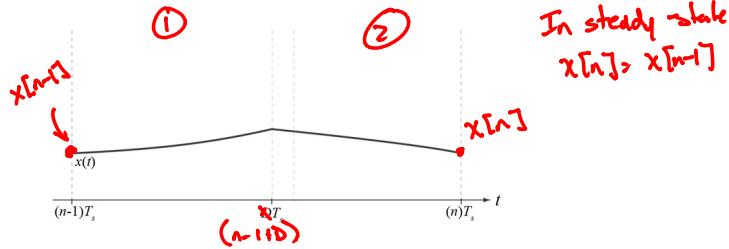


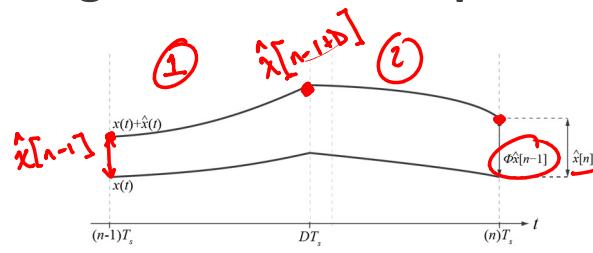
## Small Signal: Natural Response



D. Maksimovic and R. Zane, "Small-signal discrete-time modeling of digitally controlled PWM converters," IEEE Trans. Power Electron., vol. 22, no. 6, pp. 2552 –2556, nov. 2007.



## Small Signal: Natural Response



$$\hat{x}[n] = e^{A_1 t_1} \underbrace{e^{A_1 t_2} \hat{x}[n-1]}_{\hat{x}[n-1]}$$

$$t_1 = D\bar{T}_s$$

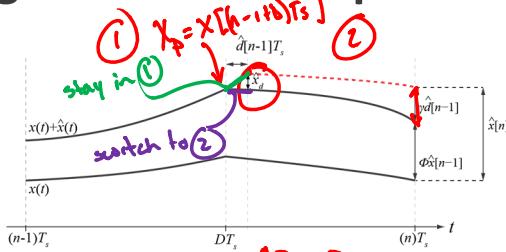
$$t_2 = D^T T_s$$

$$\hat{x}[n] = \Phi \hat{x}[n-1]$$

$$\Phi = e^{A_2 t_2} e^{A_1 t_1}$$



## Small Signal: Forced Response



$$\hat{x}[n] = \Phi \hat{x}[n-1] + \gamma \hat{d}[n-1]$$

$$\gamma \hat{d}[n-1] = e^{A_1 \hat{d} T_s} \hat{x}_d$$

$$\begin{aligned} \hat{x}_d = & \left[ e^{A_1 \hat{d} T_s} X_p + A_1^{-1} (e^{A_1 \hat{d} T_s} - I) B_1 u \right] - \\ & \left[ e^{A_2 \hat{d} T_s} X_p + A_2^{-1} (e^{A_2 \hat{d} T_s} - I) B_2 u \right] \end{aligned}$$

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## Complete Small-Signal DT Model

Approximate  $e^{At} \approx I + At$ , then

$$\begin{aligned} \hat{x}_d = & \left[ (I + A_1 \hat{d} T_s) X_p + A_1^{-1} ((I + A_1 \hat{d} T_s) - I) B_1 u \right] - \\ & \left[ (I + A_2 \hat{d} T_s) X_p + A_2^{-1} ((I + A_2 \hat{d} T_s) - I) B_2 u \right] \end{aligned}$$

$$\hat{x}_d = (A_1 - A_2) X_p \hat{d} T_s + (B_1 - B_2) u \hat{d} T_s$$

$$\gamma \hat{d}[n-1] = e^{A_2 \hat{d} T_s} [ (A_1 - A_2) X_p + (B_1 - B_2) u ] \hat{d}[n-1] T_s$$

$$\hat{x}[n] = \Phi \hat{x}[n-1] + \gamma \hat{d}[n-1]$$

$$\Phi = e^{A_2 \hat{d} T_s} e^{A_1 T_1}$$

$$\gamma = e^{A_2 \hat{d} T_s} [ (A_1 - A_2) X_p + (B_1 - B_2) u ] T_s$$

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## Control-to-Output Transfer Function

