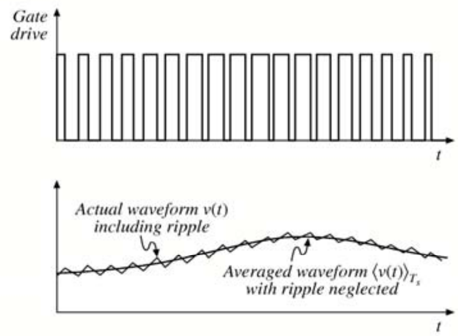
Neglecting the switching ripple

Suppose the duty cycle is modulated sinusoidally:

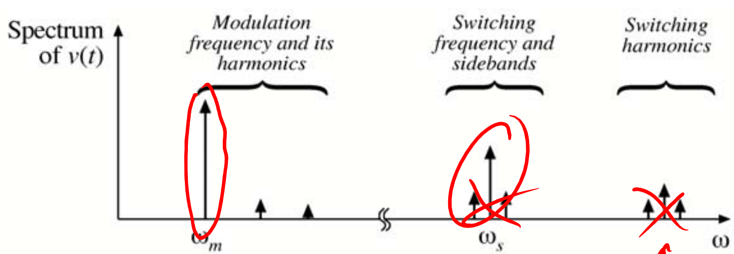
$$d(t) = D + D_m \cos \omega_m t$$

where D and D_m are constants, $|D_m| \ll D$, and the modulation frequency ω_m is much smaller than the converter switching frequency $\omega_s = 2\pi f_s$.

The resulting variations in transistor gate drive signal and converter output voltage:



Output voltage spectrum
with sinusoidal modulation of duty cycle



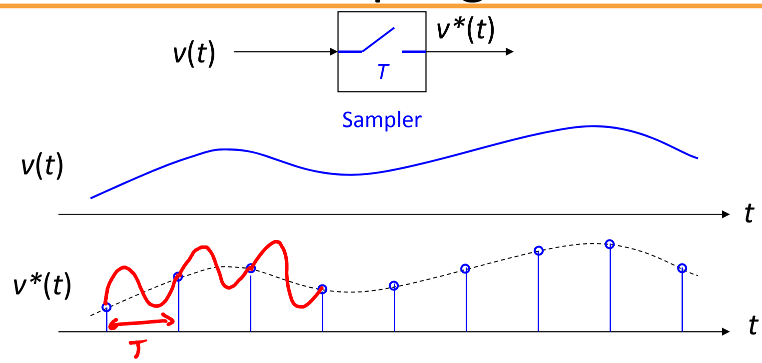
Contains frequency components at:

- Modulation frequency and its harmonics
- Switching frequency and its harmonics
- Sidebands of switching frequency

With small switching ripple, high-frequency components (switching harmonics and sidebands) are small.

If ripple is neglected, then only low-frequency components (modulation frequency and harmonics) remain.

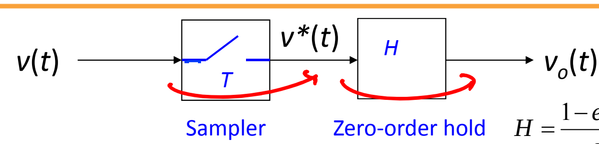
Sampling



$$v^*(t) = v(t)\sum_{-\infty}^{+\infty}\delta(t-nT)$$

Unit impulse (Dirac)

Sampled-data system example: frequency domain



$$H = \frac{1-e^{-sT}}{s}$$

$$v^*(s) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} v(s - jk\omega_s)$$

