Neglecting the switching ripple

Suppose the duty cycle is modulated sinusoidally:

\[ d(t) = D + D_m \cos \omega_m t \]

where \( D \) and \( D_m \) are constants, \( |D_m| < D \), and the modulation frequency \( \omega_m \) is much smaller than the converter switching frequency \( \omega_s = 2\pi f_s \).

The resulting variations in transistor gate drive signal and converter output voltage:

![Diagram of gate drive signal and averaged waveform]

Output voltage spectrum with sinusoidal modulation of duty cycle

Contains frequency components at:
- Modulation frequency and its harmonics
- Switching frequency and its harmonics
- Sidebands of switching frequency

With small switching ripple, high-frequency components (switching harmonics and sidebands) are small.

If ripple is neglected, then only low-frequency components (modulation frequency and harmonics) remain.
Sampling

\[ v(t) \rightarrow \text{Sampler} \rightarrow v^*(t) \]

\[ v(t) \]

\[ v^*(t) \]

\[ \rightarrow v^*(t) = v(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT) \]

Unit impulse (Dirac)

Sampled-data system example: frequency domain

\[ v(t) \rightarrow \text{Sampler} \rightarrow v^*(t) \rightarrow \text{Zero-order hold} \rightarrow v_o(t) \]

\[ H = \frac{1 - e^{-sT}}{s} \]

\[ v^*(s) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} v(s - jk\omega_c) \]

\[ v_o(s) = \frac{1 - e^{-sT}}{s} v^*(s) = \frac{1 - e^{-sT}}{sT} \sum_{k=-\infty}^{+\infty} v(s - jk\omega_c) \]

Consider only low-frequency signals: \[ v_o(s) \approx \frac{1 - e^{-sT}}{sT} v(s) \]

System “transfer function” \[ \frac{v_o}{v} = \frac{1 - e^{-sT}}{sT} \]
Zero-order hold: frequency responses

Zero-Order Hold magnitude and phase responses

\[ H / T = \frac{1-e^{-sT}}{sT} \]

How does any of this apply to converter modeling?
PWM is a small-signal sampler!

PWM sampling occurs at $t_p$ (i.e., at $dT_s$, periodically, in each switching period)

General sampled-data model

- Sampled-data model valid at all frequencies
- *Equivalent hold* describes the converter small-signal response to the sampled duty-cycle perturbations [Billy Lau, PESC 1986]
- State-space averaging or averaged-switch models are low-frequency continuous-time approximations to this sampled-data model
Application to DCM high-frequency modeling

\[ i_L \]

\[ c \]

\[ dT_s \quad d_2T_s \]

\[ \frac{T_s}{T_s} \]
DCM inductor current high-frequency response

\[ i_l(s) = \frac{V_1 + V_2}{L} T_s \frac{1-e^{-sT_s}}{s} \hat{d}(s) = \frac{V_1 + V_2}{L} T_s \frac{1-e^{-sT_s}}{s} \sum_{k=\pm\infty} \hat{d}(s-jk\omega) \]

\[ \hat{i}_l(s) \approx \frac{V_1 + V_2}{L} D_2 T_s \frac{1-e^{-sT_s}}{D_2 T_s s} \hat{d}(s) \]

\[ \hat{i}_l(s) \approx \frac{V_1 + V_2}{L} D_2 T_s \frac{1}{1+s/\omega_2} \]

\[ \omega_2 = \frac{2}{D_2 T_s} \]

\[ f_2 = \frac{f_s}{\pi D_2} \]

High-frequency pole due to the inductor current dynamics in DCM, see (11.77) in Section 11.3

Example DAB Application

- Goal: Include effects of both output sampling and PWM sampling in converter model
- Considering only control perturbations, model will take the form
  \[ \ddot{x}[n] = F\ddot{x}[n-1] + G\phi_{in}[n-1] \]
Example DAB Application

- Goal is to develop a model of the form
  \[ \dot{x}[n] = F[n-1] + G\hat{\phi}_{n-1} \]
- State vector is sampled and control updated at \( t = nT_s \)
  \[ x = \begin{bmatrix} v_p & i & v_{out} \end{bmatrix}^T \]

Continuous Time State Space

\[ \dot{x}(t) = A_x x(t) + B_i V_g \]
Continuous Time State Representation

\[ \dot{x}(t) = Ax(t) + BV_g \]

• Models change slightly depending on operating mode
• In all cases, consider dynamics at boundary just below ZVS

State Propagation

• Within each subinterval, states propagate according to continuous time model

\[ \dot{x}(t) = Ax(t) + BV_g \]

• Solution given by

\[ x(t) = e^{At}x_0 + \int_0^t e^{-A(t-s)} (Bv_g(s)) \, ds. \]

