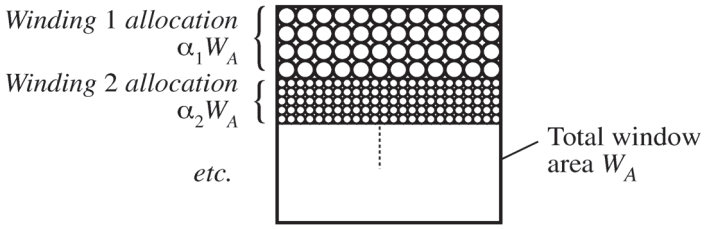
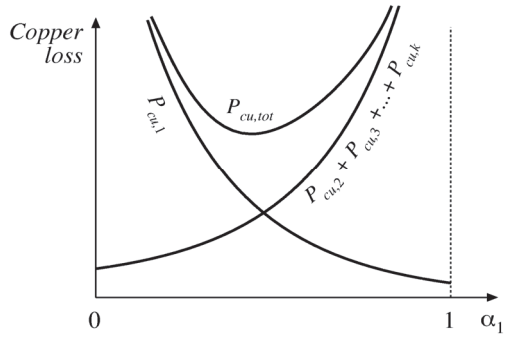
K_g Method: Multi-Winding Magnetics





$0 < \alpha_j < 1$

$\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1$

$$\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$$

Apparent power in winding j is

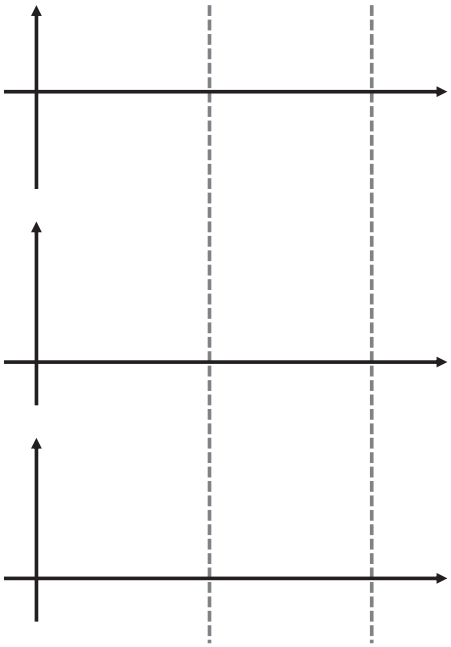
$V_j I_j$

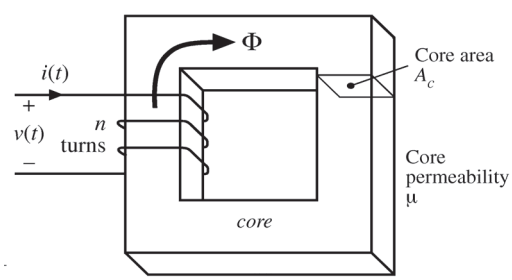
where V_j is the rms or peak applied voltage

