

## Measurement and Loss Model of Ferrites with Non-sinusoidal Waveforms

Alex Van den Bossche  
Ghent University, EELAB  
Sint-Pietersnieuwstraat 41  
Gent 9000, Belgium  
alex.vandenbossche@ugent.be

Vencislav Cekov Valchev  
Technical University of Varna  
Department of Electronics  
Studentska Str.1  
Varna 9010, Bulgaria

Georgi Bogomilov Georgiev  
Technical University of Varna  
Department of Electronics  
Studentska Str.1  
Varna 9010, Bulgaria

**Abstract** — The usual data of commercial ferrite grades are given for sinusoidal waveforms, although the voltage in the typical applications in power electronics resemble to square waves. Firstly, an accurate two wire, oscilloscope power measurement is presented. Special wide frequency current and voltage transducers were designed, with a very low phase difference up to 50MHz. Secondly, a ferrite loss model named Natural Steinmetz Extension (*NSE*) is presented. The model is checked with measurements on two different ferrite grades, with square waves with a large variation in duty ratio. The proposed model is compared with Modified Steinmetz Equation (*MSE*). The two methods with a different mathematical formulation give comparable but different results.

**Keywords:** Ferrite losses, magnetics, loss models

### I. INTRODUCTION

Losses in magnetic cores have been studied because of their particular significance to the component design. The macroscopic mechanisms responsible for losses are discussed in classical books [1,2,3] and also summarized and updated in [4]. A practical approach for computing high-frequency ferrite core losses for arbitrary voltage waveforms is presented in [5,6]. The articles [7,8] present a technique to predict a more accurate magnetic core loss for pulsed operation. Models using square dependence on the flux density and a more complex frequency dependence are proposed in [9,10]. Practical oriented approaches for optimizing losses in the design of magnetic components are presented in [11, 12].

Practical disadvantages of most of the proposed methods are the required additional measurements with a given material and parameter calculations. The most popular formula for core loss is known as the 'Steinmetz equation [13][14]:

$$P_{loss} = k f^{\alpha} \hat{B}^{\beta} \quad (1)$$

where  $\hat{B}$  is the peak induction,  $P_{loss}$  is the average power loss per unit volume and  $f$  is the frequency of the sinusoidal excitation. For common used power ferrites,  $\alpha=1.2-1.9$  and  $\beta=2.3-3$ . For square waves of 50% duty ratio the equation (1) loses in accuracy, but remains still a good approximation. When the same peak induction is

maintained, but with a duty ratio  $D$  of 5% (or 95%), our calorimetric experiments did show more than doubled losses

compared to sine wave and the prediction of (1)! This was the starting point for actual investigations.

This paper gives more accurate modeling of the losses under non-sinusoidal waveforms. The models are validated by experimental data measurements made with a set-up for oscilloscope based accurate power measurements.

### II. POWER MEASUREMENT

#### A. Classical four wire method

The traditional solution to measure the core loss is to use a transformer like, four wire measurement set-up, where the current is measured at the exciting winding and an e.m.f. is sensed at a secondary winding [14], [15]. When this measurement method is used in realistic wave shapes with fast edges, such as square waves with variable duty ratio, typical errors may occur and its accuracy is reduced.

#### B. Two wire method

Theoretically, the electrical power through a surface is obtained by integrating the contribution of the Poynting vector. This can be simplified for the power carried through a multi-wire transmission line (cable), when no other power is exchanged. Figure 1 shows the two-wire case where

$P_{loss} = \sum V_i I_i$ . The potentials  $V_i$  and the currents  $I_i$  refer to the  $i$ -th conductor.

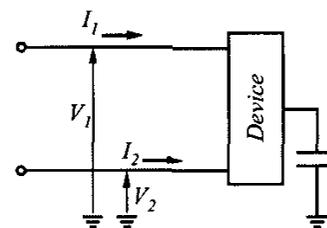


Fig.1 Two wire method. Measuring power to a device with two feeding wires and a ground plane.

The shown capacitor is a parasitic one. Care should be taken as when measuring currents or voltages at different places, errors may occur due to the phase delay of the feeding wires. A current transformer is used for measuring  $I_1$ . In the measurement  $I_2$  can be neglected when  $V_2$  is zero.

