Effect of Feedback

Feedback moves the poles of the system transfer functions

- Good news: we can use feedback to alter the poles and improve the frequency response
- Bad news: if you’re not careful, feedback can move the poles into the right half of the complex s-plane (poles have positive real parts), leading to an unstable system

Open loop

\[ G(s) \]

Closed loop

\[
\begin{align*}
\hat{v}_d(s) & \rightarrow G(s) & \hat{v}_o(s) \\
& \downarrow & \downarrow \\
& H(s) & T(s) \\
\hat{v}_m(s) & \rightarrow & \end{align*}
\]

Determining Stability From \( T(s) \)

- Nyquist stability theorem: general result.
- A special case of the Nyquist stability theorem: the phase margin test
  
  Allows determination of closed-loop stability (i.e., whether \( 1/(1 + T(s)) \) contains RHP poles) directly from the magnitude and phase of \( T(s) \).
  
  A good design tool: yields insight into how \( T(s) \) should be shaped, to obtain good performance in transfer functions containing \( 1/(1 + T(s)) \) terms.
9.4.1 – The Phase Margin Test

A test on $T(s)$, to determine whether $1/(1+T(s))$ contains RHP poles.

The crossover frequency $f_c$ is defined as the frequency where

$$\| T(j2\pi f_c) \| = 1 \Rightarrow 0 \text{dB}$$

The phase margin $\varphi_m$ is determined from the phase of $T(s)$ at $f_c$, as follows:

$$\varphi_m = 180^\circ + \angle T(j2\pi f_c)$$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin $\varphi_m$ is positive.

Example: Unstable System

\[ \angle T(j2\pi f_c) = -230^\circ \]

\[ \varphi_m = 180^\circ - 230^\circ = -50^\circ \]
Example: Stable System

![Graph showing phase margin](image)

\[ \angle T(j2\pi f_c) = -112^\circ \]

\[ \phi_m = 180^\circ - 112^\circ = +68^\circ \]

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Selecting Phase Margin

How much phase margin is required?

A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high \( Q \). The transient response exhibits overshoot and ringing.

Increasing the phase margin reduces the \( Q \). Obtaining real poles, with no overshoot and ringing, requires a large phase margin.

The relation between phase margin and closed-loop \( Q \) is quantified in this section.

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Step Response of Second-Order System

The transfer function of a Second-Order System is given by:

\[ T(s) = \frac{1000}{\left(\frac{s}{\omega_1}\right)\left(\frac{s}{\omega_2} + 1\right)} \]

A Second-Order System

Consider the case where \( T(s) \) can be well-approximated in the vicinity of the crossover frequency as:

\[ T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)} \]
Closed-Loop Response

If
\[
T(s) = \frac{1}{\frac{s}{\omega_0}\left(1 + \frac{s}{\omega_2}\right)}
\]

Then
\[
\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{s}{T(s)}} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0\omega_2}}
\]
or,
\[
\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{s}{Q\omega_c} + \left(\frac{s}{\omega_c}\right)^2}
\]

where
\[
\omega_c = \sqrt{\omega_0\omega_2} = 2\pi f_c \\
Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}
\]

Closed-Loop Step Response vs. $Q_{CL}$
Low-$Q_{CL}$ Case

\[ Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}} \]

low-$Q$ approximation: \[ Q \omega_c = \omega_0 \quad \frac{\omega_c}{Q} = \omega_2 \]

High-$Q_{CL}$ Case

\[ \omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \]

\[ Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}} \]
\( Q_{CL} \text{ vs. } \varphi_m \)

Solve for exact crossover frequency, evaluate phase margin, express as function of \( \varphi_m \). Result is:

\[
Q = \frac{\sqrt{\cos \varphi_m}}{\sin \varphi_m}
\]

\[
\varphi_m = \tan^{-1} \sqrt{\frac{1 + \sqrt{1 + 4Q^4}}{2Q^4}}
\]
9.5 – Compensator Design

Typical specifications:

- Effect of load current variations on output voltage regulation
  This is a limit on the maximum allowable output impedance
- Effect of input voltage variations on the output voltage regulation
  This limits the maximum allowable line-to-output transfer function
- Transient response time
  This requires a sufficiently high crossover frequency
- Overshoot and ringing
  An adequate phase margin must be obtained

The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.
Design Approach

• Assume $G_c(s) = 1$, and plot the resulting uncompensated loop gain $T_u(s)$
• Examine uncompensated loop gain to determine the needs of the compensator
  - Is low-frequency loop gain amplitude $\|T(0)\|$ large enough to result in low steady-state error?
  - Is $\phi_m$ sufficient for stability and requirements on ringing/overshoot?
  - Is $f_c$ high enough for a sufficiently fast response?
• Construct compensator to address shortcomings of $T_u(s)$
  - Use “toolbox” of compensators on following slides

Example: Uncompensated Loop Gain
Proportional (P) Compensator

\[ G_c(s) = G_{c0} \]

Stabilization by (P) Compensator

\[ \| T_o \| \]

\[ \angle T_o \]
Another Example

![Graph showing frequency response](graph.png)