Changing Duty Cycle

Another View: PWM Modulator

\[ d = \frac{v_c}{V_M} \]

Basic small-signal PWM model: gain \(1/V_M\)

\[ \dot{v}_c \rightarrow \frac{1}{V_M} \rightarrow d \]
Average Modeling

\[ \langle x(t) \rangle_{T_s} = \frac{1}{T_s} \int_{t-T_s/2}^{t+T_s/2} x(\tau) d\tau \]

Averaging: Frequency Domain View

\[ G_{av}(j\omega) = \frac{e^{j\omega T_s/2} - e^{-j\omega T_s/2}}{j\omega T_s} = \frac{\sin(\omega T_s/2)}{\omega T_s/2} \]

Averaging removes switching frequency ripple and harmonics
State Space Solution

The Matrix Exponential

Application to Switching Systems

C. Moler and C. V. Loan, "Nineteen Dubious Ways to Compute the Exponential of a Matrix, Twenty-Five Years Later"
Discrete Time Model