At high frequency and high permeability materials, the copper losses can be kept low using an appropriate litz wire. This allows to use a simple total power loss measurement and to subtract the copper loss. To measure the total power (1), an oscilloscope can be used. Today oscilloscopes are capable of doing a data-acquisition with phase shifts lower than 1ns. Also multiplication of channels is a standard function. But it is important that the power measurement has a wide bandwidth and especially a negligible phase shift between current and voltage measurement.

C. Current and Voltage probes

The actual usual operation frequencies for ferrites are about 20 kHz to 1 MHz. However, due to the fast voltage edges, it is good to extend the measuring characteristic up to 50 MHz. We have designed and used a current probe by a ring core TX36/23/15-3E25. The probe is designed to accept primary currents up to 20 A.turns RMS value. Designed voltage probe has the ratio 1:100. At low frequency, a high pass characteristic has been tuned to obtain a low phase shift between voltage and current measurement

The experimental amplitude-frequency characteristic of the current and voltage probes are given in Fig.2 and Fig.3.

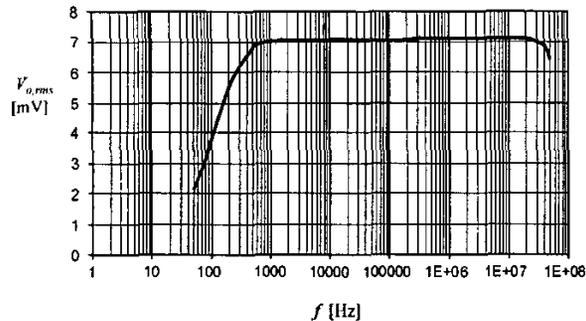


Fig.2 Amplitude-frequency characteristic of the current probe, output  $V_{o,rms}=7.071$  mV for 100 mA peak sine wave input.

The combination of both probes was tested in sine wave voltage. The obtained phase difference between the presented current and voltage probes is sufficient for measuring square wave forms in the range of 1 kHz to 1 MHz.

Flux measurement is done by a passive integrator used to estimate the flux linkage of the core. A high accuracy of the peak-peak flux measurement is required as 1% error generates about 2.5% error in the power.

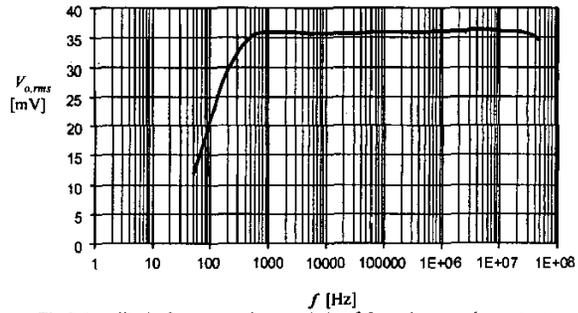


Fig.3 Amplitude-frequency characteristic of the voltage probe, output  $V_{o,rms}=35.36$  mV for 5V sine wave input voltage.

D. Ferrite power loss measurement set-up

A half bridge configuration has been used, using a measurement platform [16]. A Yokohawa DL1540 digital oscilloscope is used. The average power is computed by the multiplying capability of channel 1 and channel 2. Exactly two periods are displayed, triggering at the zero crossing of the current. An averaging factor of 32 is used in the acquisition. The duty ratio is changed to obtain a variable waveform, while maintaining a constant peak-to-peak flux.

Two different material are measured: An ETD 44 core, 3F3 material, and EE42 N67 material. A peak induction of 0.1T is maintained. The results are shown in Table I. The same current waveform (a separate inductor is put in series) is applied to the coil without ferrite to measure the copper loss. In this case, the transverse field through the litz wire is similar to the case with ferrite, and the eddy current losses have the same order of magnitude.

To have an independent check of the power loss measurement, a calorimeter (20W size) test is done for D=50%. The comparison shows a 2% overestimation of the proposed measuring method to the calorimetric measurements, and for D=95%, with a 5% underestimation. These differences are low considering the operating frequency and the special waveforms.