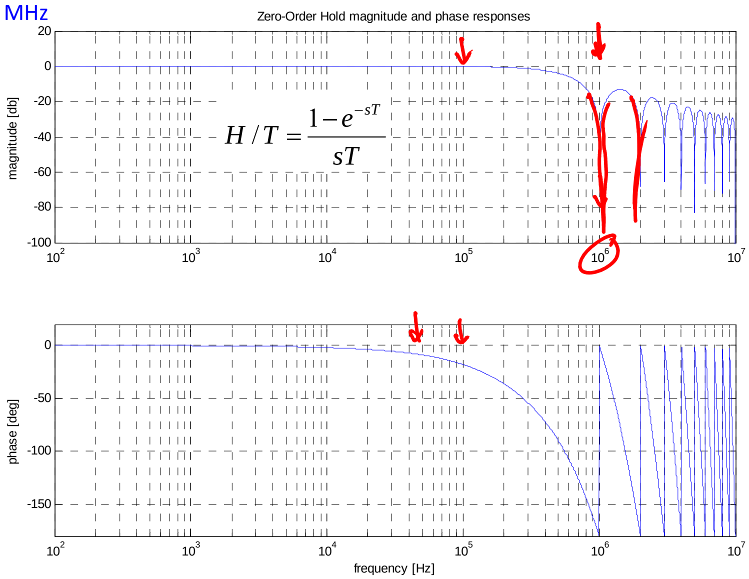
$$v_o(s) = \frac{1-e^{-sT}}{s} v^*(s) = \frac{1-e^{-sT}}{sT}\sum_{k=-\infty}^{+\infty} v(s - jk\omega_s)$$

Consider only low-frequency signals: $v_o(s) \approx \frac{1-e^{-sT}}{sT} v(s)$

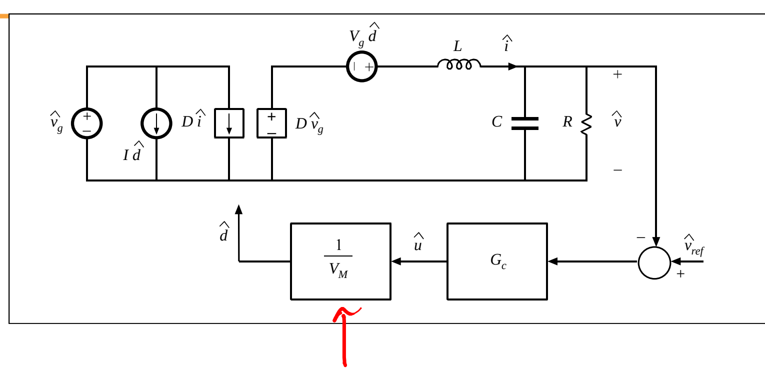
System "transfer function" = $\frac{v_o}{v} = \frac{1-e^{-sT}}{sT}$

Zero-order hold: frequency responses

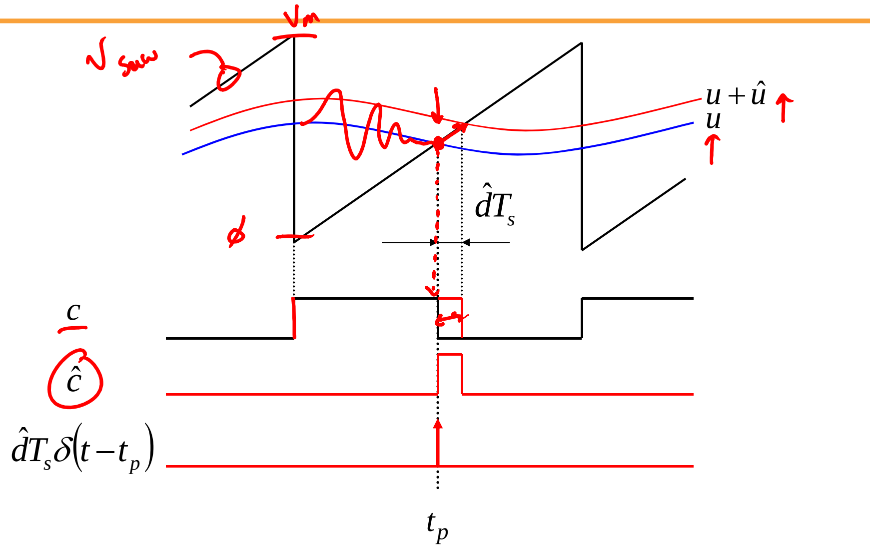
f_s = 1 MHz



How does any of this apply to converter modeling?

