Inductor Example

Simplifying assumptions:
1. Magnetic flux stays entirely within the core
2. H & B fields are uniform throughout the core
3. B = \mu H in the core (when not saturated)

Faraday:
\[ v(t) = nA_c \frac{dB(t)}{dt} \]

\[ V_{in} = A_c \frac{dB(t)}{dt} \]

Amper's:
\[ H \cdot l_m = \int_{Enclosed} B \cdot dl \]
\[ \int_{Enclosed} B \cdot dl = n \cdot i(t) \]

\[ V(t) = nA_c \frac{di(t)}{dt} \]

Core material:
if \[ B = \mu H \] (and \( B_l \) << \( B_{Sat} \))

Magnetic Circuits

Based on conservative fields (\( \vec{E}/\vec{F} \))
Potential independent of path

KVL

Electric \[ V \rightarrow \vec{F} \]
Magnetic \[ \phi \rightarrow I \]

\[ \Phi \leftrightarrow I \]

Always flow in closed loop
No single point sourcing

KCL
Inductor Magnetic Circuit Model

\[ \Phi = \frac{n \Phi_0}{R_c} = \frac{n i(t)}{L_m A_c} \]

\[ \Phi = \frac{n M A_e}{L_m} i(t) \]

\[ n(L) = n \frac{\frac{d}{dt} \left( \frac{n M A_e}{L_m} i(t) \right)}{i(t)} \]

\[ n(L) = \frac{n^2 M A_e}{L_m} \frac{di}{dt} \]

Saturation Limits

Process: first assume inductor is not saturated, analyze using \( B = nH \) → afterwards check if \( |B(t)| < B_{sat} \)

Typical \( B_{sat} \) values:
- 0.2-0.8T for ferrite
- 1-2.2T for laminated iron
- Nanocrystalline

\[ L = \frac{n A_e}{L_m} \]

Saturation \( B \) at \( I \sim 1000 \)

So \( L \sim 1000 \)

\[ n(t) = n A_e \frac{dB}{dt} \]

\[ n(t) = L \frac{di}{dt} = \frac{n A_e}{L_m} \frac{dB}{dt} \]

\[ I_{sat} = \frac{1}{n \Phi} \int_0^t n(t) \, dt \]

\[ B_{sat} = \frac{L_f}{n A_e} \]

\[ I_{sat} = \frac{1}{n \Phi} \int_0^t n(t) \, dt \]

For uniaxially oriented

\( 1 \uparrow \Phi_0 \)

\( 2 \uparrow L_m \)

\( 3 \downarrow M \)

\( 4 \uparrow L \)
Example: Gapped Inductor

Additional assumption/simplification

(1) No fringing flux

Faraday:
\[ v(t) = n \frac{d\Phi}{dt} = n A_c \frac{dB}{dt} \]

Ampere:
\[ \oint_A \mathbf{H} \cdot d\mathbf{l} = \text{enclosed} = n i(t) = A_c (l_m - l_g) + H_g l_g \]

Material characteristics:

in core \( B = m \mu_0 H_c \) (unsaturated), in air \( B = \frac{B}{m_0} H_g \)

\[ v(t) = n A_c \frac{1}{m} \left( \frac{\frac{n i(t)}{m_0} l_m}{m + l_g} \right) \]

\[ n i(t) = \frac{B}{m} (l_m - l_g) + \frac{B}{m_0} l_g \]

\[ B(t) = \frac{n i(t)}{l_m} \]

\[ n i(t) = \frac{n^2 A_c}{l_m} \frac{di}{dt} \]