TABLE I  
POWER LOSS MEASUREMENTS AT 0.1T, 100kHz, 100°C

D [%]	$P_{meas}$ [W]	$P_{cu}$ [mW]	$P_{fe}$ [W]	V [V] rms	I [mA] rms	I [A] p-p
50	0.983	3.332	0.979	35.02	350	1.208
55	1.004	3.349	1.001	35.39	349.6	1.208
60	1.015	3.35	1.012	35.68	349.6	1.208
65	1.058	3.373	1.055	37.01	349.4	1.208
70	1.113	3.453	1.110	38.5	349.4	1.208
75	1.189	3.496	1.186	40.69	349.6	1.208
80	1.332	3.55	1.328	44.09	348.9	1.208
85	1.622	3.757	1.618	49.03	348.4	1.208
90	2.154	3.944	2.150	57.49	349.5	1.232
95	4.144	4.483	4.140	77.64	368.1	1.328

### III. FERRITE LOSS MODEL

#### A. Identification of the Steinmetz Equation

In ferrite materials, there is some dependency of the batch and also manufacturer data change in time. We measured the samples with sine waves to avoid this problem.

To define a working area, a reference frequency is defined with a reference power and induction:

$$k_{ref} = \frac{P_{ref}}{B_{ref}^{\beta} f_{ref}^{\alpha}} \quad (2)$$

$$P = k_{ref} B^{\beta} f^{\alpha} = P_{ref} \left( \frac{B}{B_{ref}} \right)^{\beta} \left( \frac{f}{f_{ref}} \right)^{\alpha} \quad (3)$$

The parameter  $\beta$  is fitted at the reference frequency (100kHz), for the reference induction (0.1T), and other levels (0.05 and 0.15T). The parameter  $\alpha$  is determined using the losses at the reference induction at higher frequency. A second frequency of 250kHz was taken as it lies between the second and third harmonic.

Here we give the values  $\alpha$  and  $\beta$  for two ferrite grades: 3F3 and N67, obtained after measuring the corresponding cores. The found value of  $\beta$  at 100°C is higher than the value at 25°C. The value of  $\alpha$  is higher at 100 kHz than at 25 kHz. The values are shown in Table II.

TABLE II  
MEASURED MATERIAL CONSTANTS AT THE REFERENCE POINTS

Material Grade	$K_{ref}$	$\alpha$	$\beta$	Operational conditions
3F3	0.0482	1.842	3.06	100°C, 100 kHz
N67	0.1127	1.76	2.94	100°C, 100 kHz
3F3	17.26	1.31	2.9	100°C, 25 kHz

The parameters  $\alpha$  and  $\beta$  are quite close to the actual data sheets (internet sites).

#### B. Modified Steinmetz Equation

The 'Modified Steinmetz Equation' (MSE) is a known prediction of losses with non-sinusoidal waveforms. The modified Steinmetz equation [13,14,15] uses the root mean square (rms) value of  $dB/dt$ , and defines an equivalent frequency for which this rms value would correspond to a sinusoidal  $B$ . This derivative  $dB/dt$  is usually proportional with a voltage of a winding, which can easily be measured or calculated.

First, the rms value of  $dB/dt$  is calculated:

$$\left( \left\langle \frac{dB}{dt} \right\rangle_{rms} \right)^2 = \frac{1}{T} \int_0^T \left| \frac{dB}{dt} \right|^2 dt \quad (4)$$

Then, an equivalent frequency is defined as

$$f_{eq} = \frac{2}{\left( 2\pi \frac{B_{pp}}{2} \right)^2} \int_0^T \left( \frac{dB}{dt} \right)^2 dt \quad (4)$$

where  $B_{pp}$  is peak-to-peak induction value;

$T = 1/f$  is the period of the operating frequency.

Note that  $f_{ed} = f$  for sine waves. The equivalent frequency is used to apply the original Steinmetz equation to compute the losses.

The losses in MSE are then given as

$$P_{MSE} = k f_{eq}^{\alpha} \hat{B}^{\beta} f \quad (6)$$

where  $f_{eq}$  is an 'equivalent frequency';

$f$  is the operating frequency;

$\alpha$  and  $\beta$  are the exponents, derived under sine excitation.

The origin of the losses in ferrite is not pure eddy current, nor pure hysteresis but can be attributed to excess losses. The modified Steinmetz equation is clearly a good model when the losses depend on the square of  $dB/dt$ , i.e. when  $\alpha = 2$ . But the question remains for other values of  $\alpha$ .

