Lead (PD) Compensator

\[ G_c(s) = G_{c0} \left( \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right) \]

Improves phase margin

Maximum Phase Lead

\[ f_{\text{ymax}} = \sqrt{f_p f_z} \]

\[ \angle G_c(f_{\text{ymax}}) = \tan^{-1} \left( \frac{\sqrt{f_p}}{f_z} - \sqrt{\frac{f_p}{f_z}} \right) \]

\[ f_p = \frac{1 + \sin (\theta)}{1 - \sin (\theta)} \cdot f_z \]
Lead Compensator Design

To optimally obtain a compensator phase lead of \( \theta \) at frequency \( f_c \), the pole and zero frequencies should be chosen as follows:

\[
\begin{align*}
    f_c &= f_c \frac{1 - \sin \theta}{1 + \sin \theta} \\
    f_p &= f_c \frac{1 + \sin \theta}{1 - \sin \theta}
\end{align*}
\]

If it is desired that the magnitude of the compensator gain at \( f_c \) be unity, then \( G_{c0} \) should be chosen as

\[
G_{c0} = \sqrt{\frac{f_c}{f_p}}
\]

Example Lead Compensator Design
Combined (PID) Compensator

Example Design of Buck Compensator
## DC (Quiescent) Operating Point

- **Input voltage** \( V_i = 28 \text{V} \)
- **Output** \( V = 15 \text{V}, I_{\text{load}} = 5 \text{A}, R = 3 \Omega \)
- **Quiescent duty cycle** \( D = 15/28 = 0.536 \)
- **Reference voltage** \( V_{\text{ref}} = 5 \text{V} \)
- **Quiescent value of control voltage** \( V_c = DV_{M} = 2.14 \text{V} \)
- **Gain** \( H(s) \)
  \[ H = V_{\text{ref}}/V = 5/15 = 1/3 \]

## AC Power Stage Model

![AC Power Stage Model Diagram](image)

- **Error signal**: \( \hat{e}_r \)
- **Compensator**: \( G_c(s) \)
- **Gain**: \( H(s) \)
  \[ H = \frac{1}{3} \]

*Fundamentals of Power Electronics* 47  
Chapter 9: Controller design
System Block Diagram

\[ T(s) = G_c(s) \left( \frac{1}{V_M} \right) G_{vd}(s) H(s) \]
\[ T(s) = \frac{G_c(s) H(s)}{V_M} \frac{V}{D} \frac{1}{1 + \frac{s}{Q_c \omega_0} + \left( \frac{s}{\omega_0} \right)^2} \]

\[ \hat{v}_{\text{ref}} (= 0) \rightarrow \hat{v}_c(s) \rightarrow G_c(s) \rightarrow \frac{V_M = 4 \text{ V}}{1} \rightarrow \frac{d\hat{v}}{\text{s}}(s) \rightarrow G_{vd}(s) \rightarrow H(s) \rightarrow \frac{1}{3} \rightarrow \hat{v}(s) \]

Plotting Uncompensated Loop Gain

With \( G_c = 1 \), the loop gain is
\[ T_s(s) = T_{sd} \frac{1}{1 + \frac{s}{Q_d \omega_0} + \left( \frac{s}{\omega_0} \right)^2} \]
\[ T_{sd} = \frac{H V}{D V_M} = 2.33 \Rightarrow 7.4 \text{ dB} \]

\[ f_c = 1.8 \text{ kHz}, \quad Q_{sd} = 5^\pi \]
LTSpice Simulation – AC, Uncompensated

Transient Simulation, Uncompensated
Ringing Frequency

\[ T/(1+T) \]
Summary: Uncompensated Behavior

• Significant steady-state error
  - Need to increase low-frequency gain

• Barely stable; significant ringing
  - Need to increase $\phi_m$

• Speed: ok
  - $f_c = 1.8$ kHz
  - $(BW)_{CL} = 2.6$ kHz
  - OK for $f_s \approx 10$ kHz or above

Compensator Design

• As an example, try to
  - Increase $f_c$ to 10 kHz
  - Increase $\phi_m$ to 76° (Q=0.5)
  - Increase $\|T_0\|$ to $\infty$

• Note: Book Chooses $f_c = 5$ kHz and $\phi_m = 52°$ (Q=0.5)
PI Design

PI Simulation
PD Design

\[ f_c = f_r \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}} \]

\[ f_p = f_r \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}} \]

\[ G_{\alpha} = \sqrt{\frac{f_c}{f_p}} \]

PID Simulation

---

PID Simulation Diagram

\[
\text{Gain Function: } G(s) = \frac{K_p}{s^2 + \frac{1}{T_i}s + T_d}
\]

\[
\text{Transfer Function: } F(s) = V(s) - V_r(s)
\]

\[
\text{Frequency Response: } |V_r(j\omega)|, \arg(V_r(j\omega))
\]

\[
\text{Phase Response: } \angle(V_r(j\omega))
\]

---

Parameter List:

\[
V_p = 28 \quad V = 15 \quad R_3 = 3 \quad D = 0.536
\]

\[
\text{Gain Value: } K_p = 1 / (1 / 2 \times 1) = 2
\]

\[
\text{Time Constants: } T_i = 1 \times 10^3 \quad T_d = 1 / (6.28 \times 1.23) R_1 = 0.5
\]
$$\frac{T}{(1+T)}$$

Transient Simulation

[Diagram of transient simulation showing Bode plots and circuit diagram]