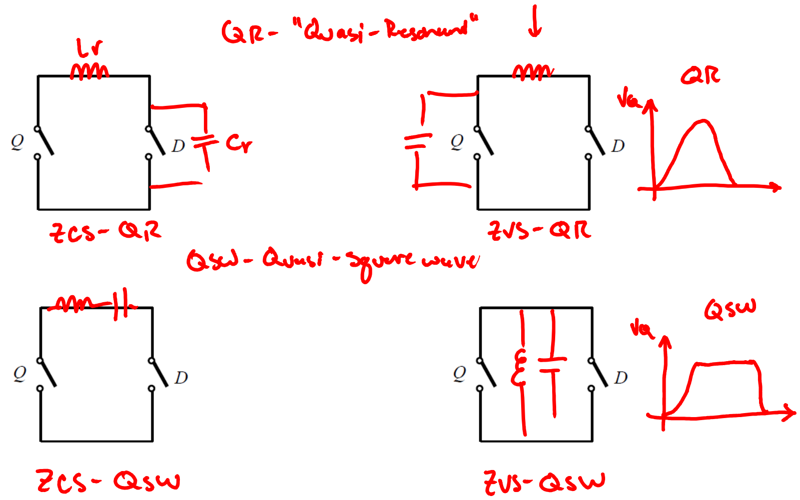
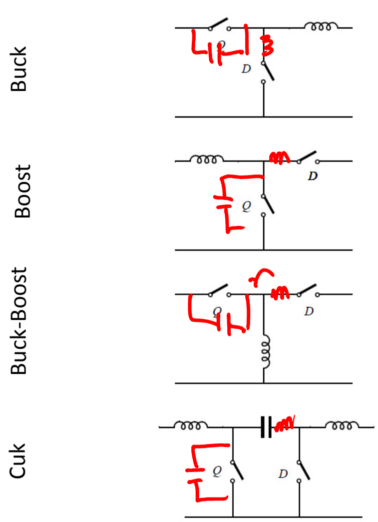
Classification of Resonant-Switch Converters

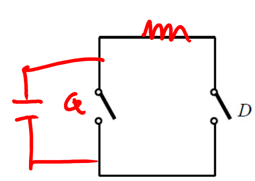


ZVS-QR

Converter examples



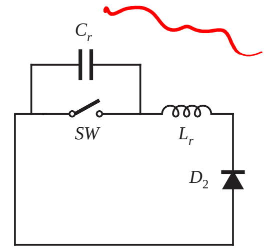
High-frequency view of the switch network



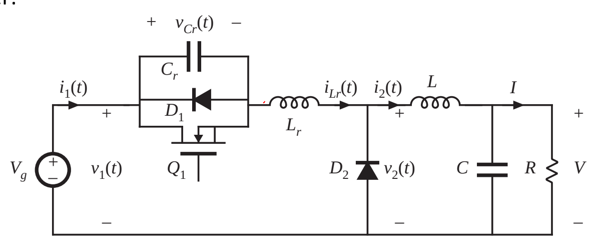
20.3.1 The zero-voltage-switching quasi-resonant switch cell

ZVS-QR

When the previously-described operations are followed, then the converter reduces to



A full-wave version based on the PWM buck converter:



ZVS-QR

$S_2 = S_3 = S_T$

Switch conversion ratio

$$\mu = 1 - FP_{\frac{1}{2}}\left(\frac{1}{J_s}\right)$$ *half-wave*

$$\mu = 1 - FP_1\left(\frac{1}{J_s}\right)$$ *full-wave*

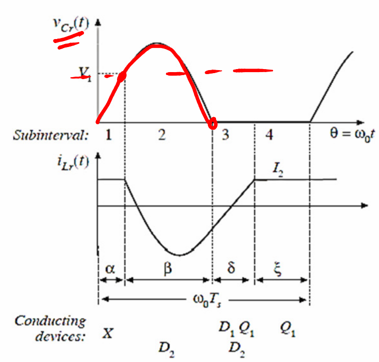
ZVS boundary

$$J_s \geq 1$$

peak transistor voltage $V_{cr,pk} = (1 + J_s)\, V_1$

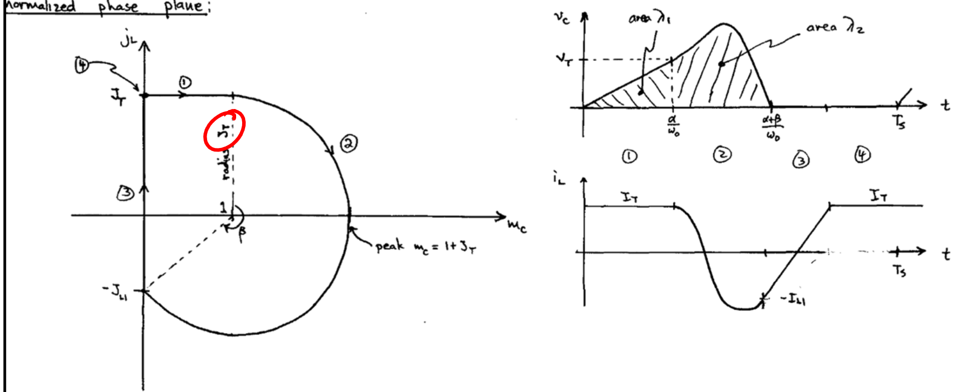
A problem with the quasi-resonant ZVS switch cell: peak transistor voltage becomes very large when zero voltage switching is required for a large range of load currents.

Tank waveforms



ZVS-QR State Plane Trajectory

normalized phase plane:



The average output voltage

Average output voltage:

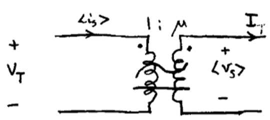
$$\langle m_s \rangle = 1 - \langle m_c \rangle$$

$$\langle v_s \rangle = \mu V_T \quad \text{with} \quad \mu = 1 - F P(J_T)$$

$$P = \frac{1}{2\pi}\left[\frac{1}{2}\frac{1}{J_T} + \pi + \sin^{-1}\frac{1}{J_T} + J_T + \sqrt{J_T^2 - 1}\right]$$

type b $P(J_T)$ = type a $P\left(\frac{1}{J_T}\right)$

$$\underbrace{\begin{bmatrix} \langle v_s \rangle \\ \langle i_s \rangle \end{bmatrix}}_{\underline{x}_s} = \mu \underbrace{\begin{bmatrix} V_T \\ I_T \end{bmatrix}}_{\underline{x}_T}$$



dc transformer
μ controlled by F, but
also depends on
V_T and I_T

Results: Quasi-resonant switches

QR

"type a" ZCS

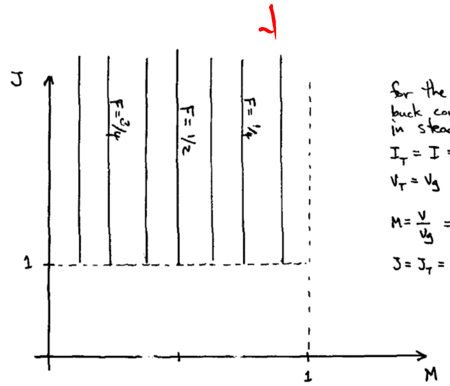
"type b" ZVS

$J_T \equiv J_\eta$

Switch	μ	$P(J_T)$	load current range	voltage conversion range
PWM	D	—	nearly infinite	$0 \le \mu \le 1$
type a $\frac{1}{2}$ wave	$FP(J_T)$	$k_{\frac{1}{2}}(J_T)$	$0 \le J_T \le 1$	$0 \le \mu \le 1$
type a full wave	$FP \approx F$	$k_1(J_T) \approx 1$	$0 \le J_T \le 1$	$0 \le \mu \le 1$
type b $\frac{1}{2}$ wave	$1 - FP(J_T)$	$k_{\frac{1}{2}}\left(\frac{1}{J_T}\right)$	$1 \le J_T \le \infty$	$0 \le \mu \le 1$
type b full wave	$1 - FP \approx 1 - F$	$k_1\left(\frac{1}{J_T}\right) \approx 1$	$1 \le J_T \le \infty$	$0 \le \mu \le 1$