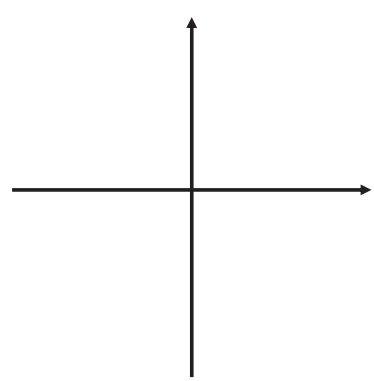
I_j is the rms current

Window area should be allocated according to the apparent powers of the windings

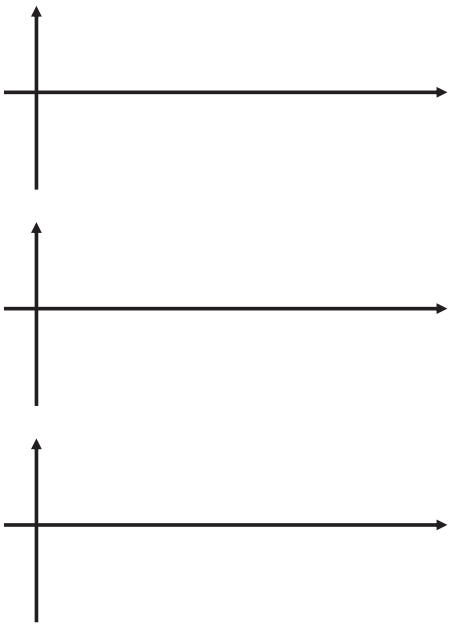
B-H Curve: Filter Inductor

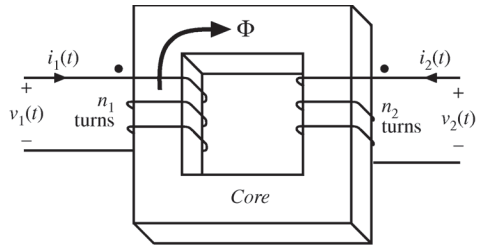


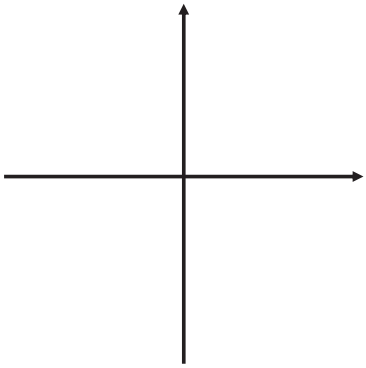




B-H Curve: Transformer

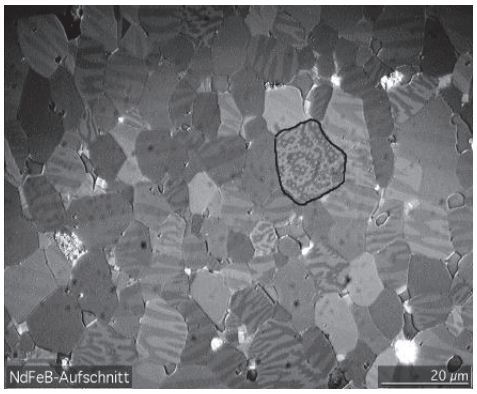




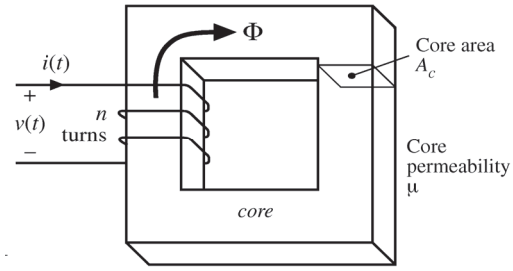


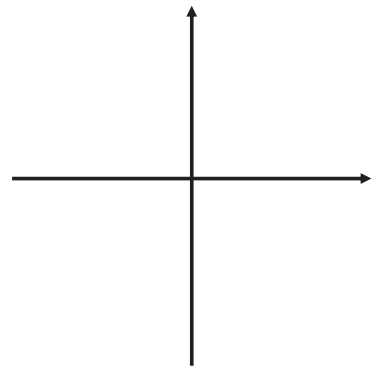
Core Loss

- Physical origin due to magnetic domains
- Modeling Approaches
  - Empirical (curve fit) models of materials
  - Direct measurement-based models
  - Physics-based models



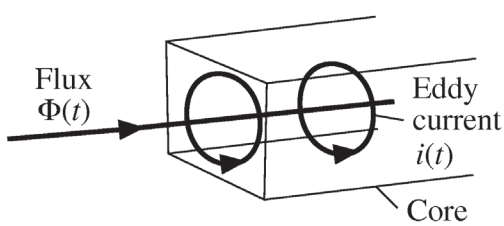
Hysteresis Loss





Eddy Currents in Magnetic Materials

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz's law, magnetic fields within the core induce currents ("eddy currents") to flow within the core. The eddy currents flow such that they tend to generate a flux which opposes changes in the core flux $\Phi(t)$. The eddy currents tend to prevent flux from penetrating the core.



