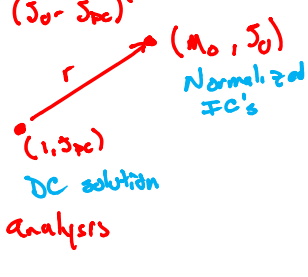
$$(m_c-1)^2 + \left(j_L - \frac{I_{DC}}{I_{base}}\right)^2 = \left(\frac{V_o}{V_{DC}}-1\right)^2 + \left[\frac{Z_o}{V_{DC}}(J_o - J_{DC})\right]^2$$

$$(m_c-1)^2 + (j_L - J_{DC})^2 = (M_o-1)^2 + (J_o - J_{DC})^2$$

→ equation for a circle with center @ $(m_c, j_L) = (1, J_{DC})$

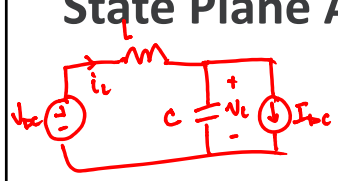
& a radius $r^2 = (M_o-1)^2 + (J_o - J_{DC})^2$



Normalization:

Diff EQs → Geometry / trig analysis

State Plane Analysis



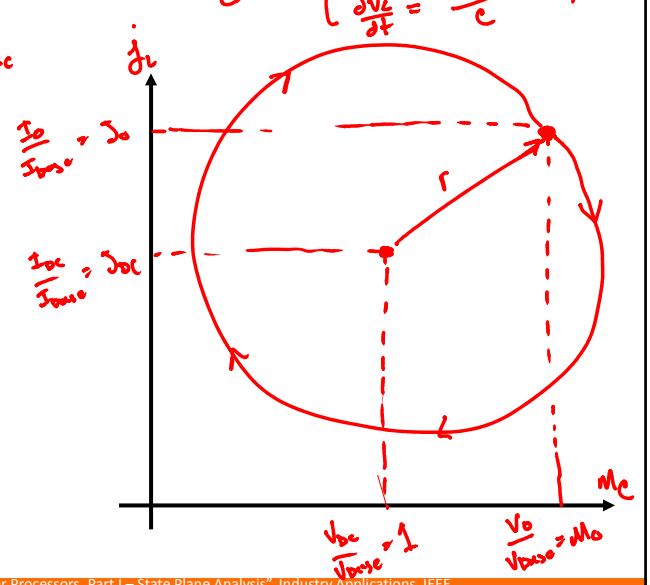
$V_{base} = V_{DC}$

$I_{base} = \frac{V_{DC}}{R_o}$

direction? @ t=0

$$\frac{di_L}{dt} = \frac{V_{DC} - V_L(0)}{L} < 0$$

$$\frac{dV_L}{dt} = \frac{i_L(0) - I_{DC}}{C} > 0$$



[1] R. Oruganti and F. C. Lee, "Resonant Power Processors, Part I – State Plane Analysis", Industry Applications, IEEE Tran. on, vol. 21, no. 6, nov 1985.

[2] D. P. Atherton, Nonlinear Control Engineering. London: Van Nostrand Reinhold, 1982, Ch. 2.

ex

what is $i_{L,pk}$? when does it occur?

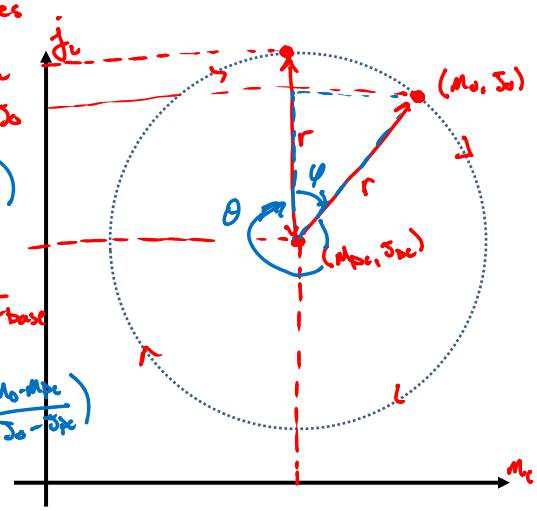
$j_{L,pk} = J_{DC} + r$

$\theta_{imax} = 2\pi - \varphi = 2\pi - \tan^{-1}\left(\frac{m_0 - m_{DC}}{J_0 - J_{DC}}\right)$