with
$$k_{\frac{1}{2}}(x) = \frac{1}{2\pi}\left[\frac{1}{2}x + \pi + \sin^{-1}x + \frac{1}{x}\left(1 + \sqrt{1-x^2}\right)\right]$$
$$k_1(x) = \frac{1}{2\pi}\left[\frac{1}{2}x + 2\pi - \sin^{-1}x + \frac{1}{x}\left(1 - \sqrt{1-x^2}\right)\right]$$

Output characteristics: Full-wave ZVS buck



for the buck converter in steady state:

$I_T = I$ = load current

$V_T = V_g$

$M = \frac{V}{V_g} = \mu$

$J = J_T = \frac{I R_0}{V_g}$

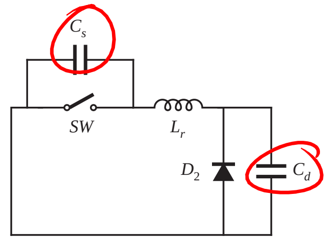
$V = \left(1 - \frac{f_s}{f_0}\right) V_g$ with $I \ge \frac{V_g}{R_0}$

with resistive load R: $I = V/R \ge V_g/R_0$

$\Rightarrow R \le \left(1 - \frac{f_s}{f_0}\right) R_0$

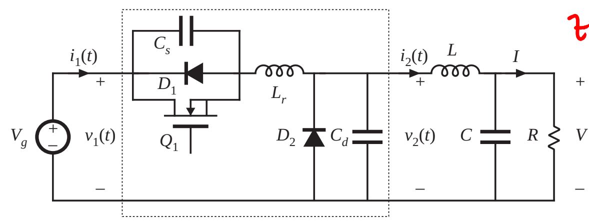
20.3.2 The ZVS multiresonant switch

When the previously-described operations are followed, then the converter reduces to



ZVS-MR

A half-wave version based on the PWM buck converter:



ZVS-MR Operating Modes

D Maksimovic, "Synthesis of PWM and Quasi-Resonant DC-to-DC Power Converters," Ph.D. thesis, California Institute of Technology, 1989.

① $I = i_{r0} + \frac{V_g}{L_r} t_1$

② $i_{r2} = I + \frac{V_g}{R_d}\sin(\omega_d t_2)$

$v_{d2} = V_g(1-\cos(\omega_d t_2))$

$\omega_d = \frac{1}{\sqrt{L_r C_d}}, \quad R_d = \sqrt{\frac{L_r}{C_d}}$

③ $v_{t3} = I R_o \omega_o t_3 - k v_{d2}$

$$\omega_o t_3 = \pi + \tan^{-1}\left(\frac{V_g - v_{d2}}{i_{r2}R_o - IR_o/(1+k)}\right) + \tan^{-1}\left(\frac{V_g - v_{t3}}{i_{r3}R_o + IR_o/(1+k)}\right)$$

$$(V_g - v_{d2})^2 + \left(i_{r2}R_o - IR_o/(1+k)\right)^2 = (V_g - v_{t3})^2 + \left(i_{r3}R_o + IR_o/(1+k)\right)^2$$

with $R_o = \sqrt{\frac{L_r}{C_t \| C_d}}, \quad k = \frac{C_d}{C_t}, \quad \omega_o = \frac{1}{\sqrt{L_r (C_t \| C_d)}}$