• If \( v_g(t) \approx V_g \) within one switching period,

\[ x_i(t_i) = e^{A_{i+1}t_i}x_0 + A_i^{-1}(e^{A_{i+1}t_i} - I)B_iV_g \]

Subinterval:

- I
- II
- III
- IV
- V
- VI
Small Signal State Propagation

- Considering steady state plus some small signal, solution becomes
  \[ \dot{X} + \dot{X}_0 = e^{A_1 t} (X_0 + \dot{X}_0) + A_1^{-1} (e^{A_1 t} - 1) B V_g \]
- Small signal portion can be separated out
  \[ \dot{X}_1 = e^{A_1 t} \dot{X}_0 \]
- And applied for each subinterval to obtain natural response
  \[ F = e^{A_1 t} e^{A_1 t} e^{A_1 t} e^{A_1 t} e^{A_1 t} \]

Small Signal Response to Phase Perturbation

- For the forced response, consider the effect of an increase in phase shift interval of magnitude \( \phi_{ab} \) around steady-state vector \( X_{p1} \)
Small Signal Response to Phase Perturbation

- Perturbation in phase shift $\phi_{ab}$ results in a state perturbation $\hat{x}_{d1}$
- In accordance with small signal modeling assumptions, affect on phase perturbations on state assumed to be instantaneous

$$\hat{x}_{d1} = (A_2 - A_3)X_p I_1 \frac{T_x}{2\pi} \hat{\phi}_{ab}$$

Subinterval:

- $\Phi_{ab}$
- $\Phi_{ab[n-1]}$
- $\Phi_a$
- $t_1$
- $t_m$
- $t_n$
- $T_x$

Small Signal Response to Phase Perturbation

- Perturbation in phase shift $\phi_{ab}$ results in a state perturbation $\hat{x}_{d1}$
- In accordance with small signal modeling assumptions, affect on phase perturbations on state assumed to be instantaneous
- State perturbation is then similarly propagated through state progression to next sampling interval

$$x_{1t_1} = e^{A_1} x_0 e^{A_1 I_1} e^{A_1 t_1} (A_2 - A_3)X_p I_1 \frac{T_x}{2\pi} \hat{\phi}_{ab}$$

Subinterval: 

- $\Phi_{ab}$
- $\Phi_{ab[n-1]}$
- $\Phi_a$
- $t_1$
- $t_m$
- $t_n$
- $T_x$
Small Signal Response to Phase Perturbation

- Perturbation in phase shift $\hat{\phi}_{ab}$ results in a state perturbation $\tilde{x}_{d1}$
- In accordance with small signal modeling assumptions, affect on phase perturbations on state assumed to be instantaneous
- Perturbations propagated to output for two independent changes in state at next sampling instant

$$G = e^{A_d t_1} e^{A_s t_2} \left( A_2 - A_1 \right) X_p + e^{A_d t_1} \left( A_5 - A_6 \right) X_p$$

Complete Discrete Time Model

- Combining forced and natural responses into
- Full model is obtained

$$\tilde{x}[n] = F\tilde{x}[n-1] + G\hat{\phi}_{ab}[n-1]$$

$$\tilde{x}[n] = \left( e^{A_d t_1} e^{A_s t_2} e^{A_d t_3} e^{A_s t_4} e^{A_d t_5} \right) \tilde{x}[n-1] + \left( e^{A_d t_1} e^{A_s t_2} \left( A_2 - A_1 \right) X_p + e^{A_d t_1} \left( A_5 - A_6 \right) X_p \right) \hat{\phi}_{ab}$$

- Model can be further simplified through recognition of half-cycle symmetry
Half-Cycle Symmetry

- All states are either symmetric or antisymmetric about the half-period
- Can sample and control at twice the switching frequency:

$$x(t + \frac{T_s}{2}) = I_{HC}x(t)$$

$$I_{HC} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{x}[n] = (e^{A_{I2}}e^{A_{I3}}e^{A_{IHC}})[n-1] + \left(e^{A_{I2}}(A_2 - A_3)X_{p1} \frac{T_s}{2\pi}\right)\hat{\phi}_{ab}[n-1]$$

DAB: Experimental Results


Frequency Response Comparison

- Bode plot compared to traditional averaged model (neglects ZVS dynamics)
- Experimental results verify very little change in dynamics other than DC gain

Transient Response Comparison

- Response in current and voltage shown at 30 W (left) and 90 W (right) for step changes in $\phi_{ab}$
- Both well matched to experimental results
Additional Readings


Power Electronics Courses at UTK

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Thank you for all your hard work, and good luck with finals!