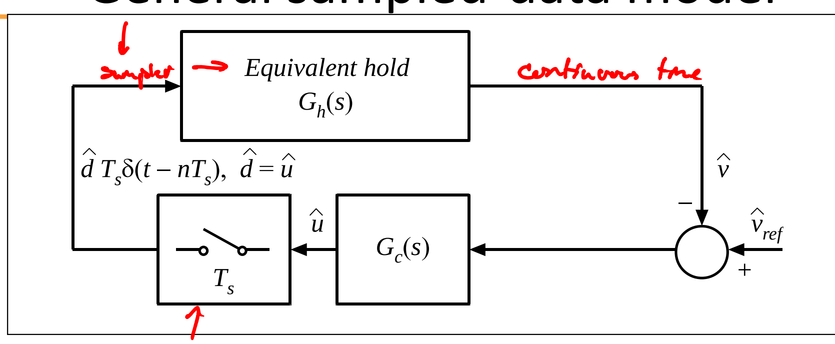


PWM is a small-signal sampler!



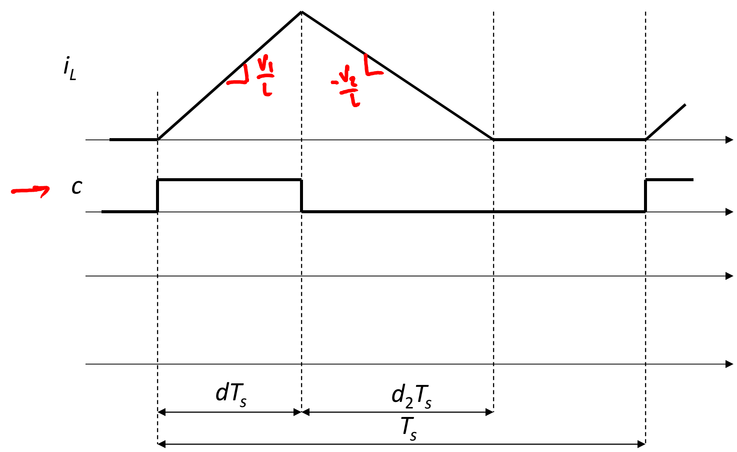
PWM sampling occurs at t_p (*i.e.* at dT_s, periodically, in each switching period)

General sampled-data model

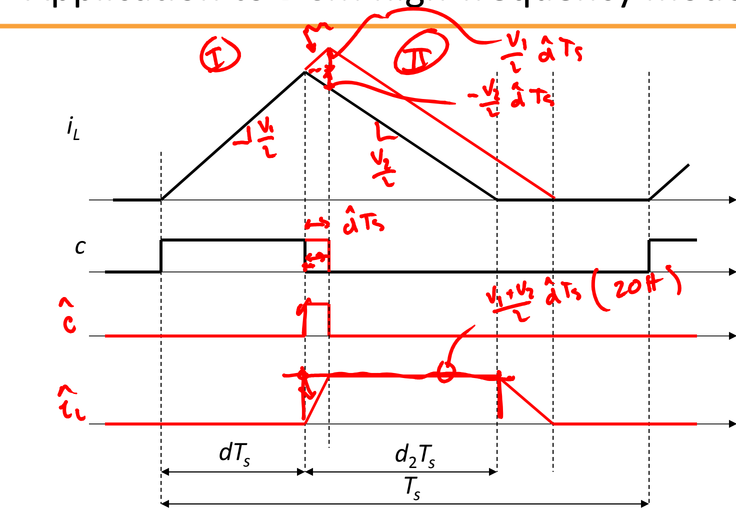


- Sampled-data model valid at all frequencies
- *<u>Equivalent hold</u>* describes the converter small-signal response to the sampled duty-cycle perturbations [Billy Lau, PESC 1986]
- State-space averaging or averaged-switch models are low-frequency continuous-time approximations to this sampled-data model