④ $(V_g - v_{t3})^2 + (i_{r3}R_t)^2 = V_g^2 + (i_{r0}R_t)^2$

$$\omega_t t_4 = \tan^{-1}\left(\frac{-i_{r3}R_t}{V_g - v_{t3}}\right) - \tan^{-1}\left(\frac{-i_{r0}R_t}{V_g}\right)$$

with $\omega_t = \frac{1}{\sqrt{L_r C_t}}, \quad R_t = \sqrt{\frac{L_r}{C_t}}$

averaging

$$\mu = \frac{t_2}{T_s} - \frac{L_r i_{r2}}{V_g T_s} + \frac{L_r I}{V_g T_s} + \frac{t_3}{(1+k)T_s} + \frac{L_r(i_{r2} - i_{r3})}{(1+k)V_g T_s} + \frac{k}{1+k}\frac{v_{d2} t_3}{V_g T_s} - \frac{I t_3^2}{(1+k)\cdot 2C_t V_g T_s}$$

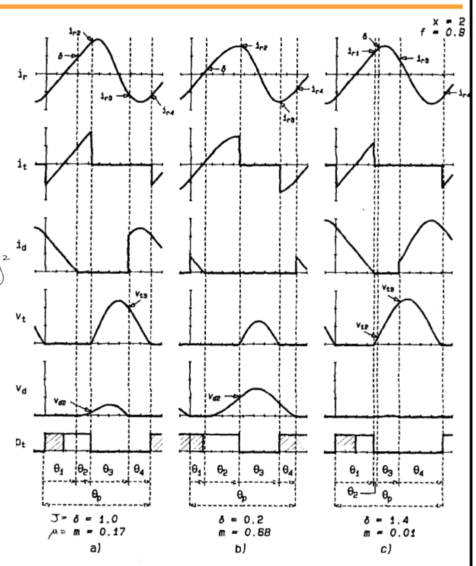
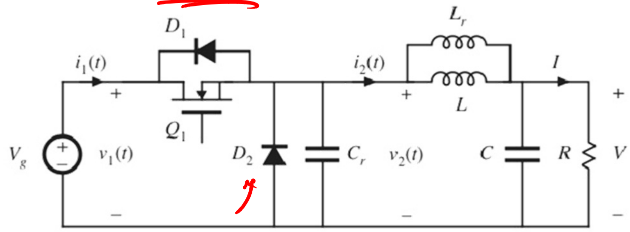


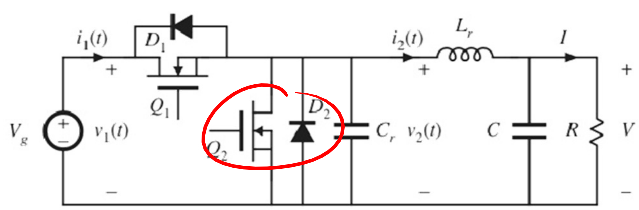
Figure 10.2: Typical waveforms for a ZV-MR converter operating in modes I_1 (a), II_1 (b) or III_1 (c)

