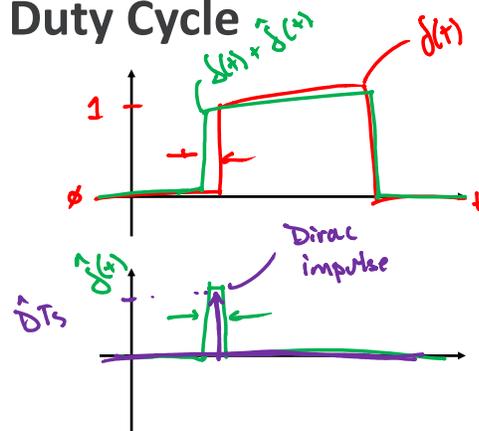


Tiny Box Challenge

- Interface Board Components:
 - Blue bag:
 - 0.1uF Capacitors
 - Adjustable LDO
 - Silver bag:
 - Everything else
- FPGA code updated on website
 - Pin assignments modified in .ucf file to match altium layout

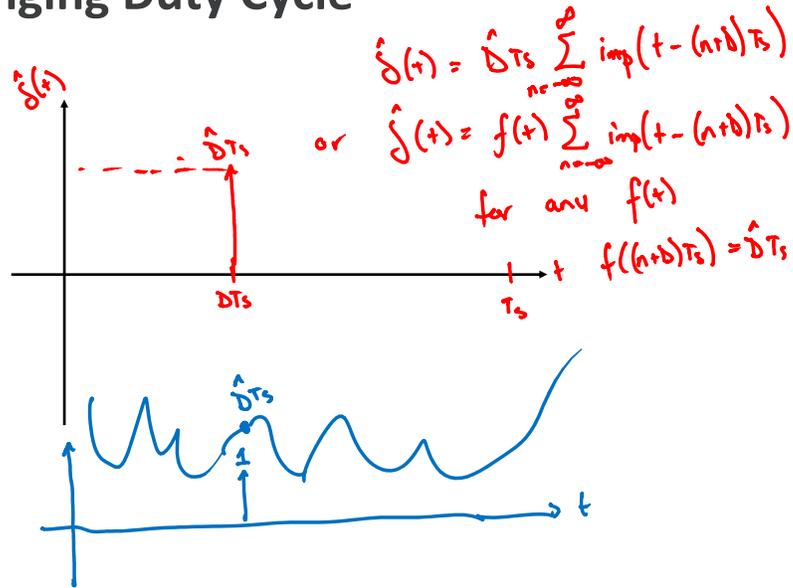
in src folder

Effect of Changing Duty Cycle



*Note: Dirac impulse usually called $\delta(t)$
we'll call it $imp(t)$ to be
different from $\delta(t)$ already used*

Changing Duty Cycle



Averaged Model

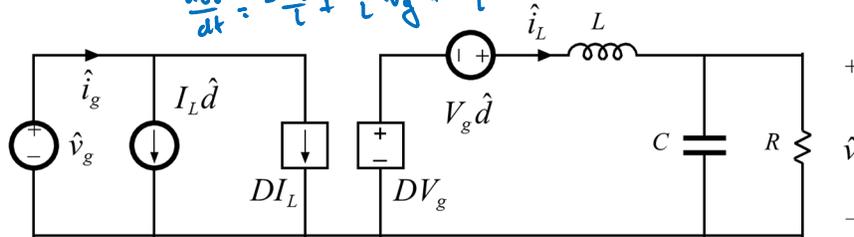
$$\langle \hat{x} \rangle_{T_s} = [DA_1 + D'A_2] \hat{x} + [DB_1 + D'B_2] \hat{u} + [(A_1 - A_2)\hat{x} + (B_1 - B_2)u] \langle \hat{d}(t) \rangle_{T_s}$$

for Buck: $A = \begin{bmatrix} -1/Rc & 1/c \\ -1/L & 0 \end{bmatrix}$

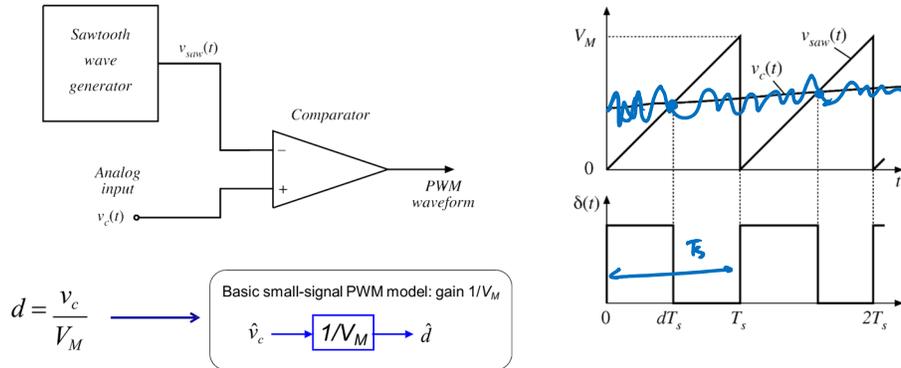
$$\langle \hat{x} \rangle_{T_s} = [A] \hat{x} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} \hat{u} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_g \hat{d}$$

$$\frac{d\hat{u}}{dt} = -\frac{\hat{u}}{RC} + \frac{\hat{i}_L}{C}$$

$$\frac{d\hat{i}_L}{dt} = -\frac{\hat{i}_L}{L} + \frac{D}{L} \hat{u} + \frac{V_g}{L} \hat{d}$$