- Difference equation  $\rightarrow z$ -transform

$$G_{\text{val}}(z) = C(zI - \Phi)^{-1}y + D$$

$$y = Cx + Du$$

↑      ↑      ↑  
states    inputs

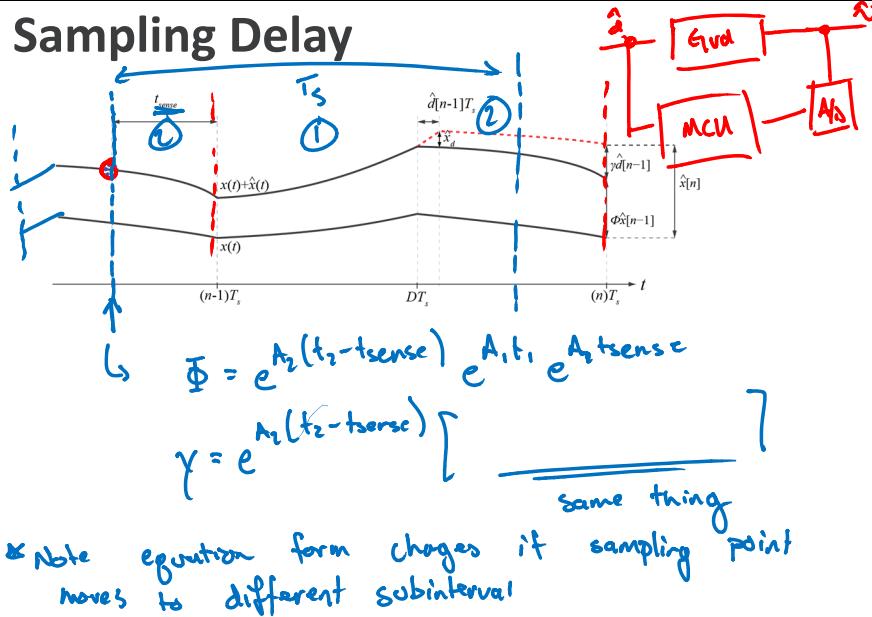
output

for example, Buck w/  $x = \begin{bmatrix} v_c \\ i_i \end{bmatrix}$

$$y = v_c = [1 \ 0] \begin{bmatrix} v_c \\ i_i \end{bmatrix} \rightarrow \begin{array}{l} C = [1 \ 0] \\ D = \emptyset \end{array}$$

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## Sampling Delay

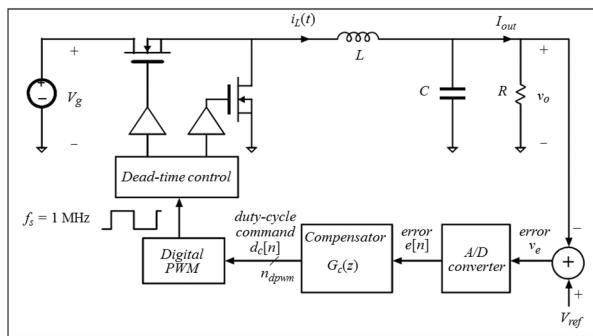


D. Maksimovic and R. Zane, "Small-signal discrete-time modeling of digitally controlled PWM converters," IEEE Trans. Power Electron., vol. 22, no. 6, pp. 2552–2556, nov. 2007.