Application to DCM high-frequency modeling



Application to DCM high-frequency modeling



DCM inductor current high-frequency response

$$\hat{i}_L(s) = \frac{V_1+V_2}{L} T_s \frac{1-e^{-sD_2T_s}}{s} \hat{d}^*(s) = \frac{V_1+V_2}{L} T_s \frac{1-e^{-sD_2T_s}}{s} \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \hat{d}(s-jk\omega_s)$$

$$\hat{i}_L(s) \approx \frac{V_1+V_2}{L} D_2 T_s \frac{1-e^{-sD_2T_s}}{D_2 T_s s} \hat{d}(s)$$

$$\frac{\hat{i}_L(s)}{\hat{d}(s)} \approx \frac{V_1+V_2}{L} D_2 T_s \frac{1}{1+\frac{s}{\omega_2}} \qquad \omega_2 = \frac{2}{D_2 T_s}$$

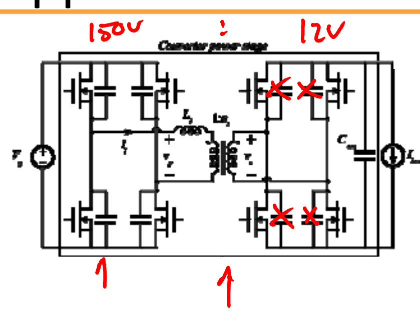
$$f_2 = \frac{f_s}{\pi D_2}$$

High-frequency pole due to the inductor current dynamics in DCM, see (11.77) in Section 11.3

Example DAB Application

- Goal: Include effects of both output sampling and PWM sampling in converter model
- Considering only control perturbations, model will take the form

$$\hat{x}[n] = F\hat{x}[n-1] + G\hat{\varphi}_{ab}[n-1]$$



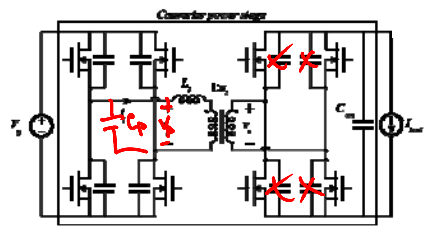
Example DAB Application

- Goal is to develop a model of the form

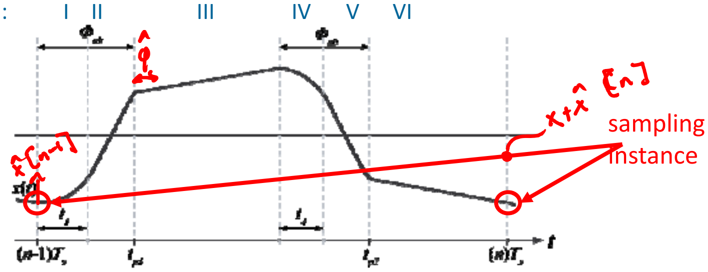
$$\hat{x}[n] = F\hat{x}[n-1] + G\hat{\varphi}_{ab}[n-1]$$

