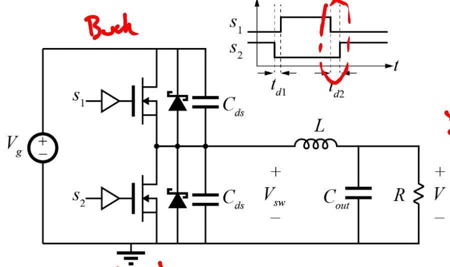
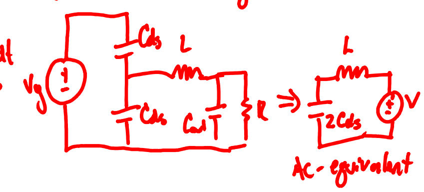
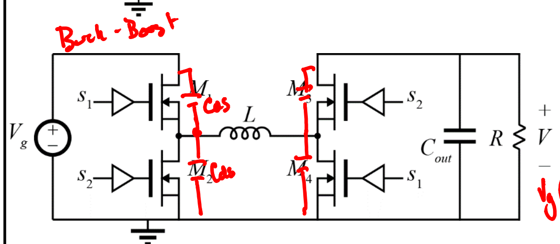
Time-Domain Analysis of Switching Transitions

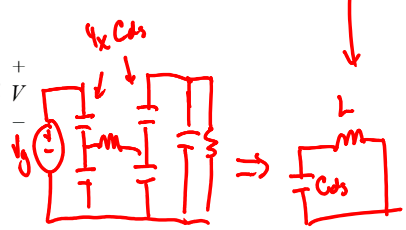
Assume: (1) C_{ds} is linear (through an equiv. or otherwise)
(2) $V \approx$ const during t_{dt}



During t_{dt}

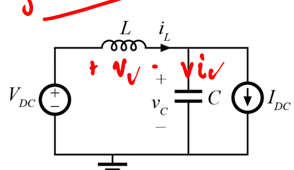






Resonant Circuit Solution

general example



(1) $V_{DC} - v_C = L\frac{di_L}{dt}$

(2) $i_L - I_{DC} = C\frac{dv_C}{dt}$

$$V_{DC} - v_C = L\frac{d}{dt}\left(C\frac{dv_C}{dt} + \cancel{I_{DC}}\right)$$

$$V_{DC} - v_C = LC\frac{d^2v_C}{dt^2}$$

$$LC\frac{d^2v_C}{dt^2} + v_C - V_{DC} = 0$$

ICs:
@ $t=0$: $v_C(0) = V_0$, $i_L(0) = I_0$

$$v_C(t) = V_{DC} + (V_0 - V_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + (I_0 - I_{DC})\sqrt{\frac{L}{C}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$i_L(t) = \underbrace{I_{DC}}_{\text{DC solution}} + (I_0 - I_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + \sqrt{\frac{C}{L}}(V_{DC} - V_0)\sin\left(\frac{t}{\sqrt{LC}}\right)$$

Define:
$\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad f_0 = \frac{1}{2\pi\sqrt{LC}}$ $\quad R_0 = \sqrt{\frac{L}{C}}$

Normalization and Notation

$$v_c(t) = V_{DC} + (V_0 - V_{DC})\cos(\omega_0 t) + (I_0 - I_{DC}) R_0 \sin(\omega_0 t)$$

$$i_L(t) = I_{DC} + (I_0 - I_{DC})\cos(\omega_0 t) + \frac{(V_{DC} - V_0)}{R_0}\sin(\omega_0 t)$$

→ Normalizing $v_c(t)$ & $i_L(t)$ according to:

- $m_c(t) = \frac{v_c(t)}{V_{base}}$, $\underline{V_{base}}$ is an ("arbitrary") constant voltage
- $j_L(t) = \frac{i_L(t)}{I_{base}}$, $I_{base} = \frac{V_{base}}{R_0}$ ←
- $\theta = \omega_0 t$

$$v_c(t) = V_{DC} + (V_0 - V_{DC})\cos\left(\frac{\omega_0 t}{\cancel{\sqrt{LC}}}\right) + (I_0 - I_{DC})\overset{R_0}{\cancel{\sqrt{\frac{L}{C}}}}\sin\left(\frac{\omega_0 t}{\cancel{\sqrt{LC}}}\right)$$

$$i_L(t) = I_{DC} + \underline{(I_0 - I_{DC})}\cos\left(\frac{\omega_0 t}{\cancel{\sqrt{LC}}}\right) + \frac{(V_{DC} - V_0)}{R_0}\cancel{\sqrt{\frac{C}{L}}}\sin\left(\frac{\omega_0 t}{\cancel{\sqrt{LC}}}\right)$$