ZVS QSW Converters: Already Studied

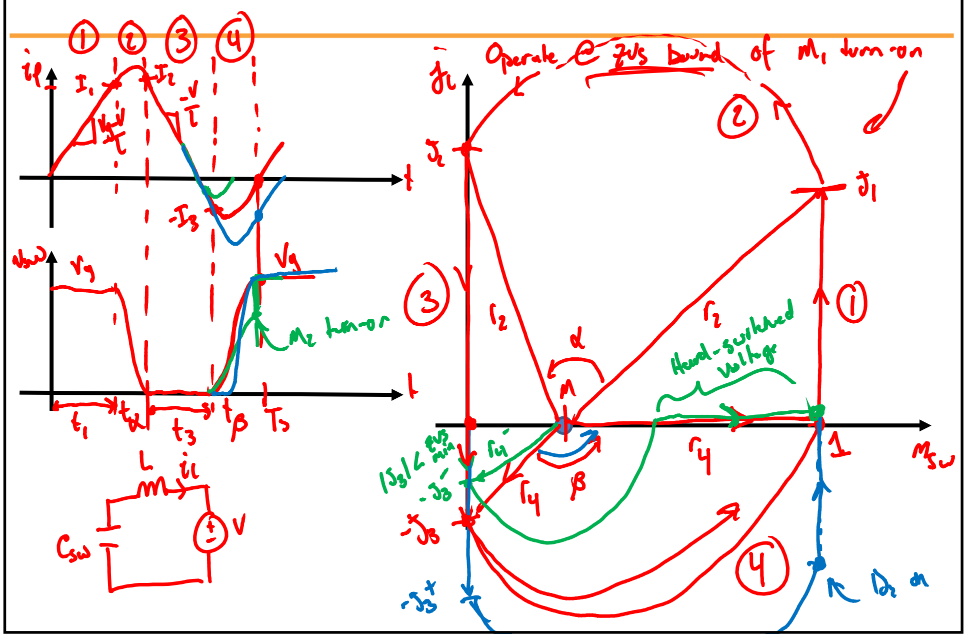
Original one-switch version



Add synchronous rectifier

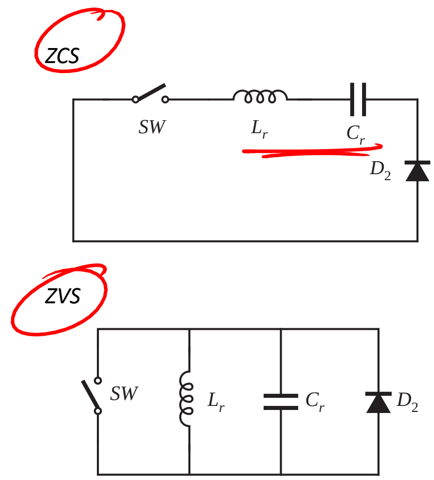


Lecture 21

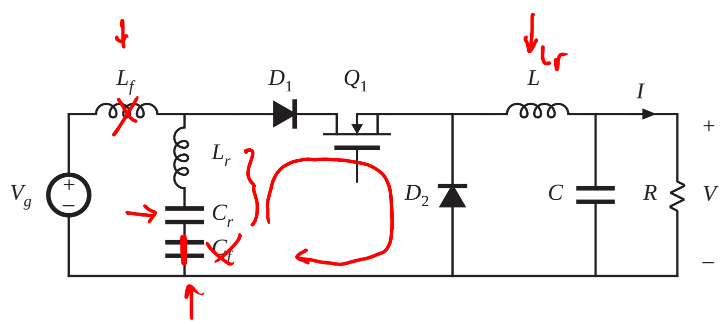


20.2.3 Quasi-square-wave resonant switches

When the previously-described operations are followed, then the converter reduces to



A quasi-square-wave ZCS buck with input filter



- The basic ZCS QSW switch cell is restricted to $0 \le \mu \le 0.5$
- Peak transistor current is equal to peak transistor current of PWM cell
- Peak transistor voltage is increased
- Zero-current switching in all semiconductor devices

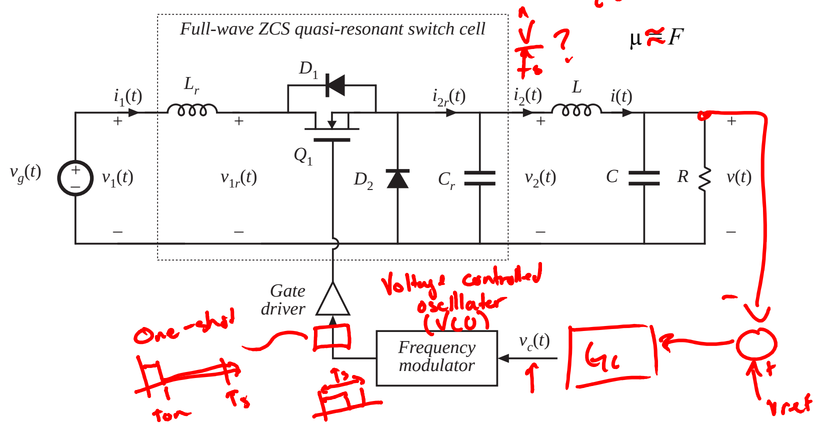
Ac modeling of quasi-resonant converters

Use averaged switch modeling technique: apply averaged PWM model, with d replaced by µ

Buck example with full-wave ZCS quasi-resonant cell:

ZCS-QR

$\mu \approx F$



Ac modeling of QR converters

Quasi-resonant converters inherit properties of PWM parents, with switch conversion ratio µ playing the role of the PWM switch duty cycle *d*

AC modeling approach:

- Start from $\mu(v,i,f_s)$ found for the resonant switch
- Perturb and linearize

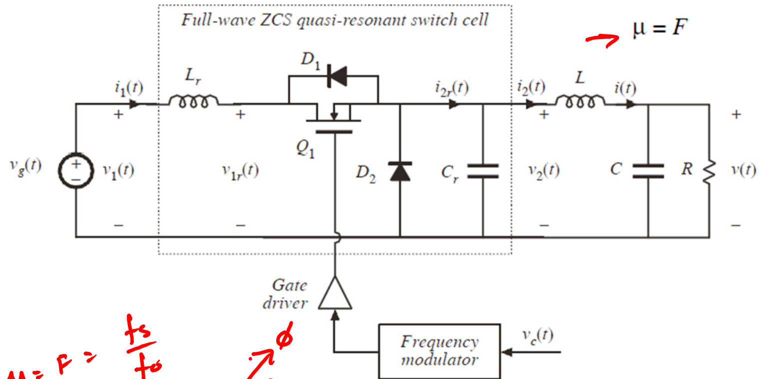
$$\hat{\mu} = \left.\frac{\partial \mu}{\partial v}\right|_{\text{DC op point}} \hat{v} + \left.\frac{\partial \mu}{\partial i}\right|_{\text{DC op}} \hat{i} + \left.\frac{\partial \mu}{\partial f_s}\right|_{\text{DC op pt.}} \hat{f}_s$$

- Replace *d* with $\hat{\mu}$ in the small-signal AC dynamic model of the PWM parent converter

$$\hat{d} \to \hat{\mu}$$

Example 1: Half-wave ZCS quasi-resonant buck

$\rightarrow \mu = F$



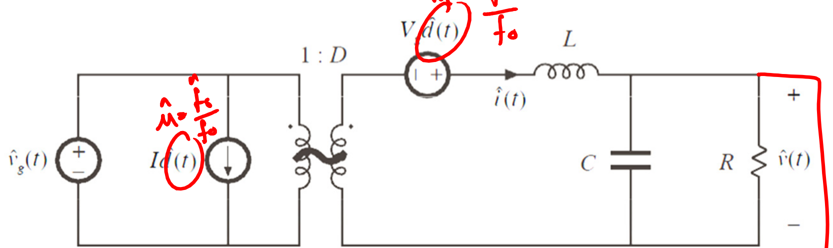
$\mu = F = \frac{f_s}{f_0}$

$\hat{\mu} = \frac{\partial \mu}{\partial f_s}\Big|_{DC} \hat{f}_s + \ldots \rightarrow \emptyset$

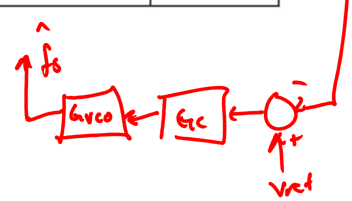
$\hat{\mu} = \frac{1}{f_0}\hat{f}_s \rightarrow$ replace $\hat{d}$ in normal PWM SSM

Buck Converter Small Signal Model

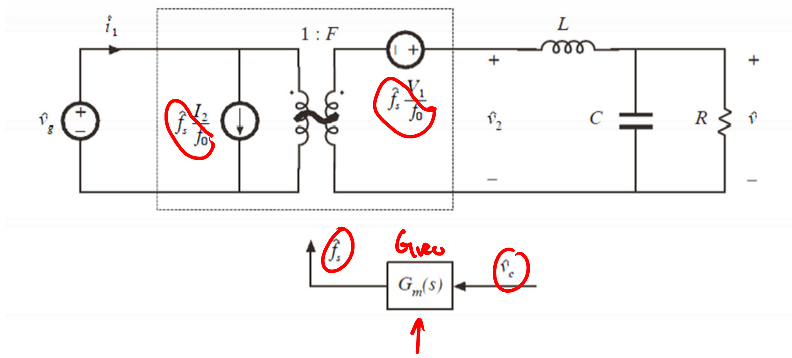
$\hat{\mu} = \frac{\hat{f}_s}{f_0}$



Averaged SSM model by:
- Waveform averaging
- state space averaging
- switch averaging



