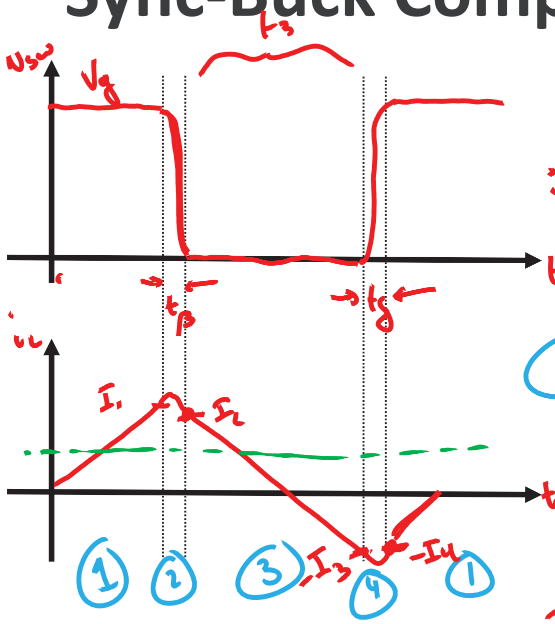
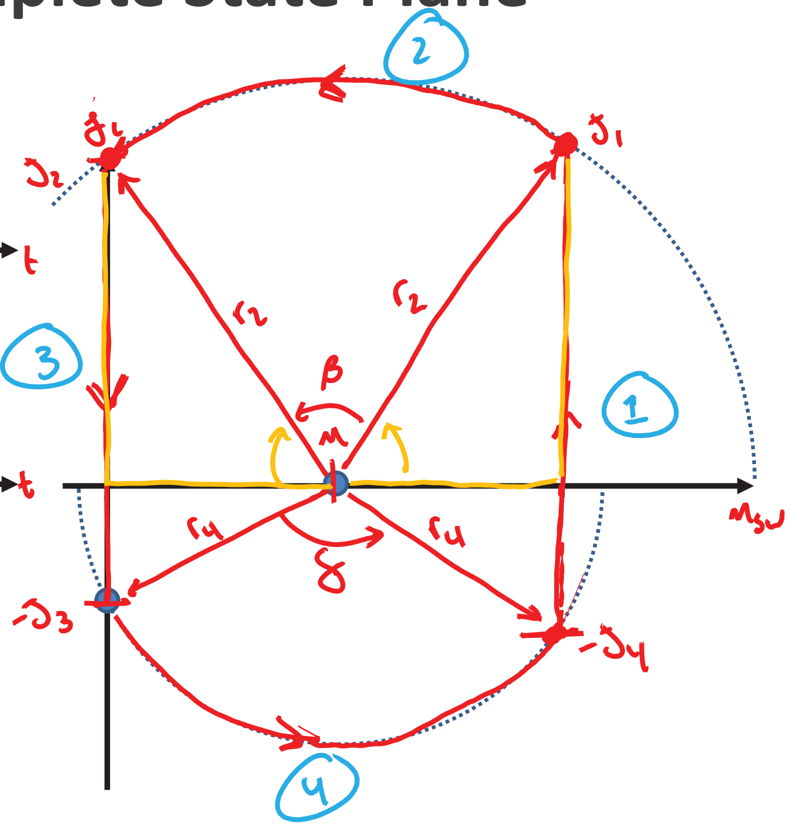
Sync-Buck Complete State Plane





④ ZVS condition:

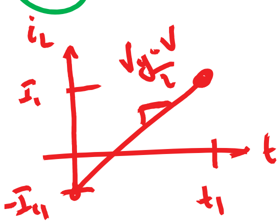
$$\sqrt{J_3^2 + M^2} + M \geq 1$$

② ZVS condition

$$\sqrt{J_2^2 + M^2} \geq M$$

State Plane Solution: Intervals 1 & 2

① Non-resonant → equation comes from time-domain



$$\left(\frac{1}{I_{base}}\right)\frac{V_g - V}{L}\, t_1 = (I_1 + I_4)\left(\frac{1}{I_{base}}\right)$$

$$\frac{V_g - V}{V_g}\,\frac{R_0}{L}\, t_1 = J_1 + J_4$$

$$\boxed{(1-M)\,\theta_1 = J_1 + J_4}$$

$$\frac{R_0}{L} = \frac{\sqrt{L/C_{sw}}}{L} = \frac{1}{\sqrt{LC_{sw}}} = \omega_0$$

