\[
\begin{align*}
(m_c - 1)^2 + (j_e - j_{DC})^2 &= (m_0 - 1)^2 + (j_0 - j_{DC})^2 \\
(m_c - 1)^2 + (j_e - j_{DC}) &= (m_0 - 1)^2 + (j_0 - j_{DC})^2 \\
\rightarrow \text{equation for a circle with center } &\mathcal{O} \\
&\text{and a radius } r^2 = (m_0 - 1)^2 + (j_0 - j_{DC})^2 \\
&\text{Normalized for DC condition}
\end{align*}
\]

```
Normalization:
Diff Eq. \rightarrow Geometry/Trig. analysis
```

---

State Plane Analysis

\[
\begin{align*}
\text{State Plane Analysis (SfC)} \\
\text{Diff Eqns: } & \frac{dV_c}{dt} = \frac{V_{in} - V_{oc}}{R_0} \\
& \frac{dI_{in}}{dt} = \frac{I_{in} - I_{oc}}{R_0} \\
\text{DC Equations: } & V_c = V_{oc} \\
& I_{in} = I_{oc}
\end{align*}
\]


\[ i(t) = \frac{V_{base}}{R_0} \]

Algorithm Summary:

Start: Resonance between \( L \) & \( C \) & (possibly) DC sources

1. Normalize: select \( V_{base} \): anything (but some choices better than others)
   \[ i(t) = \frac{V_{base}}{R_0} \]

2. Plot & examine \( i-t \) plane
   - Resonance forms a circle
   - Center is DC solution
   - Initial Conditions from starting point
   - Direction determined by \( iC \) on 1st derivative of states

3. Solve for parameters of interest using geometry/trig

4. Denormalize
DCM Buck Converter Example

\[ V_{\text{in}}(t) = \begin{cases} V_1 & 0 \leq t < T_1 \\ V_2 & T_1 \leq t < 2T_1 \end{cases} \]

\[ i_L(t) = \begin{cases} -I & 0 \leq t < T_1 \\ I & T_1 \leq t < 2T_1 \end{cases} \]

\[ v_C(\phi) = \Phi \]

\[ i_C(\phi) = \Phi \]

\[ \frac{\text{d}i_C}{\text{d}t} \bigg|_{t=0} = \frac{1}{L} \Phi \]

\[ \frac{\text{d}i_L}{\text{d}t} \bigg|_{t=0} = \frac{1}{L} \Phi \]
DCM Buck State Plane

\[ v_{\text{base}} = v \]
\[ i_{\text{base}} = \frac{v}{R} \]

DC solution
\[(V, 0) \rightarrow (M, 0)\]

\[ m_{\text{max}} = 2M \]
\[ n_{\text{so}, \text{max}} = 2V \]