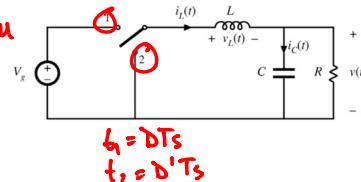


## Application to Switching Systems

for any subinterval

$$x(t) = e^{At} x(0) + A^{-1}(e^{At} - I) Bu$$

$$\xrightarrow{\substack{1 \\ x(0) \\ (A, B_1)}} \xrightarrow{\substack{2 \\ x(\Delta t_1) \\ (A_1, B_1)}} \xrightarrow{\substack{1 \\ x(\Delta t_1) \\ (A_1, B_1)}} \xrightarrow{\substack{2 \\ x(T_s) \\ (A_2, B_2)}}$$



$$t_1 = \Delta t_1$$

$$t_2 = \Delta t_2$$

$$\textcircled{1} \quad x(\Delta t_1) = e^{A_1 t_1} x(0) + A_1^{-1}(e^{A_1 t_1} - I) B_1 u$$

$$\textcircled{2} \quad x(T_s) = e^{A_2 t_2} x(\Delta t_1) + A_2^{-1}(e^{A_2 t_2} - I) B_2 u$$

combining:

$$x(T_s) = e^{A_2 t_2} \left[ e^{A_1 t_1} x(0) + A_1^{-1}(e^{A_1 t_1} - I) B_1 u \right] + A_2^{-1}(e^{A_2 t_2} - I) B_2 u$$

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## Steady-State Model

$$x(T_s) = e^{A_2 t_2} e^{A_1 t_1} x(0) + e^{A_2 t_2} A_1^{-1}(e^{A_1 t_1} - I) B_1 u + A_2^{-1}(e^{A_2 t_2} - I) B_2 u$$

for  $n$ -subintervals:

$$x(T_s) = \left( \prod_{i=1}^n e^{A_i t_i} \right) x(0) + \sum_{l=1}^n \left( \prod_{k=i+1}^n e^{A_k t_k} \right) A_i^{-1}(e^{A_i t_i} - I) B_i u$$

$$\text{in steady-state } x(T_s) = x(0)$$

$$x(0) = x(T_s) = \left( I - \prod_{i=1}^n e^{A_i t_i} \right)^{-1} \sum_{i=1}^n \left( \prod_{k=i+1}^n e^{A_k t_k} \right) A_i^{-1}(e^{A_i t_i} - I) B_i u$$

Note: Does not inherently account for uncontrolled (e.g. Diode turn-off in DCM) switching

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## Discrete Time Model

In one subinterval:

$$x(t) = e^{At} x(0) + A^{-1}(e^{At} - I) B u$$

$$x(t) + \hat{x}(t) = e^{At} (x(0) + \hat{x}(0)) + A^{-1}(e^{At} - I) B u$$

Neglect line-to  
output for  
now

$\hat{x}(t) = e^{At} \hat{x}(0)$

Small  
signal  
only

for n subintervals

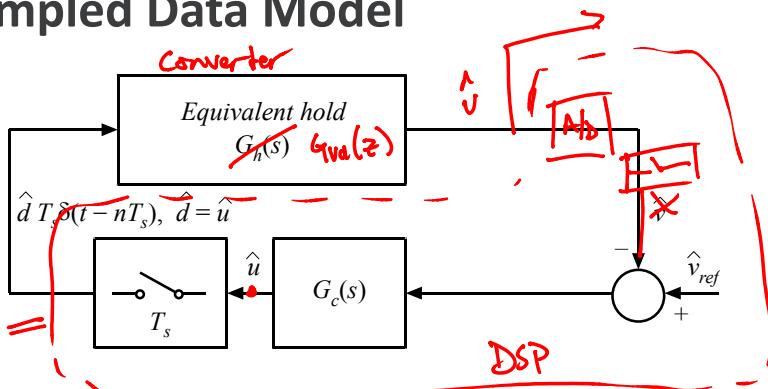
$$\hat{x}(T_s) = \left( \prod_{i=1}^n e^{A t_i} \right) \hat{x}(0)$$

$$\hat{x}[n] = \Phi \hat{x}[n-1], \quad \Phi = \prod_{i=1}^n e^{A t_i}$$

D. J. Packard, "Discrete modeling and analysis of switching regulators," Ph.D. dissertation, California Institute of Technology, Nov. 1976.

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## Sampled Data Model



- Sampled-data model valid at all frequencies
- Equivalent hold describes the converter small-signal response to the sampled duty-cycle perturbations [Billy Lau, PESC 1986]
- State-space averaging or averaged-switch models are low-frequency continuous-time approximations to this sampled-data model

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