- State vector is sampled and control updated at $t=nT_s$

$$x = \begin{bmatrix} v_p & i_l & v_{out} \end{bmatrix}^T$$

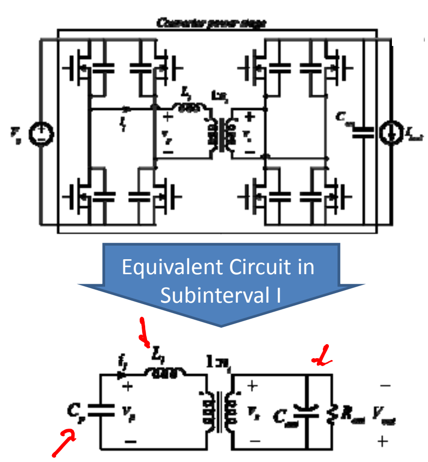


Subinterval: I II III IV V VI



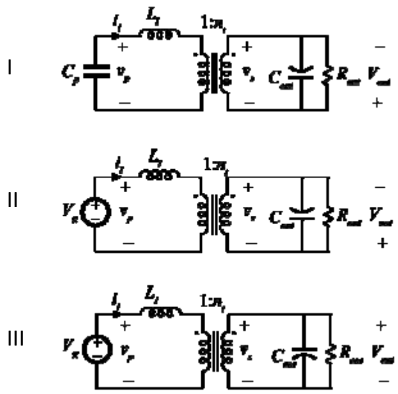
Continuous Time State Space

$$\dot{x}(t) = A_i x(t) + B_i V_g$$



Continuous Time State Representation

$$\dot{x}(t) = A_i x(t) + B_i V_g$$



$$A_1 = \begin{bmatrix} 0 & 0 & \frac{-1}{C_p} \\ 0 & \frac{-1}{C_{out}R_{out}} & \frac{1}{C_{out}n_t} \\ \frac{1}{L_l} & \frac{-1}{L_l n_t} & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \frac{-1}{2R_pC_p} & 0 & 0 \\ 0 & \frac{-1}{C_{out}R_{out}} & \frac{1}{C_{out}n_t} \\ 0 & \frac{-1}{L_l n_t} & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} \frac{-1}{2R_pC_p} \\ 0 \\ \frac{-1}{L_l} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \frac{-1}{2R_pC_p} & 0 & 0 \\ 0 & \frac{-1}{C_{out}R_{out}} & \frac{-1}{C_{out}n_t} \\ 0 & \frac{1}{L_l n_t} & 0 \end{bmatrix} \quad B_3 = \begin{bmatrix} \frac{-1}{2R_pC_p} \\ 0 \\ \frac{-1}{L_l} \end{bmatrix}$$

- Models change slightly depending on operating mode
- In all cases, consider dynamics at boundary just below ZVS

State Propagation

- Within each subinterval, states propagate according to continuous time model

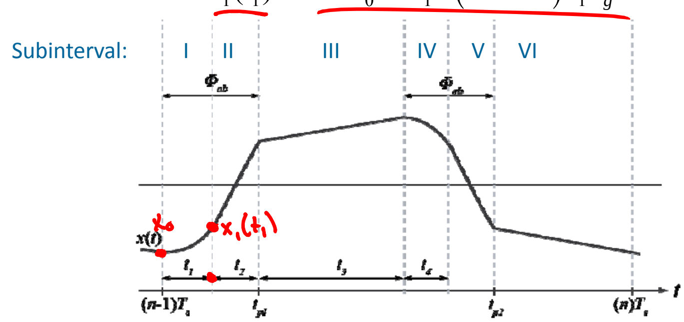
$$\dot{x}(t) = A_i x(t) + B_i V_g$$

- Solution given by

$$x(t) = e^{A_i t} x_0 + \int_0^{t_i} e^{-A_i(t-\tau)} \left(B_i v_g(\tau) \right) d\tau \; .$$

- If $v_g(t) \approx V_g$ within one switching period,

$$x_1(t_1) = e^{A_1 t_1} x_0 + A_1^{-1}\left(e^{A_1 t_1} - I\right) B_1 V_g$$



Small Signal State Propagation

- Considering steady state plus some small signal, solution becomes

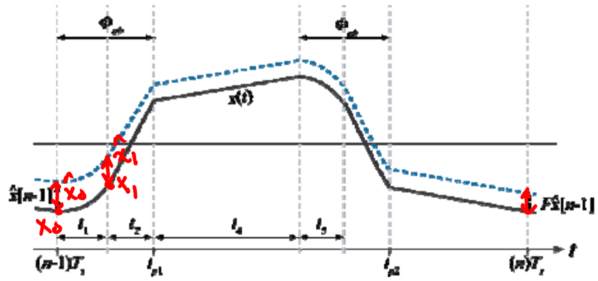
$$\cancel{X_1} + \hat{x}_1 = e^{A_1 t_1}\left(\cancel{X_0} + \hat{x}_0\right) + \cancel{A_1^{-1}\left(e^{A_1 t_1} - I\right)B_1 V_g}$$