ZVS-QR buck AC model



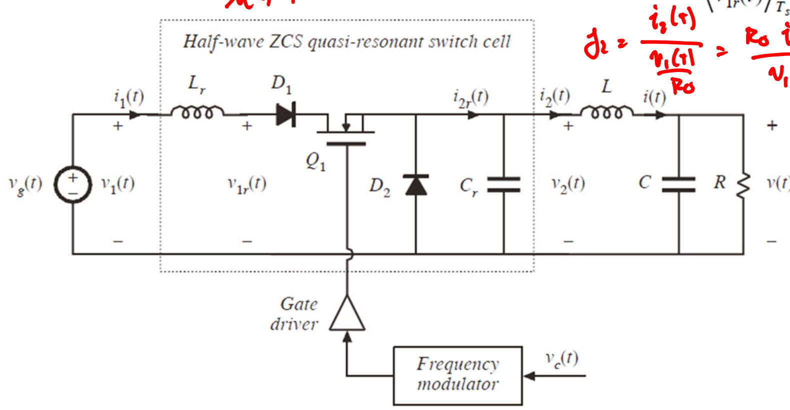
Example 2: Half-wave ZCS-QR buck

Now, μ depends on j_s: $\mu(t) = \frac{f_s(t)}{f_0} P_{\frac{1}{2}}\big(j_s(t)\big)$ $\qquad j_s(t) = R_0 \frac{\langle i_{2r}(t)\rangle_{T_s}}{\langle v_{1r}(t)\rangle_{T_s}}$

$\mu \neq P$

$j_2 = \frac{i_2(t)}{\frac{v_1(t)}{R_0}} = \frac{R_0\, i_2(t)}{v_1(t)}$

Very slow



Perturb and Linearize

Perturbation and linearization of $\mu(v_{1r}, i_{2r}, f_s)$:

$$\hat{\mu}(t) = K_v \hat{v}_{1r}(t) + K_i \hat{i}_{2r}(t) + K_c \hat{f}_s(t)$$

with

$$K_v = -\frac{\partial \mu}{\partial j_s}\frac{R_0 I_2}{V_1^2}$$

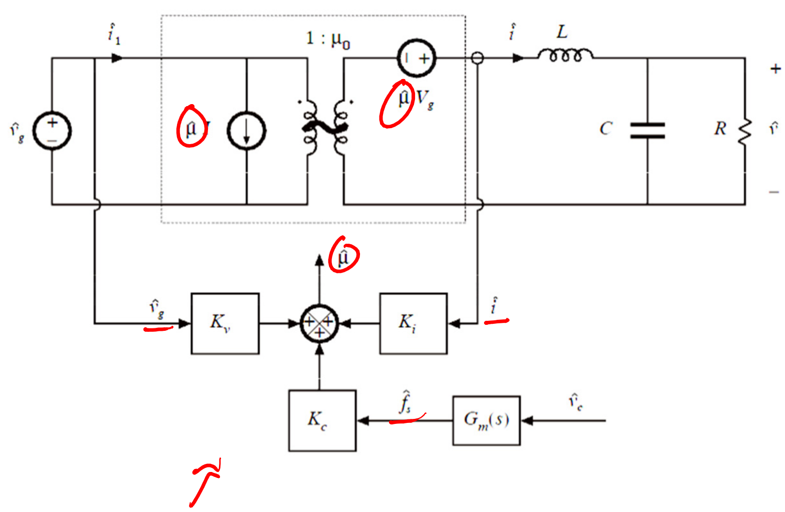
$$K_i = -\frac{\partial \mu}{\partial j_s}\frac{R_0}{V_1}$$

$$K_c = \frac{\mu_0}{F_s}$$

$$\frac{\partial \mu}{\partial j_s} = \frac{F_s}{2\pi f_0}\left(\frac{1}{2} - \frac{1+\sqrt{1-J_s^2}}{J_s^2}\right)$$

$$K_v = \frac{\partial \mu}{\partial v}\Big|_{v=V} = \frac{\partial \mu}{\partial i_2}\frac{\partial i_2}{\partial v_1} = \frac{\partial \mu}{\partial j_s}\cdot\left(-R_0\frac{I_2}{V_1^2}\right)$$

