**Kg Method: Multi-Winding Magnetics**

**Winding 1 allocation**

\[ \alpha_1 W_A \]

**Winding 2 allocation**

\[ \alpha_2 W_A \]

etc.

Total window area \( W_A \)

\[ 0 < \alpha_j < 1 \]

\[ \alpha_1 + \alpha_2 + \cdots + \alpha_i = 1 \]

\[ \alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j} \]

Apparent power in winding \( j \) is

\[ V_j I_j \]

where

- \( V_j \) is the rms or peak applied voltage
- \( I_j \) is the rms current

Window area should be allocated according to the apparent powers of the windings.

---

**B-H Curve: Filter Inductor**

Diagram of a filter inductor with core, core area, core permeability, and symbols for current and magnetic flux.
B-H Curve: Transformer

Core Loss

• Physical origin due to magnetic domains
• Modeling Approaches
  – Empirical (curve fit) models of materials
  – Direct measurement-based models
  – Physics-based models
Hysteresis Loss

Eddy Currents in Magnetic Materials

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz’s law, magnetic fields within the core induce currents (“eddy currents”) to flow within the core. The eddy currents flow such that they tend to generate a flux which opposes changes in the core flux $\Phi(i)$. The eddy currents tend to prevent flux from penetrating the core.
Eddy Current Losses

- Ac flux $\Phi(t)$ induces voltage $v(t)$ in core, according to Faraday’s law. Induced voltage is proportional to derivative of $\Phi(t)$. In consequence, magnitude of induced voltage is directly proportional to excitation frequency $f$.

- If core material impedance $Z$ is purely resistive and independent of frequency, $Z = R$, then eddy current magnitude is proportional to voltage: $i(t) = v(t)/R$. Hence magnitude of $i(t)$ is directly proportional to excitation frequency $f$.

- Eddy current power loss $i^2(t)R$ then varies with square of excitation frequency $f$.

- Ferrite core material impedance is capacitive. This causes eddy current power loss to increase as $f^4$.

The Steinmetz Equation

Empirical equation, at a fixed frequency:

$$P_{fe} = K_{fe} (\Delta B)^\nu A_c \ell_m$$

Alternately:

$$P_v = Kmf^\alpha (\Delta B)^\beta$$

![Graph showing the Steinmetz Equation]
Steinmetz Equation: Notes

- Purely empirical; not physics-based
- Parameters $\alpha, \beta, K$ vary with frequency
- Correct only for sinusoidal excitation
  - Nonlinear; Fourier expansion of waveforms cannot be used
- Modified empirical equations perform better with nonsinusoidal waveforms
  - MSE
  - GSE
  - $iGSE$
  - $i^2GSE$

Some Example Core Materials

<table>
<thead>
<tr>
<th>Core type</th>
<th>$B_{sat}$</th>
<th>Relative core loss</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminations iron, silicon steel</td>
<td>1.5 - 2.0 T</td>
<td>high</td>
<td>50-60 Hz transformers, inductors</td>
</tr>
<tr>
<td>Powdered cores</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>powdered iron, molypermalloy</td>
<td>0.6 - 0.8 T</td>
<td>medium</td>
<td>1 kHz transformers, 100 kHz filter inductors</td>
</tr>
<tr>
<td>Ferrite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manganese-zinc, Nickel-zinc</td>
<td>0.25 - 0.5 T</td>
<td>low</td>
<td>20 kHz - 1 MHz transformers, ac inductors</td>
</tr>
</tbody>
</table>
Skin Effect

For sinusoidal currents: current density is an exponentially decaying function of distance into the conductor, with characteristic length $\delta$ known as the penetration depth or skin depth.

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$

For copper at room temperature:

$$\delta = \frac{7.5}{\sqrt{f}} \text{ cm}$$
Proximity Effect

Ac current in a conductor induces eddy currents in adjacent conductors by a process called the **proximity effect**. This causes significant power loss in the windings of high-frequency transformers and ac inductors.

A multi-layer foil winding, with \( h \gg \delta \). Each layer carries net current \( i(t) \).

Two-Winding Transformer Example

Cross-sectional view of two-winding transformer example. Primary turns are wound in three layers. For this example, let’s assume that each layer is one turn of a flat foil conductor. The secondary is a similar three-layer winding. Each layer carries net current \( i(t) \). Portions of the windings that lie outside of the core window are not illustrated. Each layer has thickness \( h \gg \delta \).
Current Distribution

Skin effect causes currents to concentrate on surfaces of conductors.

Surface current induces equal and opposite current on adjacent conductor.

This induced current returns on opposite side of conductor.

Net conductor current is equal to $i(t)$ for each layer, since layers are connected in series.

Circulating currents within layers increase with the numbers of layers.