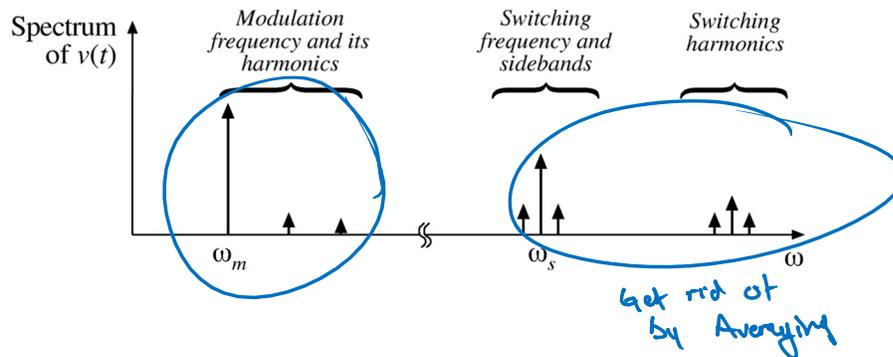


Another View: PWM Modulator



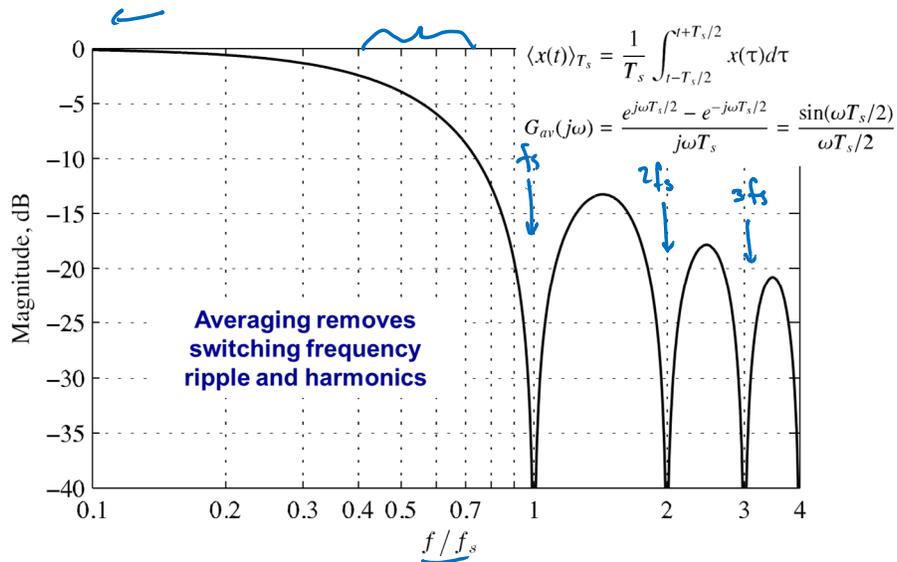
→ Every switching converter has sampling behavior!

Average Modeling



$$\langle x(t) \rangle_{T_s} = \frac{1}{T_s} \int_{t-T_s/2}^{t+T_s/2} x(\tau) d\tau$$

Averaging: Frequency Domain View



D. Maksimovic, "Analysis, Modeling and Design of Digitally Controlled Switched-Mode Power Converters", IEEE COMPEL 2013



State Space Solution

$\dot{x} = Ax + Bu$
 Multiply both sides by e^{-At} ← matrix exponential
 $e^{-At} \dot{x} - e^{-At} Ax = e^{-At} Bu$
 $y = e^{-At}$
 $\dot{y} = -e^{-At} A$
 looks like $y\dot{x} + \dot{y}x = \frac{d}{dt}(yx)$
 $\frac{d}{dt}(e^{-At} x) = e^{-At} Bu \rightarrow$ integrate

A. R. Brown and R. D. Middlebrook, "Sampled-Data Modeling of Switching Regulators", 1981



$$\int_0^t \frac{d}{d\tau} (e^{-A\tau} x(\tau)) d\tau = \int_0^t e^{-A\tau} B u(\tau) d\tau$$

$$e^{-At} x(t) - x(0) = \int_0^t e^{-A\tau} B u(\tau) d\tau$$

$$x(t) = e^{At} x(0) + e^{At} \int_0^t e^{-A\tau} B u(\tau) d\tau$$

$$x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$$

if $u(t) \approx \text{constant}$ over one switching interval

$$\rightarrow x(t) = e^{At} x(0) + A^{-1} (e^{At} - I) B u$$

$(e^{At})^{-1} = e^{-At}$

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The Matrix Exponential

$$e^{At} = \sum_{k=0}^{\infty} \frac{(At)^k}{k!} = \underbrace{I + At + \frac{(At)^2}{2} + \dots}$$

matrix

in matlab:
expm(A) → matrix exponential

X $\exp(A) = \text{element-wise exponential}$

$$\begin{bmatrix} e^{A_{1,1}} & e^{A_{1,2}} \\ e^{A_{2,1}} & \dots \end{bmatrix}$$

Fig. 1 The "hump".

C. Moler and C. V. Loan, "Nineteen Dubious Ways to Compute the Exponential of a Matrix, Twenty-Five Years Later"

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