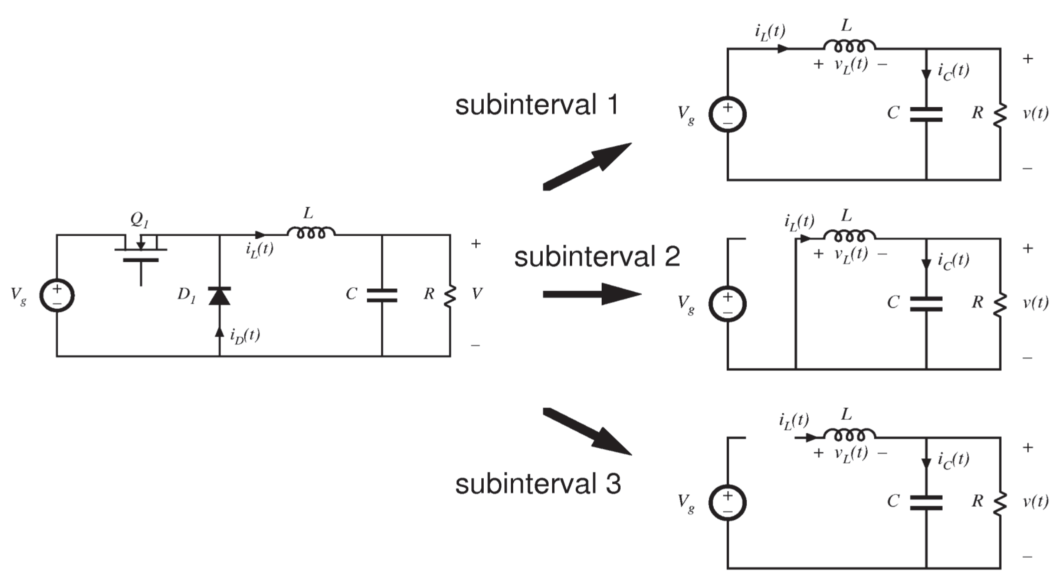
Small ripple approximation sometimes applies:

$$v(t) \approx V \quad \text{because} \quad \Delta v << V$$

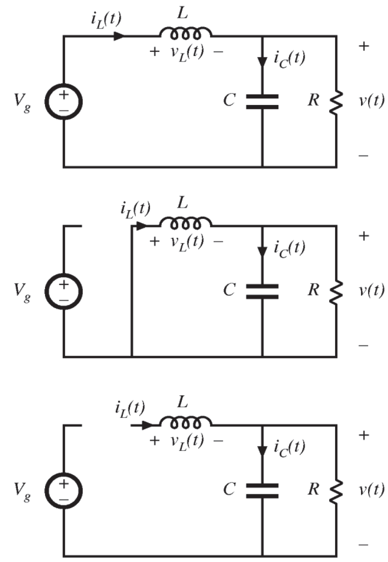
$$i(t) \approx I \;\text{ is a poor approximation when }\; \Delta i > I$$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

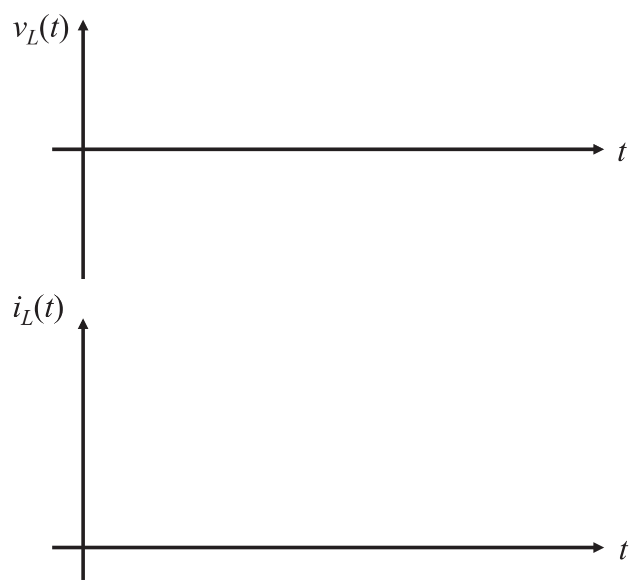
Buck Converter in DCM



Subinterval Analysis



Waveforms in DCM



Solving $M(D,K)$

Two equations and two unknowns (V and D_2):