#### C. Natural Steinmetz Extension

In a quasi-static approach, no power loss is generated during moments where  $B$  is constant. The losses can be represented as a surface in the  $B/H$  loop. A natural way to include the frequency dependence is to include a dependence on  $dB/dt$  with power  $\alpha$ , dictated by the frequency dependence. A similar type of equation has been mentioned as a possibility in [13] and other only more involved solutions have been elaborated, which do not increase the accuracy.

We propose the following loss model called 'Natural Steinmetz Extension':

$$P_{NSE} = \left( \frac{\Delta B}{2} \right)^{\beta-\alpha} \frac{k_N}{T} \int_0^T \left| \frac{dB}{dt} \right|^{\alpha} dt \quad (7)$$

This equation is consistent with the Steinmetz equation (1) for sine waves, if  $k_N$  is defined as:

$$k_N = \frac{k}{(2\pi)^{\alpha-1} \int_0^{2\pi} |\cos\theta|^\alpha d\theta} \quad (8)$$

where  $k$  comes from the equation (1). The value  $k_N/k$  is a constant, once  $\alpha$  is known. Figure 4 shows this relation.

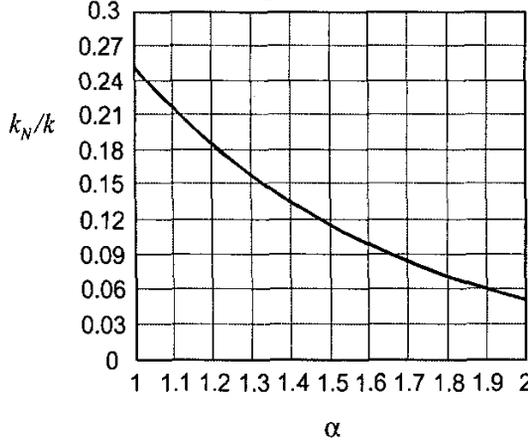


Fig. 4 Ratio of the constants  $k_N/k$ , as a function of  $\alpha$ .

For a square wave voltage with duty ratio  $D$ , the equation (7) can be simplified to:

$$P_{NSE} = k_N f^\alpha (\Delta B)^\beta \left( \left( \frac{2}{D} \right)^\alpha + \left( \frac{2}{1-D} \right)^\alpha (1-D) \right) \quad (9)$$

and then to:

$$P_{NSE} = k_N (2f)^\alpha (\Delta B)^\beta (D^{1-\alpha} + (1-D)^{1-\alpha}) \quad (10)$$

where  $f$  is the operating frequency;  
 $\Delta B / 2$  is the peak induction;  
 $D$  is the duty ratio of the square wave voltage.

Note: The second and third harmonics are dominant at moderate values of duty ratio  $D$ . For extreme values of  $D$  (95%), a higher value of  $\alpha$  could give better matching to the actual losses.

The specific loss predictions ( $P_v$ , losses per unit volume) calculated by NSE, the equation (8), are shown in Fig. 5, Fig.6 and Fig.7 for the ferrite grades 3F3 and N67 at 100 kHz and 25 KHz, 0.1 T. The same graphs show the experimental measurements for square voltage waveforms with  $D=50\%$ – $95\%$ .

The computed results of the 'Modified Steinmetz Equation' (MSE), equation (5), and the classical Steinmetz Equation (1) with corresponding  $\alpha$  and  $\beta$  for sine wave, are also shown in the same graphs. The experiments were made with an ETD 44 core, 3F3 material grade and an EE42 core, N67 material grade.