- Small signal portion can be separated out

$$\hat{x}_1 = e^{A_1 t_1}\hat{x}_0$$

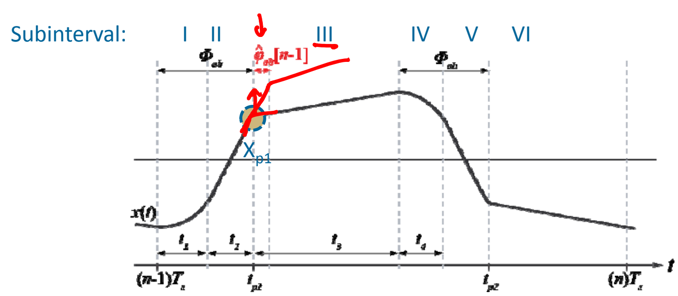
- And applied for each subinterval to obtain natural response

$$F = \underline{e^{A_6 t_6} e^{A_5 t_5} e^{A_4 t_4} e^{A_3 t_3} e^{A_2 t_2} e^{A_1 t_1}}$$



Small Signal Response to Phase Perturbation

- For the forced response, consider the effect of an increase in phase shift interval of magnitude $\hat{\varphi}_{ab}$ around steady-state vector X_{p1}

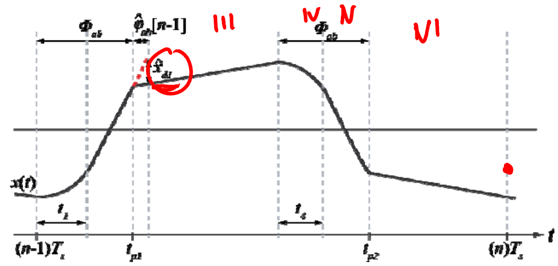


Small Signal Response to Phase Perturbation

- Perturbation in phase shift $\hat{\varphi}_{ab}$ results in a state perturbation $\hat{x}_{d1}$
- In accordance with small signal modeling assumptions, affect on phase perturbations on state assumed to be instantaneous

$$\hat{x}_{d1} = (A_2 - A_3)X_{p1}\frac{T_s}{2\pi}\hat{\varphi}_{ab}$$

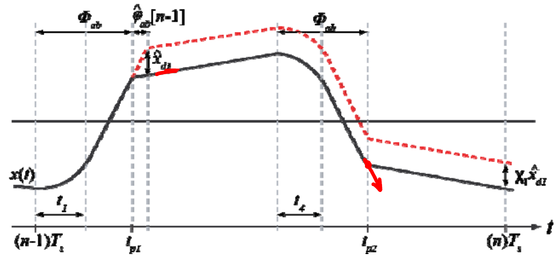
Subinterval: ~~I II~~ IV V VI VIII



Small Signal Response to Phase Perturbation

- Perturbation in phase shift $\hat{\varphi}_{ab}$ results in a state perturbation $\hat{x}_{d1}$
- In accordance with small signal modeling assumptions, affect on phase perturbations on state assumed to be instantaneous
- State perturbation is then similarly propagated through state progression to next sampling interval

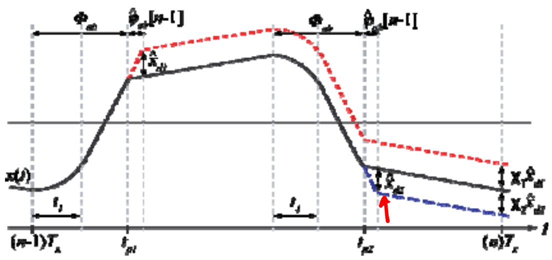
$$\chi_1\hat{x}_{d1} = \underline{e^{A_6 t_6} e^{A_5 t_5} e^{A_4 t_4}}\overbrace{(A_2 - A_3)X_{p1}\frac{T_s}{2\pi}\hat{\varphi}_{ab}}^{\hat{x}_{d1}}$$