② Resonant interval → get equations from state plane

$$\left\{\begin{array}{l} r_2^2 = J_1^2 + (1-M)^2 \\ r_2^2 = J_2^2 + M^2 \end{array}\right\} \qquad \boxed{J_1^2 + (1-M)^2 = J_2^2 + M^2}$$

$$\boxed{\beta = \pi - \tan^{-1}\left(\frac{J_1}{1-M}\right) - \tan^{-1}\left(\frac{J_2}{M}\right)}$$

State Plane Solution: Intervals 3 & 4

3 Non-resonant

$$-\frac{V}{L}t_3 = -(I_2 + I_3) \quad \text{normalize}$$

$$\boxed{M\theta_3 = (J_2 + J_3)}$$

4 Resonant

$$\boxed{J_3^2 + M^2 = J_4^2 + (1-M)^2}$$

$$\boxed{\gamma = \pi - \tan^{-1}\left(\frac{J_3}{M}\right) - \tan^{-1}\left(\frac{J_4}{1-M}\right)}$$

State Plane Solution: Averaging Step

$$I_{out} = \frac{1}{T_s}\int_0^{T_s} i_{out}(\tau)\,d\tau = \frac{1}{T_s}\int_0^{T_s} i_L\,d\tau$$

$$I_{out} = \frac{1}{T_s}\Big[\underbrace{\int_0^{t_1} i_L(\tau)\,d\tau}_{1} + \underbrace{\int_{t_1}^{t_1+t_2} i_L(\tau)\,d\tau}_{2} + \dots\Big]$$

$$I_{out} = \frac{1}{T_s}\Big[\underbrace{\frac{I_1 - I_4}{2}t_1}_{1} + \underbrace{\cancel{q_c}}_{2} + \underbrace{\frac{I_2 - I_3}{2}t_3}_{3} + \underbrace{\cancel{q_c}}_{4}\Big]$$

Multiply both sides by $\left(\frac{\omega_0}{I_{base}}\right)$ $\quad q_c = C_{sw}V_g$

$$\left(\frac{\omega_0}{I_{base}}\right)I_{out} = \left(\frac{\omega_0}{I_{base}}\right)f_s\left[\frac{I_1 - I_4}{2}t_1 + \frac{I_2 - I_3}{2}t_3\right]$$

$$J_{out}\,\omega_0 = f_s\left(\frac{J_1 - J_4}{2}\theta_1 + \frac{J_2 - J_3}{2}\theta_3\right)$$

$$\boxed{J_{out} = \frac{F}{2\pi}\left(\frac{J_1 - J_4}{2}\theta_1 + \frac{J_2 - J_3}{2}\theta_3\right)}, \quad F \equiv \frac{f_s}{f_0}$$

Normalized Period

One complete switching period is comprised of the four subintervals, 1 - 4

$$(\omega_0)\, T_s = (t_1 + t_\beta + t_3 + t_\delta)(\omega_0)$$

$$\frac{\omega_0}{f_s} = \theta_1 + \beta + \theta_3 + \delta$$

$$\boxed{\frac{2\pi}{F} = \theta_1 + \beta + \theta_3 + \delta}$$

Complete Converter Solution

Complete equation set is:

1 $(1-M)\,\theta_1 = J_1 + J_4$

2 $\begin{cases} J_1^2 + (1-M)^2 = J_2^2 + M^2 \\ \beta = \pi - \tan^{-1}\left(\frac{J_1}{1-M}\right) - \tan^{-1}\left(\frac{J_2}{M}\right) \end{cases}$

3 $M\theta_3 = J_2 + J_3$

4 $\begin{cases} J_3^2 + M^2 = J_4^2 + (1-M)^2 \\ \delta = \pi - \tan^{-1}\left(\frac{J_3}{M}\right) - \tan^{-1}\left(\frac{J_4}{1-M}\right) \end{cases}$

Averaging $J_{out} = \frac{F}{2\pi}\left(\frac{J_1 - J_4}{2}\theta_1 + \frac{J_2 - J_3}{2}\theta_3\right)$

Timing $\frac{2\pi}{F} = \theta_1 + \beta + \theta_3 + \delta$

This gives eight total equations, with unknowns

$\underbrace{\theta_1, \beta, \theta_3, \delta, J_1, J_2, J_3, J_4}_{\text{waveform details (8)}}$ and $\underbrace{F, M, J_{out}}_{\text{operating specifications}}$

If we define the operating point and converter (i.e. L, C_{sw}, V_g, V_{out}, P_{out}, f_s) we have a system of eight equations and eight unknowns.