Denormalize:

$i_{L,pk} = j_{L,pk} I_{base} = I_{DC} + r I_{base}$

$t_{imax} = \frac{\theta}{\omega_0} = \frac{2\pi}{\omega_0} - \frac{1}{\omega_0}\tan^{-1}\left(\frac{m_0 - m_{DC}}{J_0 - J_{DC}}\right)$



Algorithm Summary:

Start: Resonance between IL & IC & (possibly) DC sources

(1) Normalize: select V_{base}: anything (but some choices better than others)

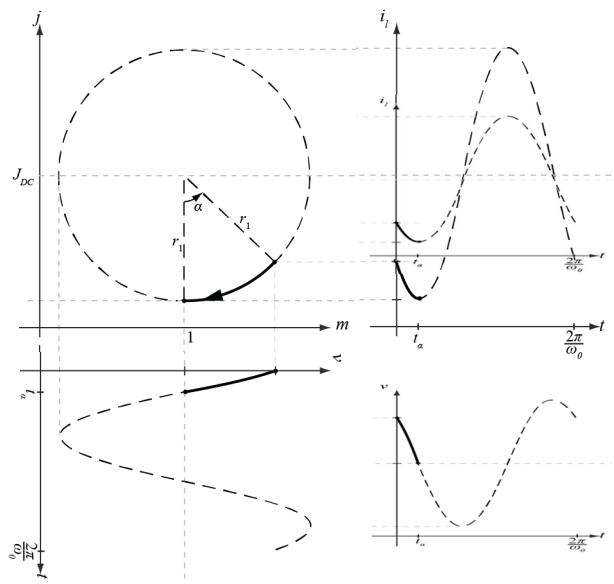
↳ $I_{base} = \frac{V_{base}}{R_0}$

(2) Plot & examine m-j state plane $m = \frac{v}{V_{base}}$ $j = \frac{i}{I_{base}}$ $\theta = \omega_0 t$

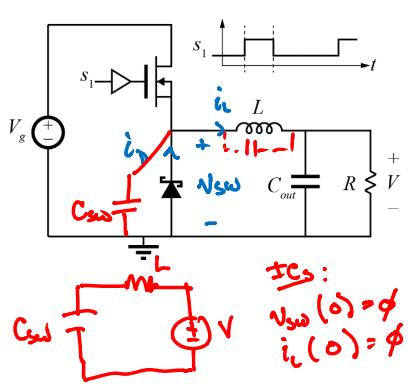
- Resonance forms a circle
- Center is DC solution
- Initial Conditions form starting point
- Direction determined by IC on 1st derivative of states

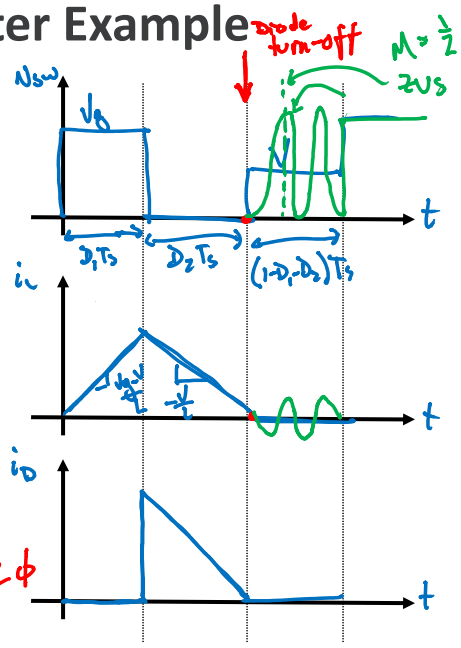
(3) Solve for parameters of interest using geometry/trig

(4) Denormalize



DCM Buck Converter Example





ICs:

$v_{SW}(0) = \phi$

$i_L(0) = \phi$

$\left.\frac{dv_{SW}}{dt}\right|_{t=0} = \phi$

$\left.\frac{di_L}{dt}\right|_{t=0} = \frac{-V}{L} \neq \phi$

DCM Buck State Plane

$V_{base} = V_g$

$I_{base} = \frac{V_g}{R_o}$

DC solution

$(V, 0) \rightarrow (M, 0)$

$M_{max} = 2M$

$\downarrow$

$v_{so,max} = 2V$