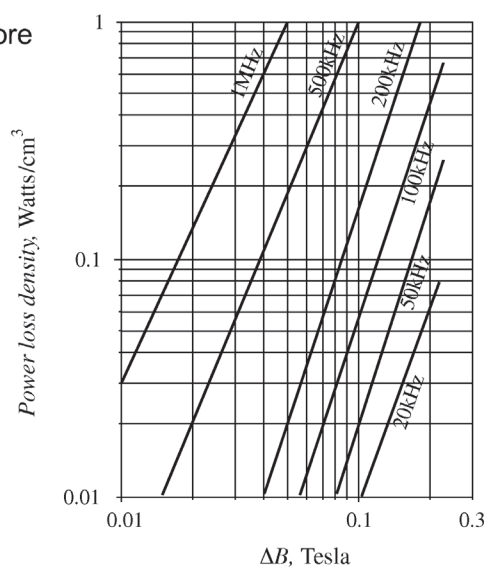
Eddy current loss $i^2(t)R$

Eddy Current Losses

- Ac flux $\Phi(t)$ induces voltage $v(t)$ in core, according to Faraday's law. Induced voltage is proportional to derivative of $\Phi(t)$. In consequence, magnitude of induced voltage is directly proportional to excitation frequency f.
- If core material impedance Z is purely resistive and independent of frequency, $Z = R$, then eddy current magnitude is proportional to voltage: $i(t) = v(t)/R$. Hence magnitude of $i(t)$ is directly proportional to excitation frequency f.
- Eddy current power loss $i^2(t)R$ then varies with square of excitation frequency f.
- Ferrite core material impedance is capacitive. This causes eddy current power loss to increase as f^4.

The Steinmetz Equation

Ferrite core material



Empirical equation, at a fixed frequency:

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

Alternately:

$$P_v = Kmf^{\alpha}(\Delta B)^{\beta}$$

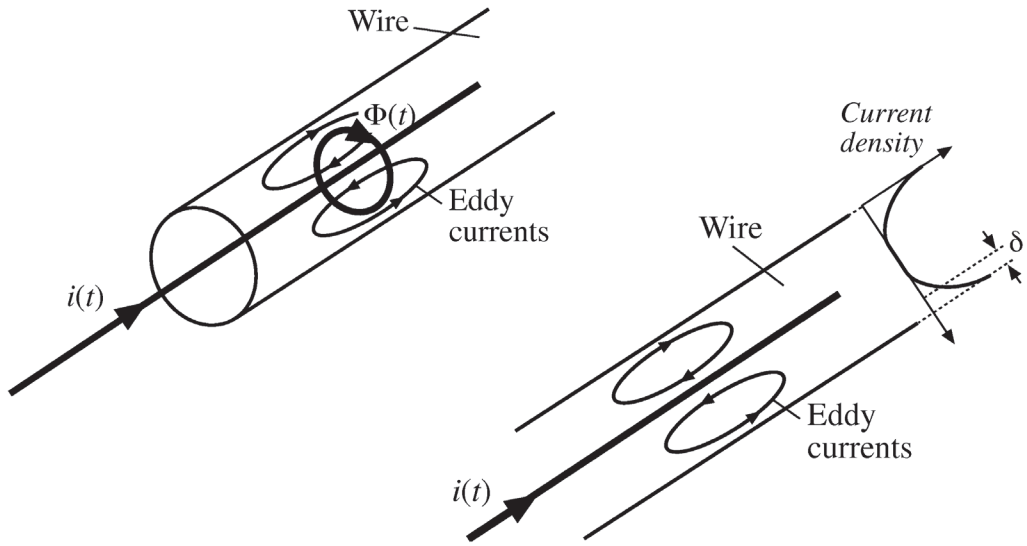
Steinmetz Equation: Notes

- Purely empirical; not physics-based
- Parameters α, β, K vary with frequency
- Correct only for sinusoidal excitation
  - Nonlinear; Fourier expansion of waveforms cannot be used
- Modified empirical equations perform better with nonsinusoidal waveforms
  - MSE
  - GSE
  - iGSE
  - i^2GSE

Some Example Core Materials

Core type	B_{sat}	Relative core loss	Applications
Laminations iron, silicon steel	1.5 - 2.0 T	high	50-60 Hz transformers, inductors
Powdered cores powdered iron, molypermalloy	0.6 - 0.8 T	medium	1 kHz transformers, 100 kHz filter inductors
Ferrite Manganese-zinc, Nickel-zinc	0.25 - 0.5 T	low	20 kHz - 1 MHz transformers, ac inductors