Example Matlab Solution

However, the equation set is transcendental because it posesses both polynomial and trigonometric terms. Generally, we won't be able to solve these in closed form. That is, we can't manipulate these equations into the form

$$\theta_x = f(J_{out}, F, m)$$

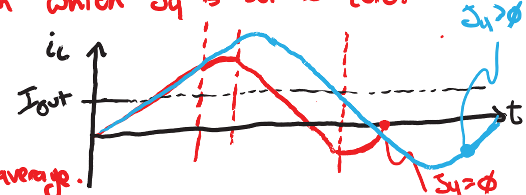
$$J_x = f(J_{out}, F, m)$$

even though it is true that these three variables uniquely determine the converter waveforms.

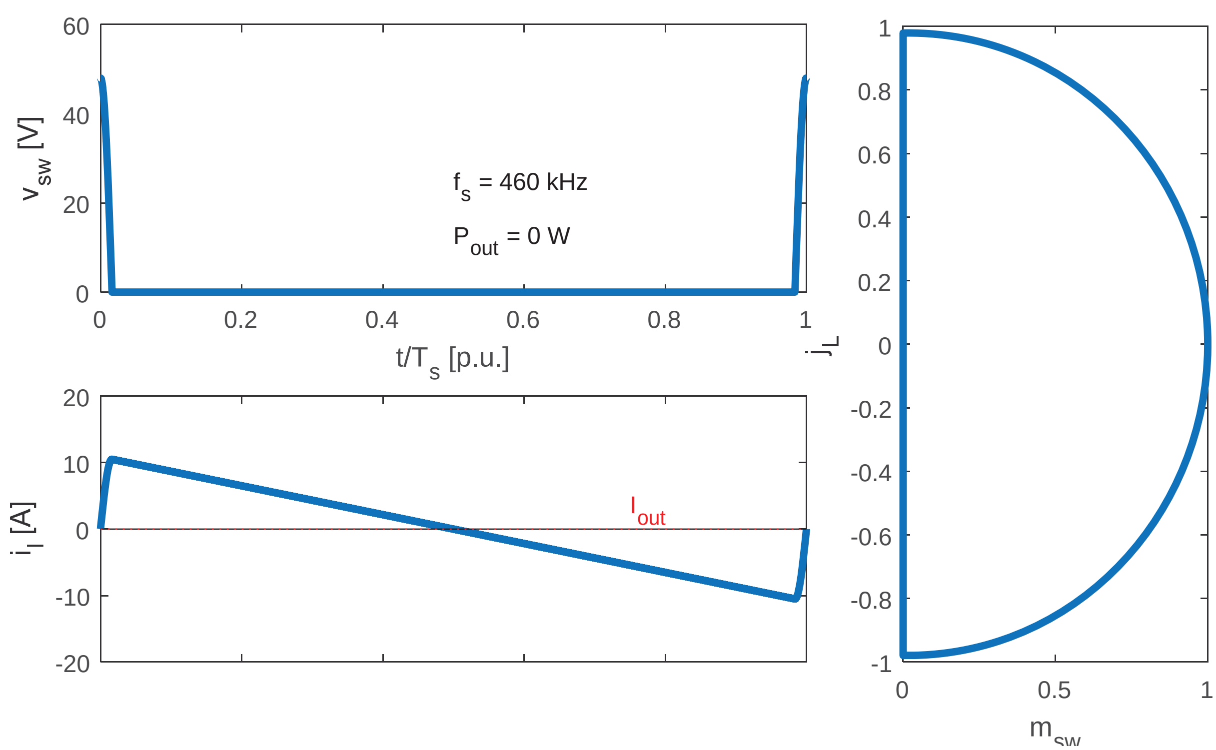
Instead, we can find solutions purely numerically. Depending on the converter, we may be able to do this just by plugging in and evaluating, or we may need to use a numerical solver.