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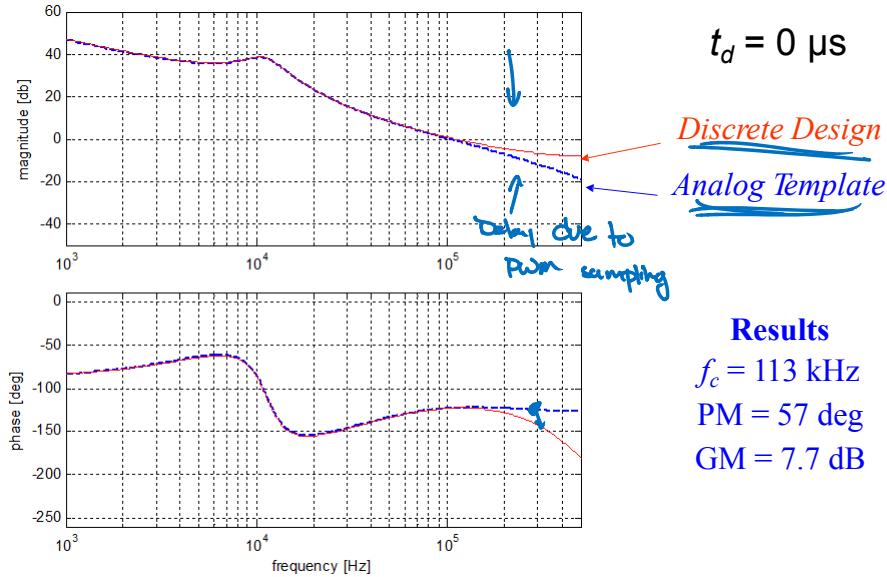
## Example Application



### Power stage parameters

- $f_s = 1/T = 1 \text{ MHz}$
- $V_{ref} = 1.8 \text{ V}$ ,
- $I_{out} = 0 \text{ to } 5 \text{ A}$
- $V_g = 5 \text{ V}$
- $L = 1 \mu\text{H}$
- $R_L = 30 \text{ m}\Omega$
- $C = 200 \mu\text{F}$
- $R_{esr} = 0.8 \text{ m}\Omega$

## TF Comparison With Averaged

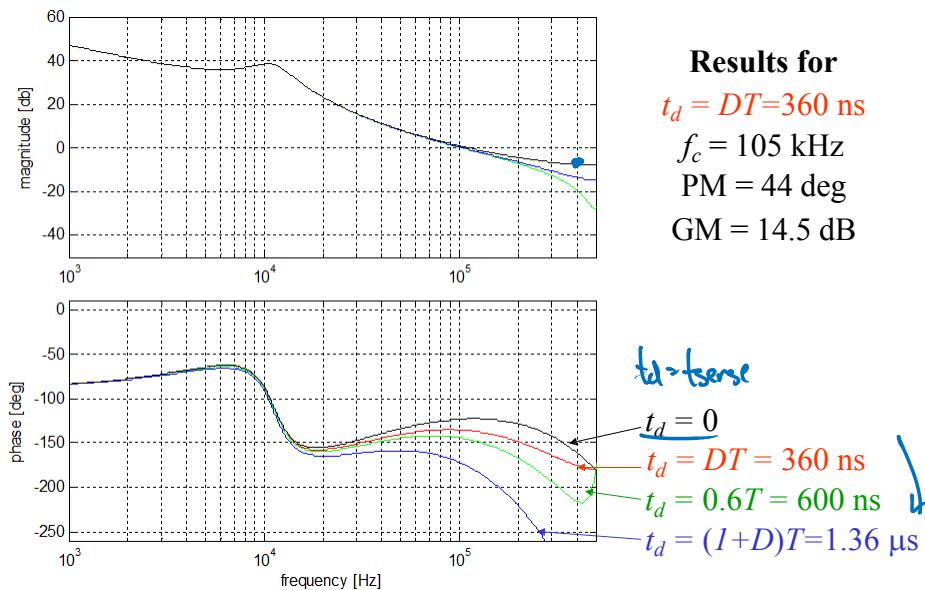


### Results

$f_c = 113 \text{ kHz}$   
 $\text{PM} = 57 \text{ deg}$   
 $\text{GM} = 7.7 \text{ dB}$



## Effect of Sampling Delay



### Results for

$t_d = DT = 360 \text{ ns}$   
 $f_c = 105 \text{ kHz}$   
 $\text{PM} = 44 \text{ deg}$   
 $\text{GM} = 14.5 \text{ dB}$