Skin Effect



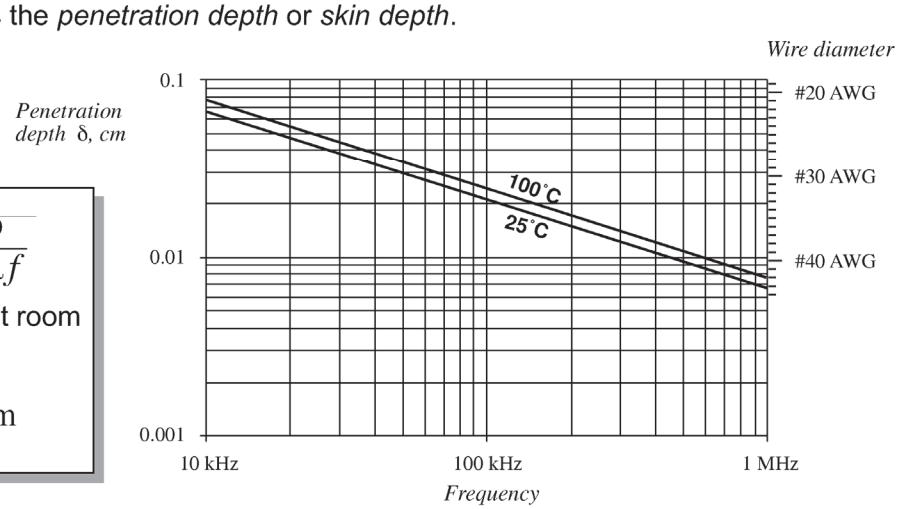
Skin Depth

For sinusoidal currents: current density is an exponentially decaying function of distance into the conductor, with characteristic length δ known as the *penetration depth* or *skin depth*.

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$

For copper at room temperature:

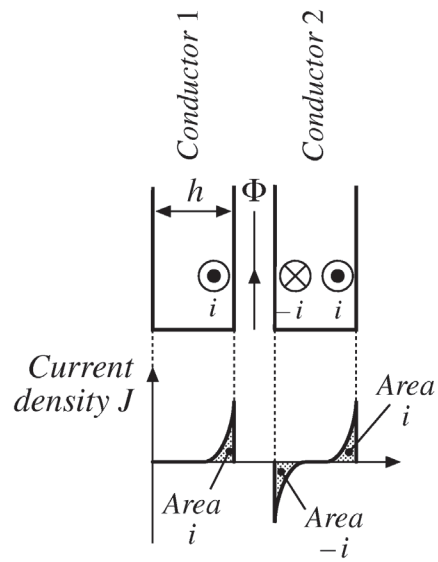
$$\delta = \frac{7.5}{\sqrt{f}} \text{ cm}$$



Proximity Effect

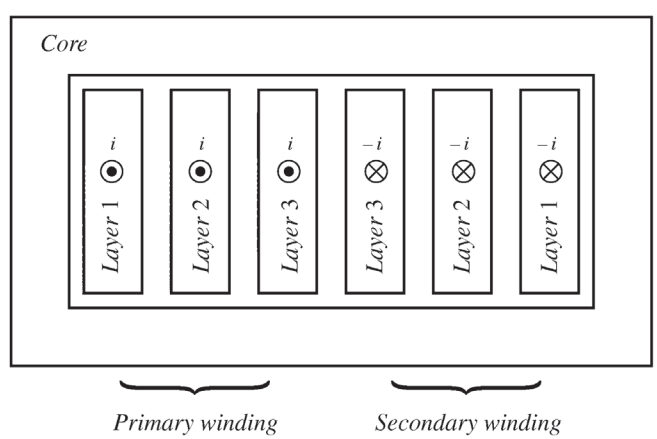
Ac current in a conductor induces eddy currents in adjacent conductors by a process called the *proximity effect*. This causes significant power loss in the windings of high-frequency transformers and ac inductors.

A multi-layer foil winding, with $h \gg \delta$. Each layer carries net current $i(t)$.



Two-Winding Transformer Example

Cross-sectional view of two-winding transformer example. Primary turns are wound in three layers. For this example, let's assume that each layer is one turn of a flat foil conductor. The secondary is a similar three-layer winding. Each layer carries net current $i(t)$. Portions of the windings that lie outside of the core window are not illustrated. Each layer has thickness $h \gg \delta$.