An example where direct evaluation of the equations is possible is posted on the website, with example results on the following slides. This example is for a 48-to-1V synchronous buck converter in which J_y is set to zero.

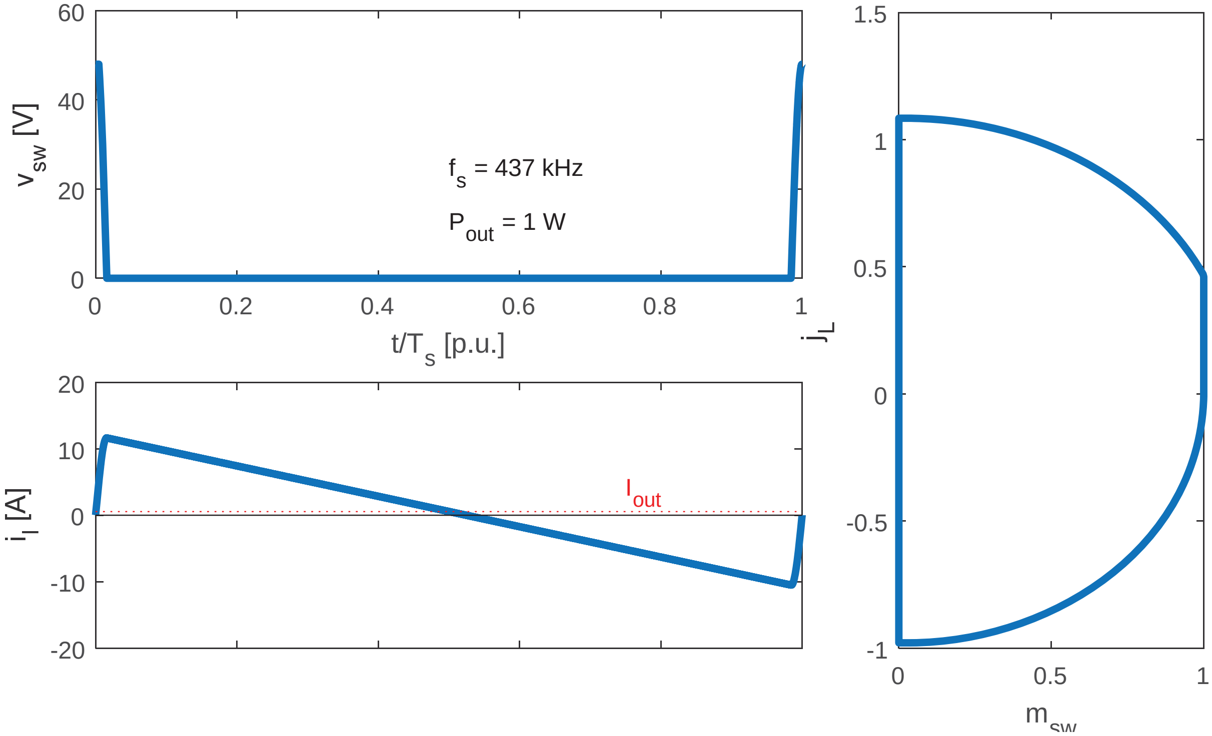
Any waveforms with $J_y > \emptyset$ result in excess conduction loss. $J_y > \emptyset$ means we have more negative current than is necessary for ZVS. This extra negative current means we need a higher peak current to keep the same average.