$$V = V_g \frac{D_1}{D_1 + D_2} \qquad \text{(from inductor volt-second balance)}$$

$$\frac{V}{R} = \frac{D_1 T_s}{2L}(D_1 + D_2)(V_g - V) \qquad \text{(from capacitor charge balance)}$$

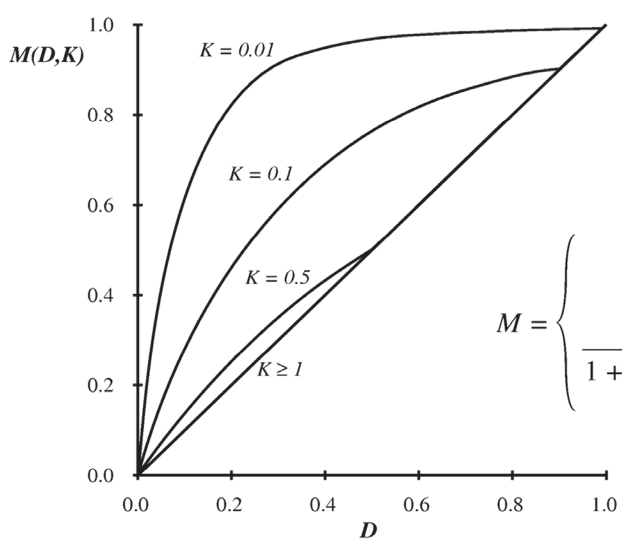
Eliminate D_2 , solve for V :

$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = 2L / RT_s$

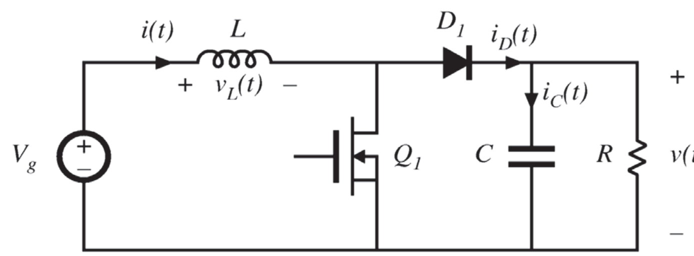
valid for $K < K_{crit}$

Buck Converter $M(D,K)$



$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit} \end{cases}$$

Boost Converter in DCM



Mode boundary:

$$I > \Delta i_L \quad for\ CCM$$
$$I < \Delta i_L \quad for\ DCM$$

Previous CCM soln:

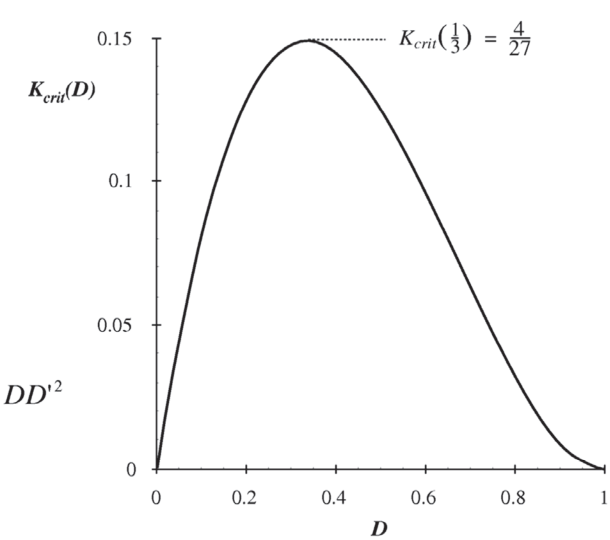
$$I = \frac{V_g}{D'^2 R} \qquad \Delta i_L = \frac{V_g}{2L} DT_s$$