Small Signal Response to Phase Perturbation

- Perturbation in phase shift $\hat{\varphi}_{ab}$ results in a state perturbation $\hat{x}_{d1}$
- In accordance with small signal modeling assumptions, affect on phase perturbations on state assumed to be instantaneous
- Perturbations propagated to output for two independent changes in state at next sampling instant

$$G = e^{A_6 t_6} e^{A_5 t_5} e^{A_4 t_4} (A_2 - A_3) X_{p1} \frac{T_s}{2\pi} + e^{A_6 t_6} (A_5 - A_6) X_{p2} \frac{T_s}{2\pi}$$



Complete Discrete Time Model

- Combining forced and natural responses into

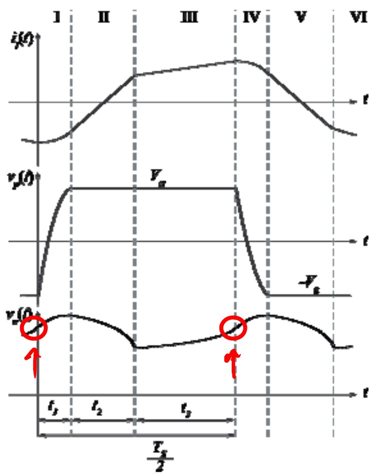
$$\hat{x}[n] = F\hat{x}[n-1] + G\hat{\varphi}_{ab}[n-1]$$

- Full model is obtained

$$\hat{x}[n] = \left(e^{A_6 t_6} e^{A_5 t_5} e^{A_4 t_4} e^{A_3 t_3} e^{A_2 t_2} e^{A_1 t_1}\right)\hat{x}[n-1] + \ldots$$
$$\left(e^{A_6 t_6} e^{A_5 t_5} e^{A_4 t_4} (A_2 - A_3) X_{p1} \frac{T_s}{2\pi} + e^{A_6 t_6} (A_5 - A_6) X_{p2} \frac{T_s}{2\pi}\right)\hat{\varphi}_{ab}$$

- Model can be further simplified through recognition of half-cycle symmetry

Half-Cycle Symmetry



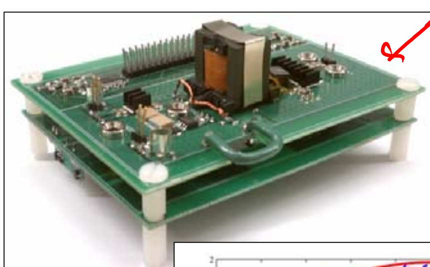
- All states are either symmetric or antisymmetric about the half-period
- Can sample and control at twice the switching frequency:

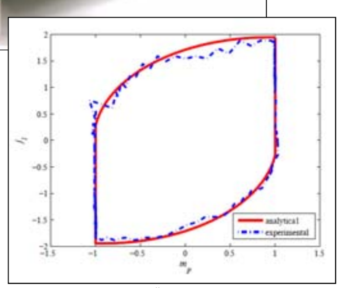
$$x\left(t+\frac{T_s}{2}\right)=\mathrm{I}_{HC}\,x(t)$$

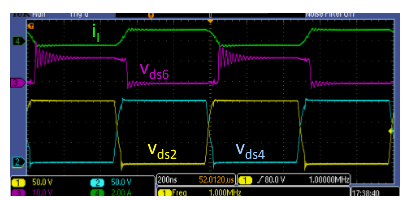
$$\mathrm{I}_{HC}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

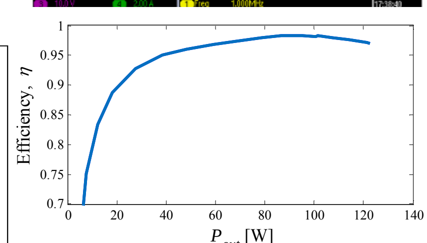
$$\hat{x}[n]=\left(e^{A_3t_3}e^{A_2t_2}e^{A_1t_1}I_{HC}\right)\hat{x}[n-1]+\left(e^{A_3t_3}\left(A_2-A_3\right)X_{p1}\frac{T_s}{2\pi}\right)\hat{\varphi}_{ab}[n-1]$$