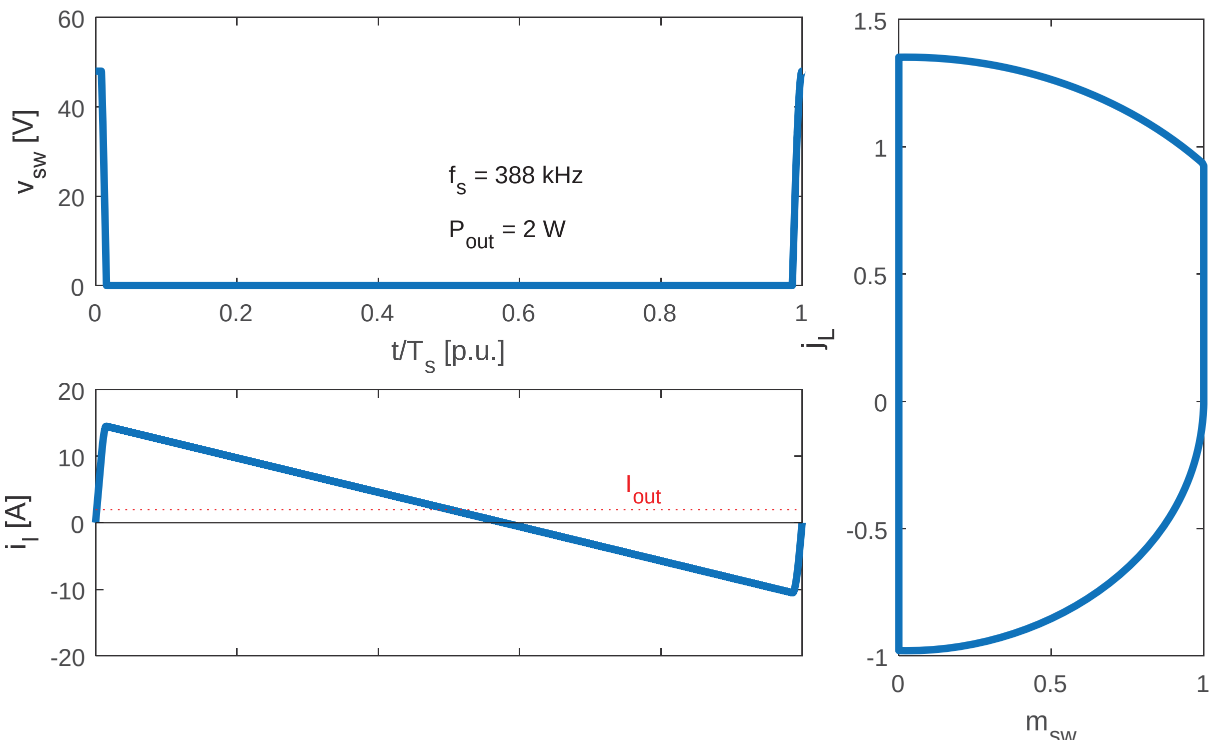
Matlab Solution



Matlab Solution



Matlab Solution



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