Current Distribution

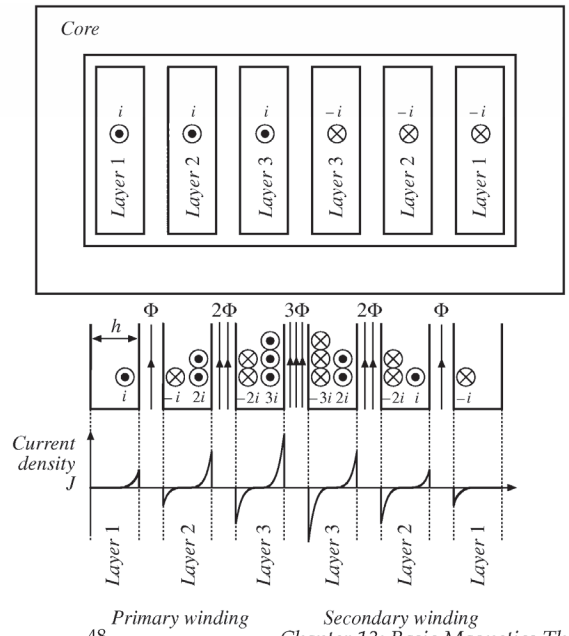
Skin effect causes currents to concentrate on surfaces of conductors

Surface current induces equal and opposite current on adjacent conductor

This induced current returns on opposite side of conductor

Net conductor current is equal to $i(t)$ for each layer, since layers are connected in series

Circulating currents within layers increase with the numbers of layers



High Frequency Estimation

The current $i(t)$ having rms value I is confined to thickness d on the surface of layer 1. Hence the effective "ac" resistance of layer 1 is:

$$R_{ac} = \frac{h}{\delta} R_{dc}$$

This induces copper loss P_1 in layer 1:

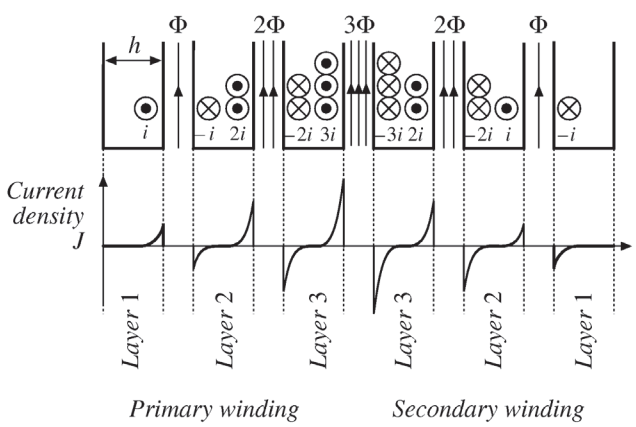
$$P_1 = I^2 R_{ac}$$

Power loss P_2 in layer 2 is:

$$P_2 = P_1 + 4P_1 = 5P_1$$

Power loss P_3 in layer 3 is:

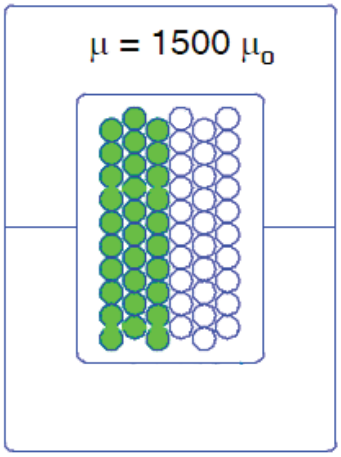
$$P_3 = \left(2^2 + 3^2\right) P_1 = 13P_1$$



Power loss P_m in layer m is:

$$P_m = I^2 \left[(m-1)^2 + m^2 \right] \left(\frac{h}{\delta} R_{dc} \right)$$

Simulation Example

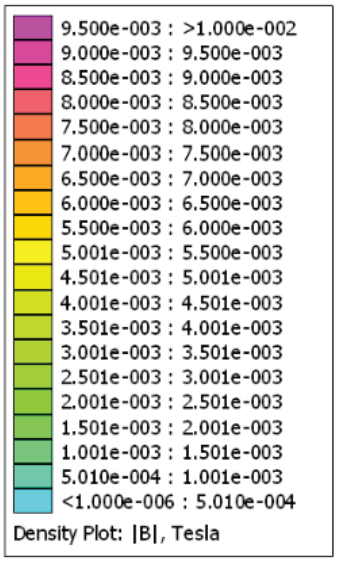


- AWG#30 copper wire
  - Diameter $d = 0.294$ mm
  - $d = \delta$ at around 50 kHz
- 1:1 transformer
  - Primary and secondary are the same, 30 turns in 3 layers
- Sinusoidal currents,