AC SSM of half-wave ZCS-QR buck



$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{1}{Q}\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$G_{vc}(s) = G_{c0} \frac{1}{1 + \frac{1}{Q}\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Full-wave: poles and zeroes are same as PWM

Half-wave: effective feedback reduces Q-factor and dc gains

$$G_{g0} = \frac{\mu_0 + K_v V_g}{1 + \frac{K_i V_g}{R}}$$

$$G_{c0} = \frac{K_c V_g}{1 + \frac{K_i V_g}{R}}$$

$$\omega_0 = \sqrt{\frac{1 + \frac{K_i V_g}{R}}{L_r C_r}}$$

$$Q = \frac{\sqrt{1 + \frac{K_i V_g}{R}}}{\frac{R_0}{R} + K_i V_g \frac{R}{R_0}}$$

$$R_0 = \sqrt{\frac{L_r}{C_r}}$$

Summary of results

Switch	μ_0	$\partial\mu/\partial J_x$	K_i	K_v	K_c
PWM	D	0	0	0	$K_d = 1$
ZCS HW	$\frac{F}{2\pi}(\frac{J_T}{2}+\pi+\sin^{-1}(J_T)+\frac{1}{J_T}(1+\sqrt{1-J_T^2}))$	$\frac{\partial\mu}{\partial J_T}=\frac{F}{2\pi}(\frac{1}{2}-\frac{(1+\sqrt{1-J_T^2})}{J_T^2})$	$-\frac{\partial\mu}{\partial J_T}\frac{R_0}{V_{T0}}$	$-\frac{\partial\mu}{\partial J_T}\frac{R_0 I_{T0}}{V_{T0}^2}$	$K_f = \frac{\mu_0}{F_{s0}}$
ZCS FW	$\frac{F}{2\pi}(\frac{J_T}{2}+2\pi-\sin^{-1}(J_T)+\frac{1}{J_T}(1-\sqrt{1-J_T^2}))$	$\frac{\partial\mu}{\partial J_T}=\frac{F}{2\pi}(\frac{1}{2}-\frac{(1-\sqrt{1-J_T^2})}{J_T^2})$	0	0	$K_f = \frac{\mu_0}{F_{s0}}$
ZVS HW	$1-\frac{F}{2\pi}(\frac{\frac{1}{2}}{J_T}+\pi+\sin^{-1}(\frac{1}{J_T})+J_T(1+\sqrt{1-\frac{1}{J_T^2}}))$	$\frac{\partial\mu}{\partial J_T}=\frac{1}{J_T^2}\frac{F}{2\pi}(\frac{1}{2}-\frac{(1+\sqrt{1-J_T^2})}{J_T^2})$	$-\frac{\partial\mu}{\partial J_T}\frac{R_0}{V_{T0}}$	$-\frac{\partial\mu}{\partial J_T}\frac{R_0 I_{T0}}{V_{T0}^2}$	$K_f=\frac{\mu_0}{F_{s0}}$
ZVS FW	$1-\frac{F}{2\pi}(\frac{\frac{1}{2}}{J_T}+2\pi-\sin^{-1}(\frac{1}{J_T})+J_T(1-\sqrt{1-\frac{1}{J_T^2}}))$	$\frac{\partial\mu}{\partial J_T}=\frac{1}{J_T^2}\frac{F}{2\pi}(\frac{1}{2}-\frac{(1-\sqrt{1-J_T^2})}{J_T^2})$	0	0	$K_f=\frac{\mu_0}{F_{s0}}$