Boost DCM Boundary

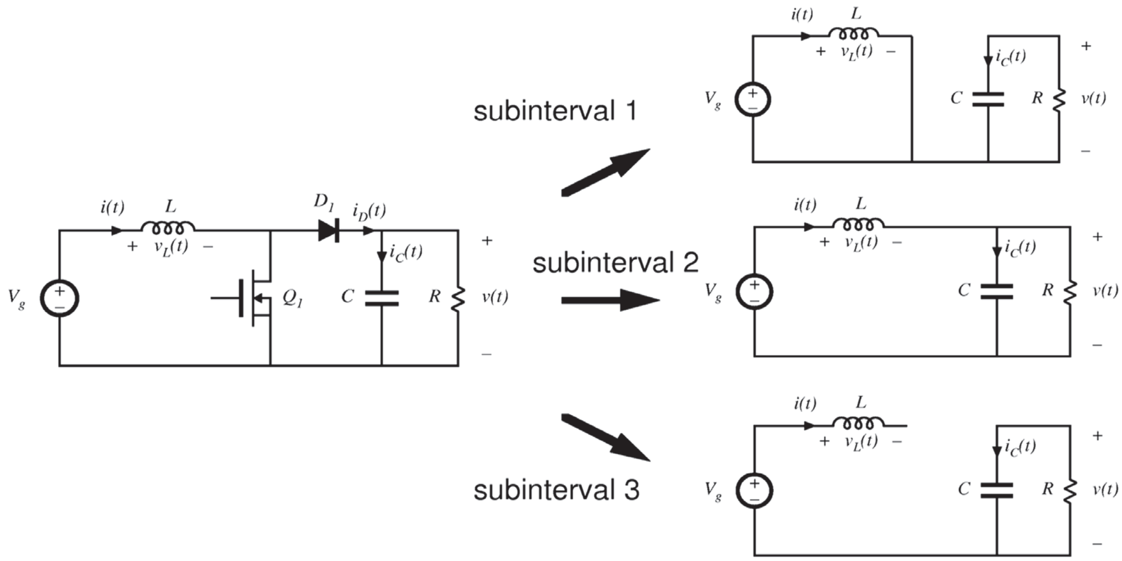
$$\frac{V_g}{D'^2 R} > \frac{DT_s V_g}{2L} \qquad for\ CCM$$

$$\frac{2L}{RT_s} > DD'^2 \qquad for\ CCM$$

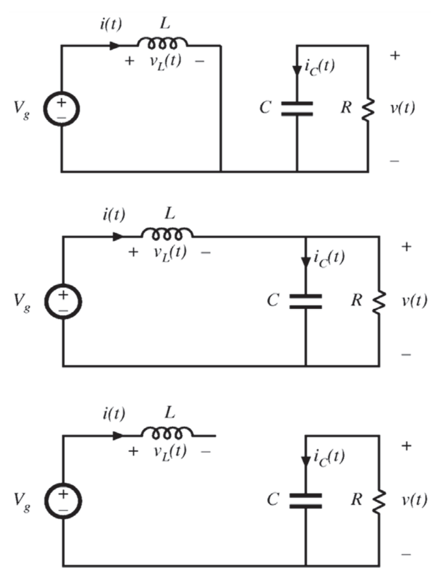
$$K > K_{crit}(D) \qquad for\ CCM$$
$$K < K_{crit}(D) \qquad for\ DCM$$
$$where \quad K = \frac{2L}{RT_s} \quad and \quad K_{crit}(D) = DD'^2$$