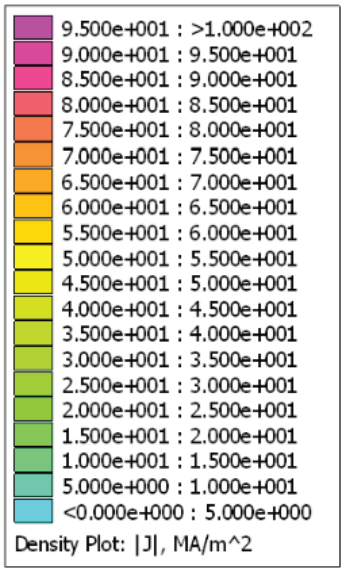
  $I_{1rms} = I_{2rms} = 1$ A

Numerical field and current density solutions using FEMM (Finite Element Method Magnetics), a free 2D solver, http://www.femm.info/wiki/HomePage

Flux density magnitude

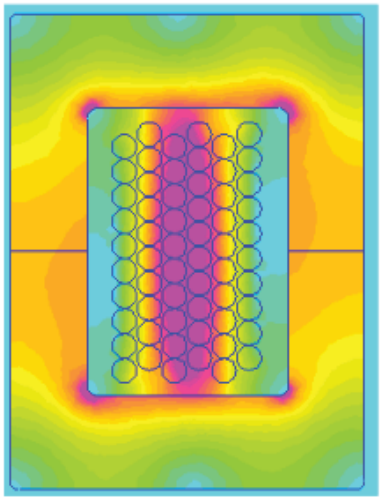


Current density magnitude

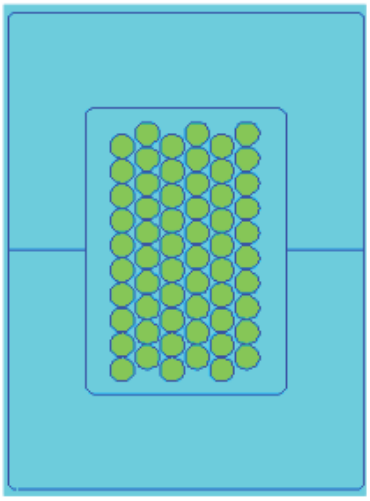


Frequency: 1 kHz

Flux density

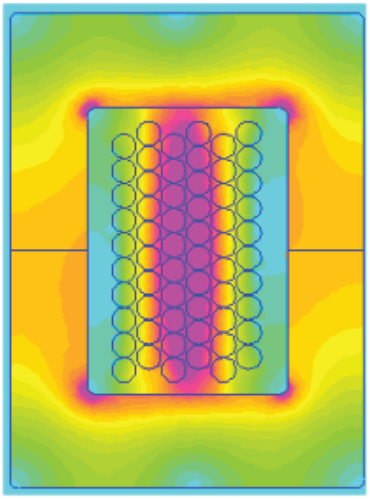


Current Density

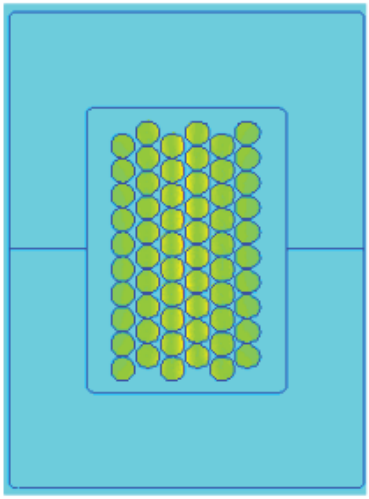


Frequency: 100 kHz

Flux density



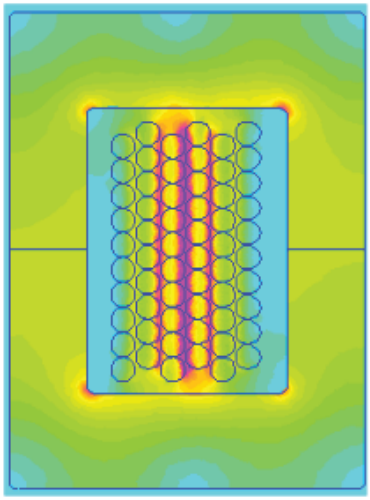
Current Density



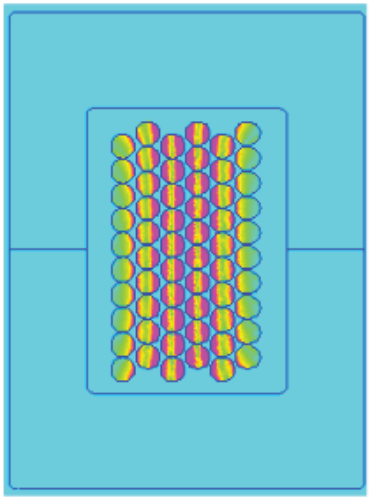
Total copper losses 1.8 larger than at 1 kHz

