Example Lag Compensator Design

Original (uncompensated) loop gain is

\[ T_c(s) = \frac{T_{d0}}{1 + \frac{s}{\omega_c}} \]

Compensator:

\[ G_c(s) = G_{c0} \left( 1 + \frac{\omega_c}{s} \right) \]

Design strategy:

Choose \( G_{c0} \) to obtain desired crossover frequency \( \omega_c \) sufficiently low to maintain adequate phase margin

Lead (PD) Compensator

\[ G_c(s) = G_{c0} \left( 1 + \frac{s}{\omega_z} \right) \quad \left( 1 + \frac{s}{\omega_p} \right) \]

Improves phase margin \( \omega_z \approx \omega_p \)
Maximum Phase Lead

\[ f_{\text{max}} = \sqrt{f_p f_z} \]

\[ \angle G_c(f_{\text{max}}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2} \right) \]

\[ f_p = \frac{1 + \sin(\theta)}{1 - \sin(\theta)} \]

Lead Compensator Design

To optimally obtain a compensator phase lead of \( \theta \) at frequency \( f_c \), the pole and zero frequencies should be chosen as follows:

\[ f_z = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}} \]

\[ f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}} \]

If it is desired that the magnitude of the compensator gain at \( f_c \) be unity, then \( G_{e0} \) should be chosen as

\[ G_{e0} = \sqrt{\frac{f_z}{f_p}} \]
Example Lead Compensator Design

Combined (PID) Compensator
Example Design of Buck Compensator

DC (Quiescent) Operating Point

- Input voltage: $V_e = 28V$
- Output: $V = 15V$, $I_{load} = 5A$, $R = 3\Omega$
- Quiescent duty cycle: $D = 15/28 = 0.536$
- Reference voltage: $V_{ref} = 5V$
- Quiescent value of control voltage: $V_c = DV_{ref} = 2.14V$
- Gain $H(s)$: $H = V_{ref}/V = 5/15 = 1/3$
AC Power Stage Model

Table 7.1

System Block Diagram

\[
\hat{u} = \hat{u}_\text{ref} \frac{1}{1 + \frac{1}{sT}} + \hat{u}_g \text{sgn} \frac{1}{1 + \frac{1}{sT}} - \hat{i}_\text{load} \frac{1}{1 + \frac{1}{sT}}
\]
Plotting Uncompensated Loop Gain

1. Low frequency gain too low → ringing, overshoot → PD compensator
2. High frequency gain too low → significant SS error → PI
3. $f_c \approx \omega_L$ → can improve bandwidth by increasing $\omega_L$

With $G_c = 1$, the loop gain is

$$T_u(s) = \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{w0} = \frac{HV}{DV_m} = 2.33 \Rightarrow 7.4\text{dB}$$

$$f_c = 1.8\text{ kHz}, \phi_m = 5^\circ$$

LTSpice Simulation – AC, Uncompensated
Transient Simulation, Uncompensated

\[ V = \frac{V_{ref}}{1 + \frac{I_{T0}}{I_{T1}}} = (15\,\text{V}) \left( \frac{2.5}{1 + 2.5} \right) \approx 10.5\,\text{V} \]
Summary: Uncompensated Behavior

- Significant steady-state error
  - Need to increase low-frequency gain
- Barely stable; significant ringing
  - Need to increase $\phi_m$
- Speed: ok
  - $f_c = 1.8$ kHz
  - $(BW)_{CL} = 2.6$ kHz
  - OK for $f_s \approx 10$ kHz or above
Compensator Design

• As an example, try to
  - Increase $f_c$ to 10 kHz
  - Increase $\phi_m$ to 76° (Q_{CL}=0.5)
  - Increase $\|T_0\|$ to $\infty$

• Note: Book Chooses $f_c = 5$ kHz and $\phi_m = 52^\circ$ (Q=0.5)

PI Design

$\|T_u\|$ $\|T_d\|$ $T_u$ $T_d$ 2.33 $\Rightarrow$ 7.4 dB

$Q_m = 9.5$ $\Rightarrow$ 19.5 dB

$\angle T_u$ $\angle T_d$

$10^\frac{1}{2} f_c = 1.1$ kHz

Select $f_1i = 50$ Hz

$\|G_{cd}\|_{dB} = 40$ dB $\Rightarrow$ 7.4 dB $\Rightarrow 2.6 B = 42.6$

PI compensator $G_{c,PC} = 42.6 \left(1 + \frac{u}{5}\right)$
**PI Simulation**

- Large LF gain
- $f_L = f_c$
- $f_p$
- $f_c = 10 \text{ kHz}$

**PD Design**

\[
\begin{align*}
    f_c &= f_c \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} \\
    f_p &= f_c \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} \\
    G_{po} &= \sqrt{\frac{f_c}{f_p}}
    \end{align*}
\]

- $\phi_m = 0^\circ$
- $\phi_m = \theta + \phi_{m_{-\text{raw}}} = 76^\circ$
- $\theta = 76^\circ$
- $f_c = 10 \text{ kHz}$
- $f_L = 1.23 \text{ kHz}$
- $f_p = 81 \text{ kHz}$
- $G_{leo} = 0.12$

\[
G_{pd} = G_{po} \left( \frac{1 + \frac{f_p}{f_c}}{1 + \frac{f_c}{f_p}} \right)
\]
$T/(1+T)$
Transient Simulation

Graph showing transient behavior with parameters and circuit diagram.

\[ \text{param } V_g = 20 \quad R = 15 \quad B = 0.336 \]
\[ \text{param } V_{ref} = 3 \quad R = 1/2 \quad V_{os} = 4 \]
\[ \text{param } l = 50\mu \quad C = 300\mu \]

\[ \text{V(out)} = 15 \text{ R(1)} - 5 \text{ V(os)} - (0^* \text{Vn}) \]