Boost Converter Subintervals



Boost Conversion Ratio



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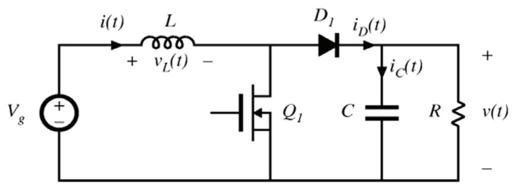
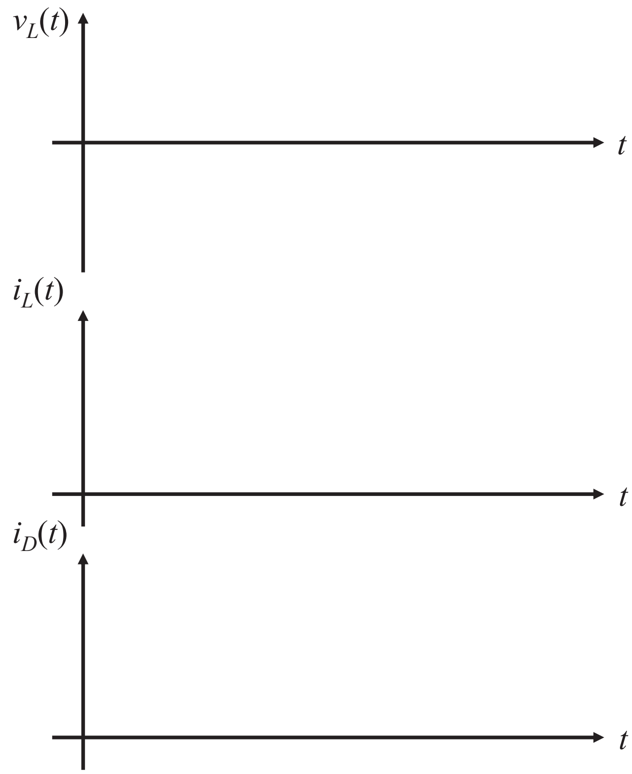
Boost Waveforms in DCM





Boost DCM Conversion Ratio

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

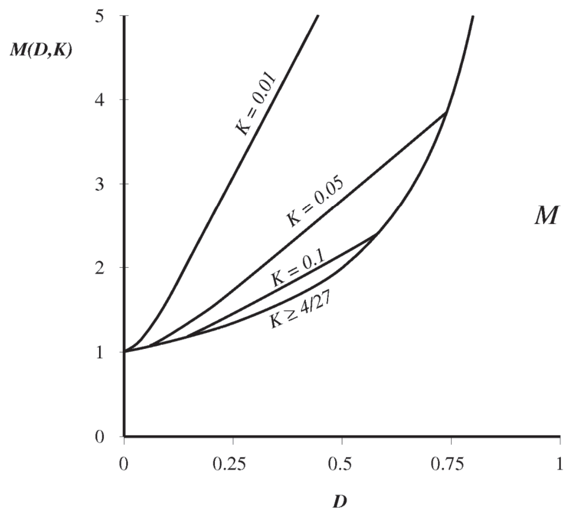
Note that one root leads to positive V, while other leads to negative V. Select positive root:

$$\frac{V}{V_g} = M(D_1,K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where $K = 2L / RT_s$
valid for $K < K_{crit}(D)$

Transistor duty cycle D = interval *1* duty cycle D_1

Boost Conversation Ratio



$$M = \begin{cases} \frac{1}{1-D} & \text{for } K > K_{crit} \\ \frac{1+\sqrt{1+4D^2/K}}{2} & \text{for } K < K_{crit} \end{cases}$$

Approximate M in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

Summary of DCM Characteristics

Table 5.2. Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	DCM $M(D,K)$	DCM $D_2(D,K)$	CCM $M(D)$
Buck	$(1-D)$	$\frac{2}{1+\sqrt{1+4K/D^2}}$	$\frac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\frac{1+\sqrt{1+4D^2/K}}{2}$	$\frac{K}{D}M(D,K)$	$\frac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\frac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\frac{D}{1-D}$

with $K = 2L/RT_s$. DCM occurs for $K < K_{crit}$.

Chapter 5 Summary

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity.
2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.
3. The dc conversion ratio M of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.
4. Extra care is required when applying the small-ripple approximation. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more inductor currents, may have large ripple that cannot be ignored.
5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load-dependent, resulting in an increase in the converter output impedance.