Frequency: 1 MHz

Flux density



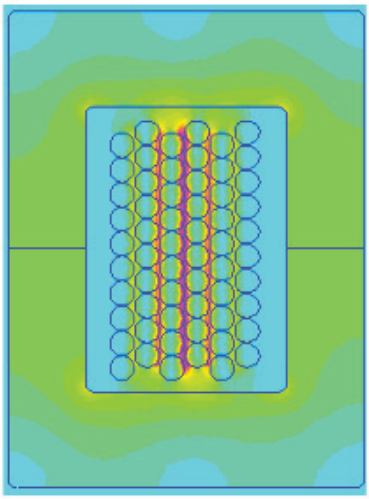
Current Density



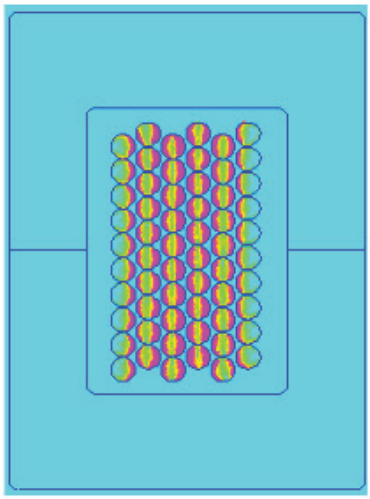
Total copper losses 20 times larger than at 1 kHz

Frequency: 10 MHz

Flux density

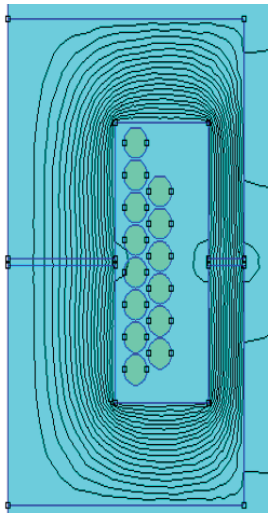


Current Density

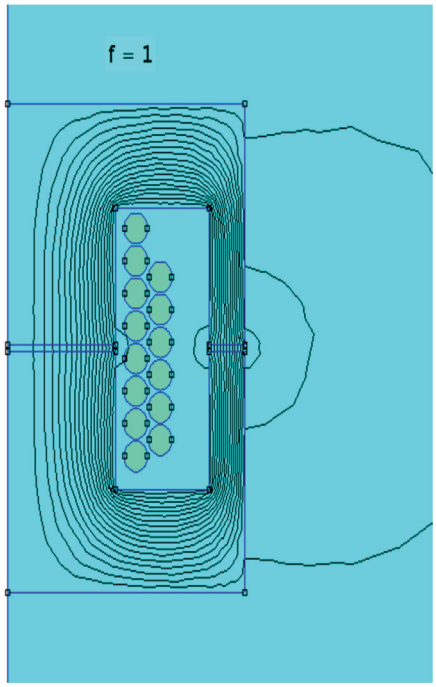


Very significant proximity effect
Total copper losses = 65 times larger than at 1 KHz

Fringing Flux



Fringing Flux Simulation



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Litz Wire

- A way to increase conductor area while maintaining low proximity losses
- Many strands of small-gauge wire are bundled together and are externally connected in parallel
- Strands are twisted, or transposed, so that each strand passes equally through each position on inside and outside of bundle. This prevents circulation of currents between strands.
- Strand diameter should be sufficiently smaller than skin depth
- The Litz wire bundle itself is composed of multiple layers
- Advantage: when properly sized, can significantly reduce proximity loss
- Disadvantage: increased cost and decreased amount of copper within core window