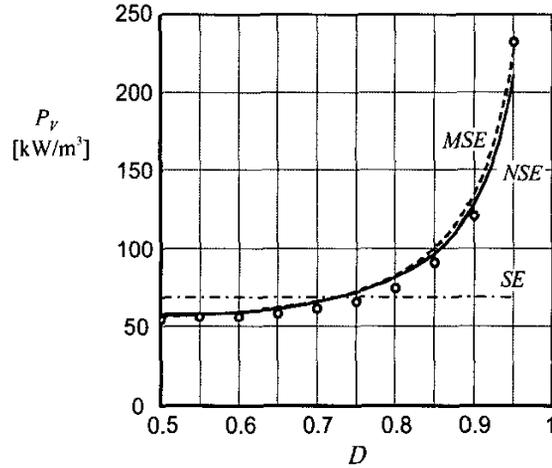


Fig.5 Specific ferrite core losses with square voltage waveforms for ferrite grade 3F3 at 100 kHz, 100°C, 0.1 T as a function of duty ratio  $D$ ; the experiments are the circles; the Natural Steinmetz Extension is the solid curve (NSE), for  $\alpha=1.842$ ;  $\beta=3.06$ ; the Modified Steinmetz Equation (5) is the dashed curve (MSE); classical Steinmetz Equation (1) is dash-dot curve (SE).

The given comparisons show that the matching with the experimental results for NSE is within 5% for duty ratio  $D$  up to 90%. The small difference for  $D=95\%$  can be explained by the high frequency content at that point and the fact that material characteristics show a higher  $\alpha$  at higher frequencies.

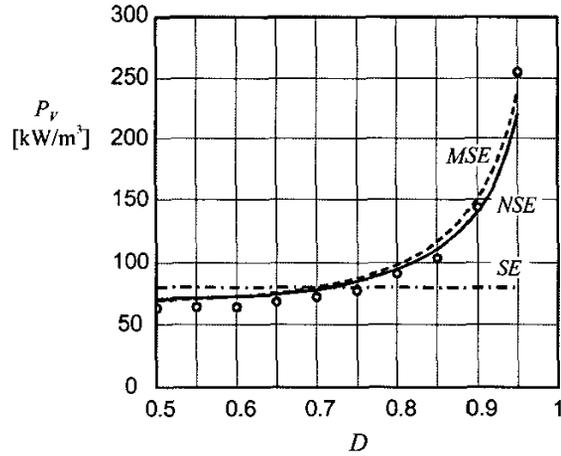


Fig.6 Specific ferrite core losses with square voltage waveforms for ferrite grade N67 at 100 kHz, 100°C, 0.1 T as a function of duty ratio  $D$ ; the experiments are the circles; the Natural Steinmetz Extension is the solid curve (NSE), for  $\alpha=1.76$ ;  $\beta=2.94$ ; the Modified Steinmetz Equation (5) is the dashed curve (MSE); classical Steinmetz Equation (1) is dash-dot curve (SE).

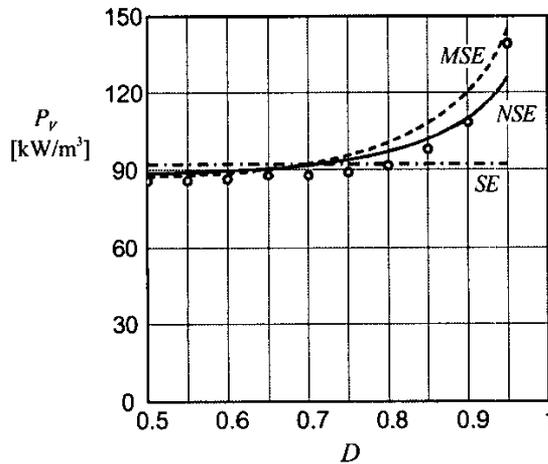


Fig.7 Specific ferrite core losses with square voltage waveforms for ferrite grade 3F3 at 25 kHz, 100°C, 0.2 T as a function of duty ratio  $D$ ; the experiments are the circles; the Natural Steinmetz Extension is the solid curve (NSE), for  $\alpha=1.31$ ;  $\beta=2.9$ ; the Modified Steinmetz Equation (5) is the dashed curve (MSE); classical Steinmetz Equation (1) is dash-dot curve (SE).

Note that NSE and MSE show the same numerical results for  $\alpha=1$  (pure hysteresis losses) and for  $\alpha=2$  (pure Foucault losses).

In Fig.8 we show the comparison between NSE and MSE as a function of the parameter  $\alpha$ . The small difference proves the validity of the proposed NSE.

Remark: DC magnetization increases the losses for the same  $B_{pp}$  induction value. Anyhow, it is not usual in practice to have large  $B_{pp}$  and large DC components combined.