High Frequency Estimation

The current $i(t)$ having rms value $I$ is confined to thickness $d$ on the surface of layer 1. Hence the effective “ac” resistance of layer 1 is:

$$R_{ac} = \frac{h}{d} R_{dc}$$

This induces copper loss $P_1$ in layer 1:

$$P_1 = I^2 R_{ac}$$

Power loss $P_2$ in layer 2 is:

$$P_2 = P_1 + 4P_1 = 5P_1$$

Power loss $P_3$ in layer 3 is:

$$P_3 = \left(2^2 + 3^2\right)P_1 = 13P_1$$

Power loss $P_m$ in layer $m$ is:

$$P_m = I^2 \left[(m-1)^2 + m^2\right] \left(\frac{h}{d} R_{dc}\right)$$
Simulation Example

- AWG#30 copper wire
  - Diameter $d = 0.294$ mm
  - $d = \delta$ at around 50 kHz
- 1:1 transformer
  - Primary and secondary are the same, 30 turns in 3 layers
- Sinusoidal currents,
  \[ I_{1\text{rms}} = I_{2\text{rms}} = 1\ A \]

Numerical field and current density solutions using FEMM (Finite Element Method Magnetics), a free 2D solver, http://www.femm.info/wiki/HomePage

Flux density magnitude

Current density magnitude

Density Plot: $|B|$, Tesla

Density Plot: $|J|$, MA/m$^2$
Frequency: 1 kHz

Frequency: 100 kHz

Total copper losses 1.8 larger than at 1 kHz
Frequency: 1 MHz

Flux density

Current Density

Total copper losses 20 times larger than at 1 kHz

Frequency: 10 MHz

Flux density

Current Density

Very significant proximity effect
Total copper losses = 65 times larger than at 1 KHz
Fringing Flux

Fringing Flux Simulation
Litz Wire

- A way to increase conductor area while maintaining low proximity losses
- Many strands of small-gauge wire are bundled together and are externally connected in parallel
- Strands are twisted, or transposed, so that each strand passes equally through each position on inside and outside of bundle. This prevents circulation of currents between strands.
- Strand diameter should be sufficiently smaller than skin depth
- The Litz wire bundle itself is composed of multiple layers
- Advantage: when properly sized, can significantly reduce proximity loss
- Disadvantage: increased cost and decreased amount of copper within core window

Chapter 15: Transformer Design

15.1 Transformer design: Basic constraints
15.2 A step-by-step transformer design procedure
15.3 Examples
15.4 AC inductor design
15.5 Summary
Transformer Design Constraints

Minimizing Total Loss

There is a value of $\Delta B$ that minimizes the total power loss

\[ P_{\text{tot}} = P_{fe} + P_{cu} \]

\[ P_{fe} = K_{fe}(\Delta B)^{6}A_c \ell_m \]

\[ P_{cu} = \left( \frac{\rho \lambda_1^2 I_{\text{tot}}^2}{4 K_u} \right) \left( \frac{MLT}{W_A A_c^2} \right) \left( \frac{1}{\Delta B} \right)^2 \]
Calculation of Total Loss

Substitute optimum $\Delta B$ into expressions for $P_{cu}$ and $P_{fc}$. The total loss is:

$$P_{tot} = \left[ A_c \ell_m K_{fe} \right]^{\frac{2}{\beta + 2}} \left[ \frac{\rho \lambda^2 I_{tot}^2}{4K_u A_c^2} \right] \left[ \frac{\beta}{\beta + 2} \right]$$

Rearrange as follows:

$$\frac{W_A(A_c)^{\left(2(\beta - 1)/\beta\right)}}{(MLT)\ell_m^{\left(2/\beta\right)}} \left[ \frac{\beta}{2} \right]^{\left(\beta + 2\right)} + \left(\frac{\beta}{2}\right)^{\left(2 + \beta\right)} = \frac{\rho \lambda^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u \left( P_{tot} \right)^{\left(\beta + 2\right)/\beta}}$$

Left side: terms depend on core geometry
Right side: terms depend on specifications of the application

The $K_{gfe}$ Method

Define

$$K_{gfe} = \frac{W_A(A_c)^{\left(2(\beta - 1)/\beta\right)}}{(MLT)\ell_m^{\left(2/\beta\right)}} \left[ \frac{\beta}{2} \right]^{\left(\beta + 2\right)} + \left(\frac{\beta}{2}\right)^{\left(2 + \beta\right)}$$

Design procedure: select a core that satisfies

$$K_{gfe} \geq \frac{\rho \lambda^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u \left( P_{tot} \right)^{\left(\beta + 2\right)/\beta}}$$

Appendix D lists the values of $K_{gfe}$ for common ferrite cores

$K_{gfe}$ is similar to the $K_g$ geometrical constant used in Chapter 14:

- $K_g$ is used when $B_{max}$ is specified
- $K_{gfe}$ is used when $\Delta B$ is to be chosen to minimize total loss
The $K_{gfe}$ Method

\[
K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{2/\beta}}{4K_u (P_{tot})^{(\beta+2)/\beta}} 10^8
\]

\[
\Delta B = \left[ 10^8 \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \frac{W_A A^3 c_m}{\beta K_{fe}} \right]^{1/\beta+2}
\]

\[
n_1 = \frac{\lambda_1}{2\Delta B A_c} 10^4 \quad n_k = n_1 \frac{n_k}{n_1}
\]

\[
\alpha_k = \frac{n_k I_k}{n_1 I_{tot}} \quad A_{wk} \leq \frac{\alpha K_u W_A}{n_2}
\]

Verify $B_{max} < B_{sat}$

Switching Frequency Vs. XF Size

- As switching frequency is increased from 25 kHz to 250 kHz, core size is dramatically reduced
- As switching frequency is increased from 400 kHz to 1 MHz, core size increases