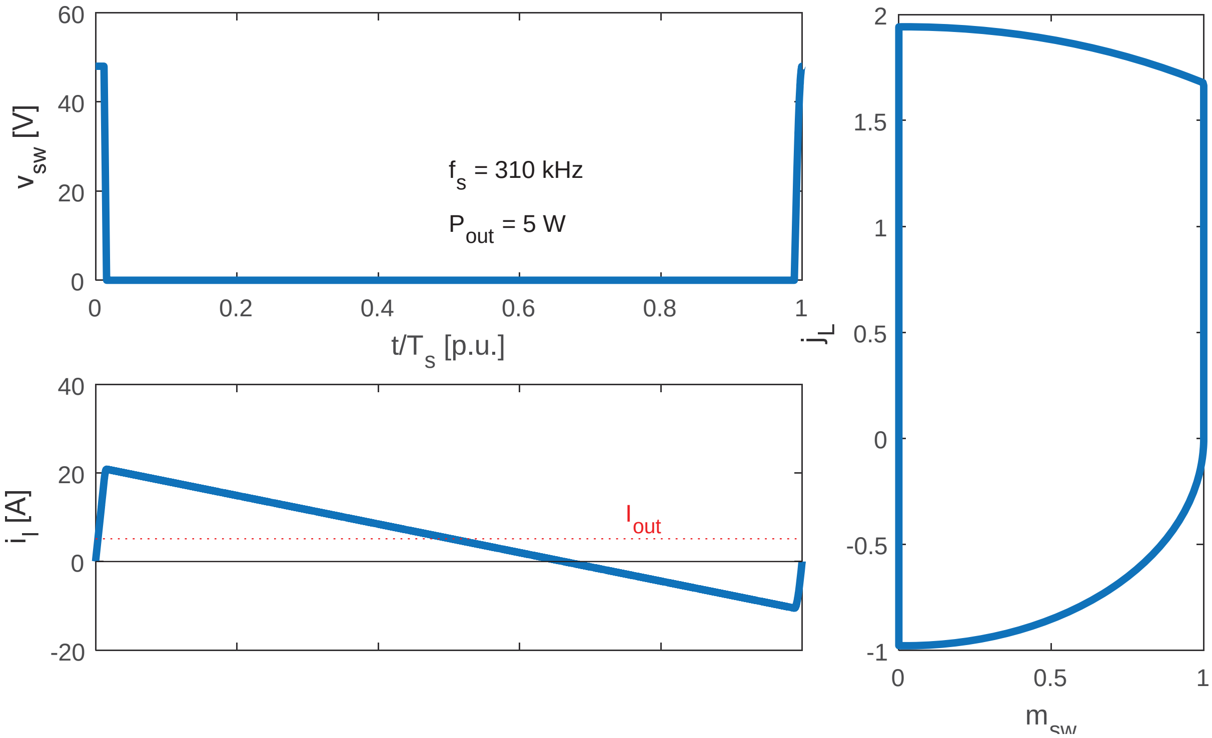
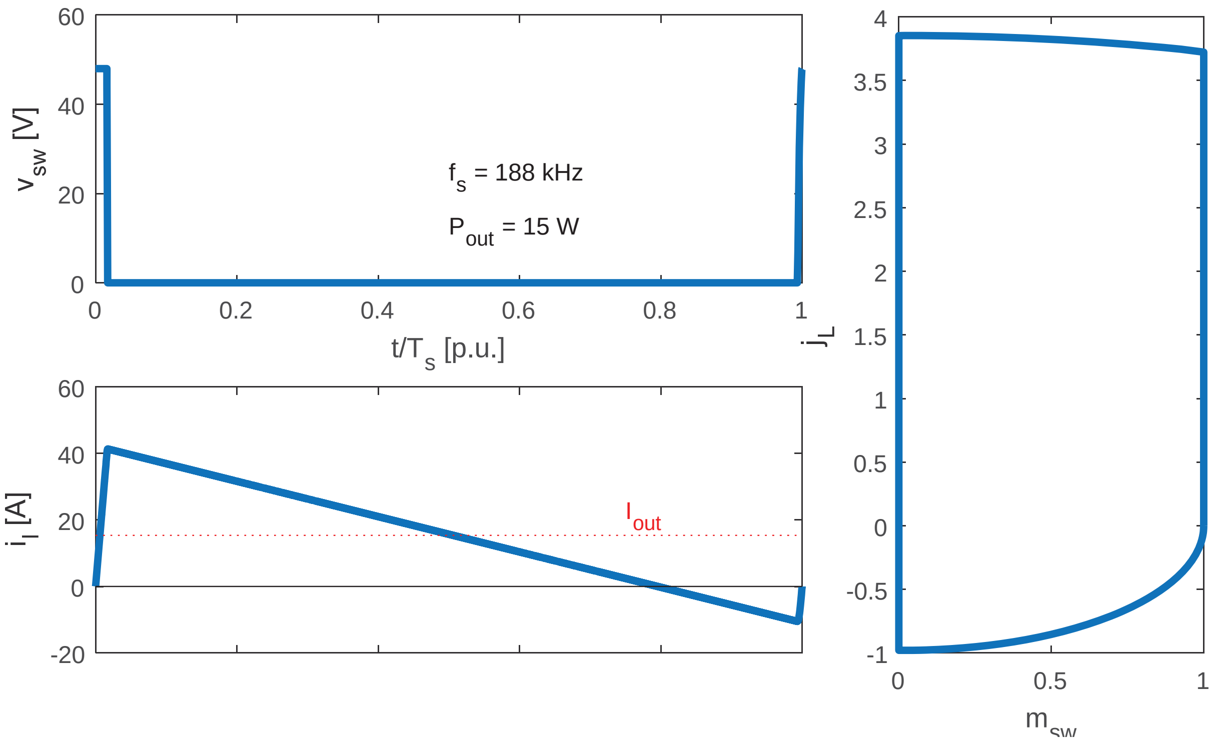


Matlab Solution



Matlab Solution