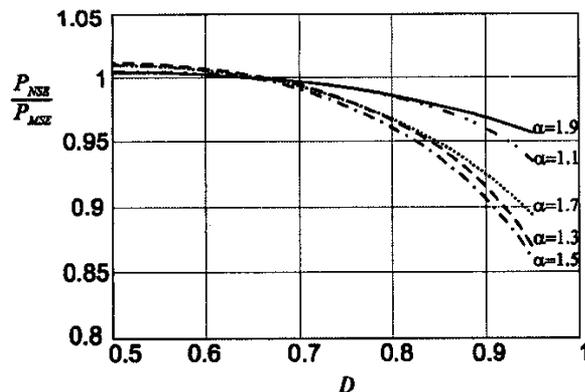


Fig.8 Comparison between NSE and MSE as a function of the duty ratio  $D$ , for different values of  $\alpha$ .

#### IV. CONCLUSION

We realized a performing measurement set-up for measuring ferrite losses of real cores, at actually used frequencies for non-sinusoidal waveforms with fast edges. A new ferrite loss model named Natural Steinmetz Extension (NSE) is proposed. The model is checked with measurements on two different ferrite grades, with square waves with a large variation in duty ratio. The presented model is compared with Modified Steinmetz Equation (MSE). The two methods with a different mathematical formulation give comparable but different results.

The formulation of the proposed NSE is quite straightforward, and it matches well the experiments of two quite different materials.

#### REFERENCES

- [1] G. Bertotti, 'Hysteresis in magnetism', Academic Press, San Diego, USA, 1998, pp. 225-429.
- [2] R. Boll, 'Soft Magnetic Materials', The Vacuumschmelze Handbook, Heyden&Son Ltd., London, UK, 1979, pp. 13-108.
- [3] Jiles, D., 'Introduction to Magnetism and Magnetic Materials', Chapman & Hall, London, UK, 1991, pp.89-174.
- [4] J. B. Goodenough, 'Summary of losses in magnetic materials', IEEE Transactions on Magnetics, vol. 38, No 5, Sept. 2002, pp. 3398-3408.
- [5] W. Roshen, 'Ferrite core loss for power magnetic component design', IEEE Transactions on Magnetics, vol. 27, No 6, Nov. 1991, pp. 4407-4415.
- [6] Devin D., 'Sorting out losses in high frequency magnetic design', Power Electronic Technology, Vol.28, No5, May 2002, pp.55-60.
- [7] Jamerson C., 'Core loss calculations', Power Electronic Technology, Vol.28, No2, Feb. 2002, pp.14-24.
- [8] Severns R., 'Additional losses in high frequency magnetics due to non ideal field distributions', APEC'92, 7th Annual IEEE Appl. Power Electronics Conf., 1992, pp. 333-338.
- [9] K.N. Carpenter, 'Simple models for dynamic hysteresis curves which add frequency dependent losses to static models', IEEE Transactions on Magnetics, vol. 34, No 3, 1997, pp. 406-413.
- [10] D.C. Jiles, Frequency dependence of hysteresis curves in non-conducting magnetic materials', K.N. Carpenter, 'Simple models for dynamic hysteresis curves which add frequency dependent losses to static models', IEEE Transactions on Magnetics, vol. 29, No 6, 1993, pp. 3490-3492.
- [11] Hurley W.G., E. Gath and J. G. Breslin, "Optimized Transformer Design: Inclusive of High-Frequency Effects", IEEE Transactions on Power Electronics, vol. 13, No 4, Yuly 1998, pp. 651-658.
- [12] Lavers J. D. and V. Bolborici "Loss Comparison in the Design of High Frequency Inductors and Transformers", IEEE Transactions on Magnetics, vol. 35, No 5, September 1999, pp. 3541-3542.
- [13] Jieli Li, Tarek Abdallah and Charles Sullivan, "Improved calculation of core loss with nonsinusoidal waveforms" IEEE, IAS 36-th Annual Meeting, Chicago, USA, September 30 - October 4, 2001, pp. 2203-2210.
- [14] Snelling, E. C., 'Soft Ferrites, Properties and Applications', second edition, London, U.K., Butterworths, 1988.
- [15] A. Brockmeyer, 'Dimensionierungswerkzeug für magnetische Bauelemente in Stromrichteranwendungen', PhD thesis, Aachen, University of Technology, Germany, 1997.