Chapter 15: Transformer Design

Transformer Design Constraints

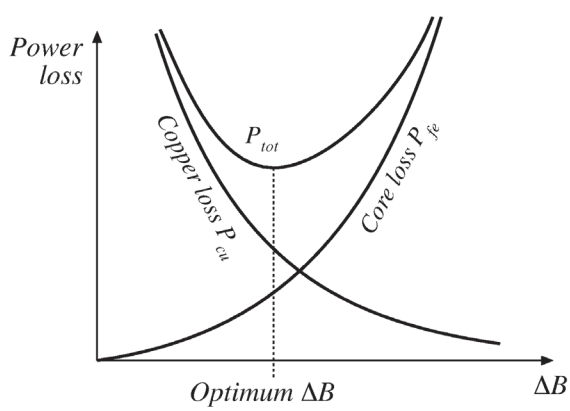
Minimizing Total Loss

There is a value of ΔB that minimizes the total power loss

$$P_{tot} = P_{fe} + P_{cu}$$

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4K_u}\right)\left(\frac{(MLT)}{W_A A_c^2}\right)\left(\frac{1}{\Delta B}\right)^2$$



Calculation of Total Loss

Substitute optimum ΔB into expressions for P_{cu} and P_{fe}. The total loss is:

$$P_{tot} = \left[A_c \ell_m K_{fe}\right]^{\left(\frac{2}{\beta+2}\right)} \left[\frac{\rho \lambda_1^2 I_{tot}^2}{4K_u} \frac{(MLT)}{W_A A_c^2}\right]^{\left(\frac{\beta}{\beta+2}\right)} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]$$

Rearrange as follows:

$$\frac{W_A \left(A_c\right)^{\left(2(\beta-1)/\beta\right)}}{(MLT)\ell_m^{(2/\beta)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)} = \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u \left(P_{tot}\right)^{\left((\beta+2)/\beta\right)}}$$

Left side: terms depend on core geometry

Right side: terms depend on specifications of the application

The K_{gfe} Method

Define $$K_{gfe} = \frac{W_A \left(A_c\right)^{\left(2(\beta-1)/\beta\right)}}{(MLT)\ell_m^{(2/\beta)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)}$$

Design procedure: select a core that satisfies

$$K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u \left(P_{tot}\right)^{\left((\beta+2)/\beta\right)}}$$

Appendix D lists the values of K_{gfe} for common ferrite cores

K_{gfe} is similar to the K_g geometrical constant used in Chapter 14:

- K_g is used when B_{max} is specified
- K_{gfe} is used when ΔB is to be chosen to minimize total loss

The K_{gfe} Method

$$K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u \left(P_{tot}\right)^{\left((\beta+2)/\beta\right)}} 10^8$$

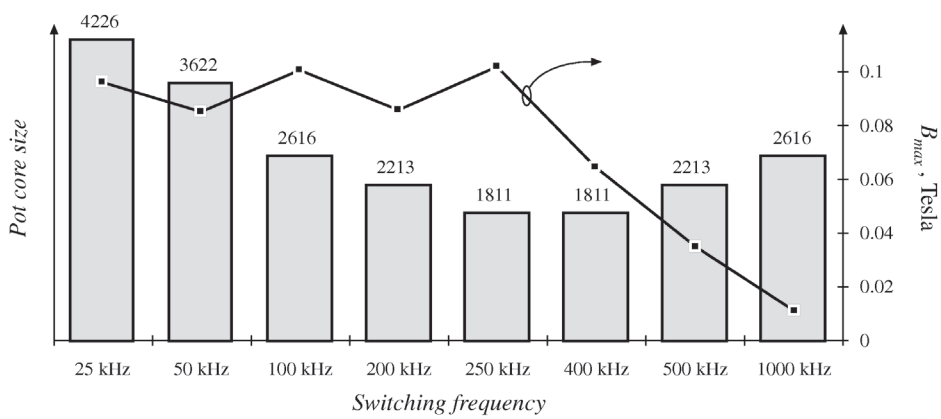
$$\Delta B = \left[10^8 \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \frac{(MLT)}{W_A A_c^3 \ell_m} \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta+2}\right)}$$

$$n_1 = \frac{\lambda_1}{2\Delta B A_c} 10^4 \qquad n_k = n_1 \frac{n_k}{n_1}$$

$$\alpha_k = \frac{n_k I_k}{n_1 I_{tot}} \qquad A_{wk} \leq \frac{\alpha_2 K_u W_A}{n_2}$$

Verify $B_{max} < B_{sat}$

Switching Frequency Vs. XF Size



- As switching frequency is increased from 25 kHz to 250 kHz, core size is dramatically reduced
- As switching frequency is increased from 400 kHz to 1 MHz, core size increases