Chapter 2: Converters in Equilibrium

Buck Converter Review

![Buck Converter Circuit Diagram]
Three Basic DC-DC PWM Converters

Buck

Boost

Buck-boost

Chapter 2: Goals

- Develop techniques for easily determining output voltage of an arbitrary converter circuit
- Derive the principles of inductor volt-second balance and capacitor charge (amp-second) balance
- Introduce the key small ripple approximation
- Develop simple methods for selecting filter element values
- Illustrate via examples
Buck Output Voltage Ripple

Actual output voltage waveform, buck converter

Buck converter containing practical low-pass filter

Actual output voltage waveform

\[ v(t) = V + v_{\text{ripple}}(t) \]

In a well-designed converter, the output voltage ripple is small. Hence, the waveforms can be easily determined by ignoring the ripple:

\[ v_{\text{ripple}} \ll V \]

\[ v(t) \approx V \]
Buck Switching Intervals: Inductor Current

Subinterval 1
Subinterval 2

Current Waveform
Transient vs. Steady-State Operation

Volt-Second Balance
Derivation of Volt-second Balance

Inductor defining relation:

\[ v_L(t) = L \frac{di_L(t)}{dt} \]

Integrate over one complete switching period:

\[ i_L(T_s) - i_L(0) = \frac{1}{T_s} \int_0^{T_s} v_L(t) \, dt \]

In periodic steady state, the net change in inductor current is zero:

\[ 0 = \int_0^{T_s} v_L(t) \, dt \]

Hence, the total area (or volt seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

An equivalent form:

\[ 0 = \frac{1}{T_s} \int_0^{T_s} v_L(t) \, dt = \langle v_L \rangle \]

The average inductor voltage is zero in steady state.
Current Ripple Magnitude

\[ i_L(t) \]

\[ i_L(0) \]

\[ i_L(DT_s) \]

\[ \Delta i_L \]

\[ 0 \]

\[ DT_s \]

\[ T_s \]

\[ t \]

\[ (\text{change in } i_L) = (\text{slope})(\text{length of subinterval}) \]

Capacitor Charge Balance

\[ V_s \]

\[ i_s(t) \]

\[ L \]

\[ v_L(t) \]

\[ i_c(t) \]

\[ C \]

\[ R \]

\[ v(t) \]

\[ v_L(t) \]

\[ v(t) \]

\[ i_L(t) \]

\[ v(t) \]

\[ v(t) \]
Derivation of Capacitor Charge Balance

Capacitor defining relation:

\[ i_c(t) = C \frac{d v_c(t)}{dt} \]

Integrate over one complete switching period:

\[ v_c(T_s) - v_c(0) = \frac{1}{C} \int_0^{T_s} i_c(t) \, dt \]

In periodic steady state, the net change in capacitor voltage is zero:

\[ 0 = \frac{1}{T_s} \int_0^{T_s} i_c(t) \, dt = \langle i_c \rangle \]

Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.