Pick $V_{base} = V_{DC}$ → $I_{base} = \frac{V_{base}}{R_0} = \frac{V_{DC}}{\sqrt{L/C}} = \frac{V_{DC}}{R_0}$ ←

$$m_c(t) = \frac{v_c(t)}{V_{base}} = \underline{1} + \frac{(V_0 - V_{DC})}{V_{DC}}\cos(\omega_0 t) + \frac{(I_0 - I_{DC})R_0}{V_{DC}}\sin(\omega_0 t)$$

$$j_L(t) = \frac{i_L(t)}{I_{base}} = \frac{I_{DC}}{\underline{I_{base}}} + \frac{(I_0 - I_{DC})R_0}{V_{DC}}\cos(\omega_0 t) + \frac{V_{DC} - V_0}{\cancel{R_0}}\frac{\cancel{R_0}}{V_{DC}}\sin(\omega_0 t)$$

try: $\left(m_c - 1\right)^2 + \left(j_L - \frac{I_{DC}}{I_{base}}\right)^2$

$$= \left(\frac{V_0 - V_{DC}}{V_{DC}}\cos(\omega_0 t) + \frac{(I_0 - I_{DC})R_0}{V_{DC}}\sin(\omega_0 t)\right)^2 + \left(\frac{I_0 - I_{DC}}{V_{DC}}R_0\cos(\omega_0 t) - \frac{V_0 - V_{DC}}{V_{DC}}\sin(\omega_0 t)\right)^2$$

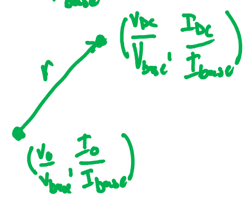
$$= \left(\frac{V_0 - V_{DC}}{V_{DC}}\right)^2 \overset{1}{\cancel{\left(\cos^2\omega_0 t + \sin^2\omega_0 t\right)}} + \left(\frac{(I_0 - I_{DC})R_0}{V_{DC}}\right)^2 \overset{1}{\cancel{\left(\sin^2\omega_0 t + \cos^2\omega_0 t\right)}}$$

$$\left(m_c(t)-1\right)^2 + \left(j_L(t) - \frac{I_{DC}}{I_{base}}\right)^2 = \underbrace{\left(\frac{V_0 - V_{DC}}{V_{DC}}\right)^2 + \left(\frac{I_0 - I_{DC}}{I_{base}}\right)^2}$$

↓

Circle centered @ $m_c = 1$, $j_L = \frac{I_{DC}}{I_{base}}$ w/ radius of r

$$\to \left[\sqrt{\left(\frac{V_0-V_{DC}}{V_{DC}}\right)^2 + \left(\frac{I_0 - I_{DC}}{I_{base}}\right)^2}\right]^2 = r^2 \leftarrow$$



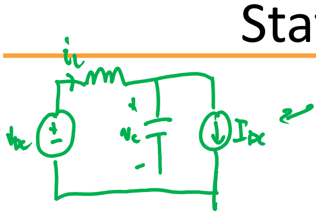
Normalization

Differential equations → geometric equation

- for linear 1L, 1C circuits (w/ DC voltage & current sources)

State Plane Analysis

if lossless, r is constant



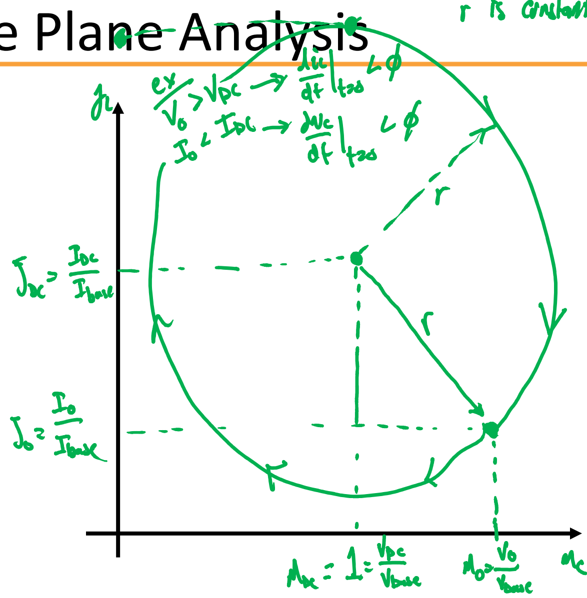
ex what is $i_{L,max}$?

$j_{L,max} = J_{DC} + r$

Denormalize

$i_{L,max} = j_{L,max} I_{base}$

$$\boxed{= I_{DC} + r I_{base}}$$



[1] R. Oruganti and F. C. Lee, "Resonant Power Processors, Part I – State Plane Analysis", Industry Applications, IEEE Tran. on, vol. 21, no. 6, nov 1985.
[2] D. P. Atherton, Nonlinear Control Engineering. London: Van Nostrand Reinhold, 1982, Ch. 2.