DAB: Experimental Results







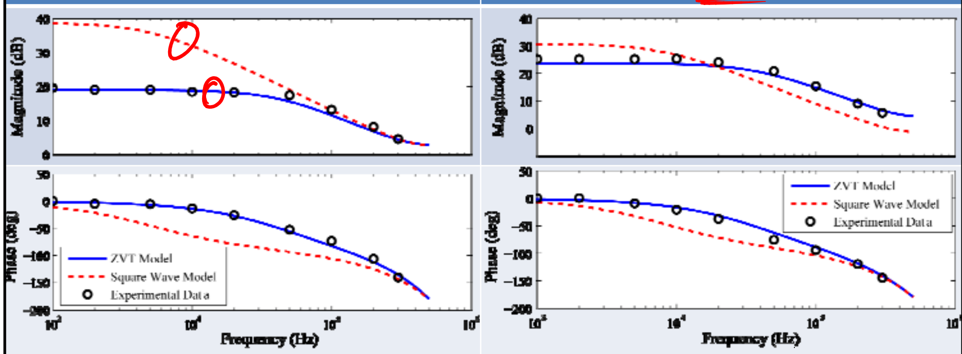


D. Costinett, H. Nguyen, R. Zane, and D. Maksimovic, "GaN-FET based dual active bridge DC-DC converter," in Proc. Appl. Power Electron. Conf. (APEC), march 2011, pp. 1425 –1432.

D. Costinett, D. Maksimovic, and R. Zane, "Design and control for high efficiency in high step-down dual active bridge converters operating at high switching frequency," IEEE Trans. Power Electron., vol. PP, no. 99, p. 1, 2012.

Frequency Response Comparison

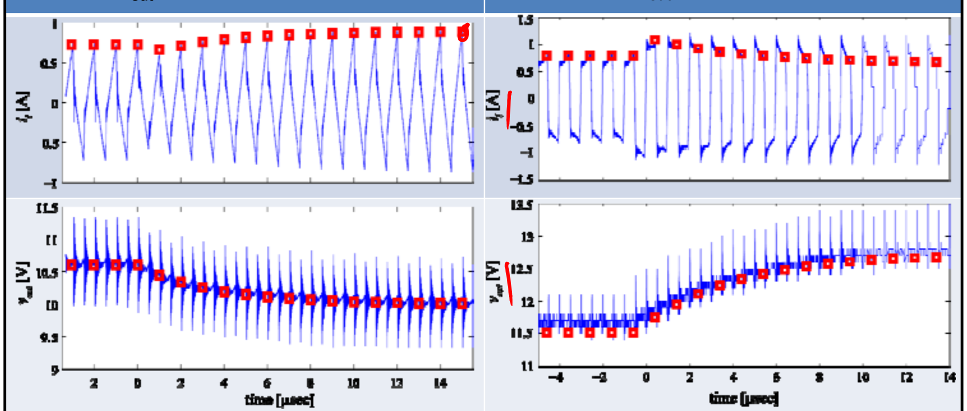
P_{out} = 30 W	P_{out} = 90 W



- Bode plot compared to traditional averaged model (neglects ZVS dynamics
- Experimental results verify very little change in dynamics other than DC gain

Transient Response Comparison

P_{out} = 30 W	P_{out} = 90 W



- Response in current and voltage shown at 30 W (left) and 90 W (right) for step changes in φ_{ab}
- Both well matched to experimental results

Additional Readings

[1] D. J. Packard, "Discrete modeling and analysis of switching regulators," Ph.D. dissertation, California Institute of Technology, Nov. 1976.

[2] A. R. Brown and R. D. Middlebrook, "Sampled-data modeling of switching regulators," p. 349 369, 1984.

[3] D. Maksimovic and R. Zane, "Small-signal discrete-time modeling of digitally controlled PWM converters," IEEE Trans. Power Electron., vol. 22, no. 6, pp. 2552 –2556, nov. 2007.

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ECE 525 Alternative Energy Sources

ECE 626 Solid State Power Semiconductors

ECE 581 High Frequency Power Electronics

ECE 691/2 Advanced Graduate Seminar 2

Thank you for all your hard work, and good luck with finals!