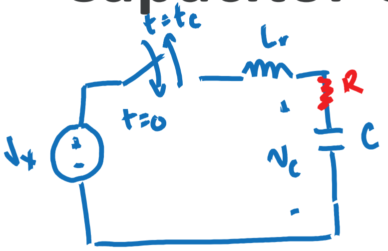
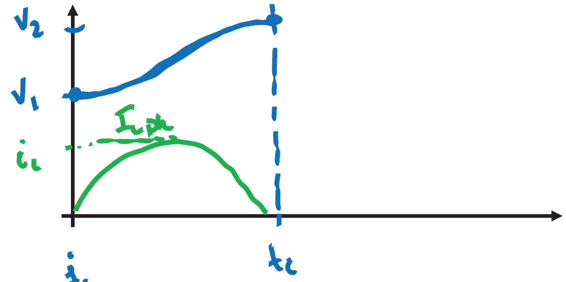
Capacitor Charging: Resonant



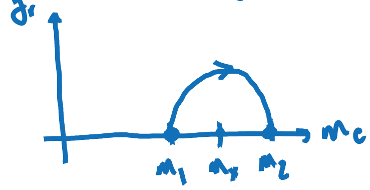
$v_c(t=0) = V_1$

$v_c(t=t_c) = V_2$



$V_x = \frac{V_1 + V_2}{2}$

$I_{Lpk} = \frac{V_x - V_1}{R_0} = \frac{V_2 - V_1}{2R_0}$



$\Delta E_c = \frac{1}{2} C (V_2^2 - V_1^2)$

Assume i_L doesn't change when adding R

$E_R = E_{loss} = \int_0^{t_c} i_L^2 R \, dt = i_{L,rms,t_c}^2 R t_c = \frac{I_{Lpk}^2}{2} R t_c$

$= \left(\frac{V_2 - V_1}{2R_0} \right)^2 \frac{R t_c}{2} = \frac{\pi R C}{2} \cdot \frac{(V_2 - V_1)^2}{4 R_0}$

$t_c = \frac{1}{2 f_0} = \frac{\pi}{\omega_0}$

$R_0 \omega_0 = \sqrt{\frac{L}{C}} \cdot \frac{1}{\sqrt{LC}} = \frac{1}{C}$

$E_R = \frac{\pi R C}{2} \frac{(V_2 - V_1)^2}{4 R_0} = \underbrace{\frac{R\pi}{4 R_0}}_{\text{scale factor}} \underbrace{\left[\frac{1}{2} C (V_2 - V_1)^2 \right]}_{E_{loss} \text{ in Voltage-source case}}$

$\frac{R\pi}{4R_0} = \frac{R\pi}{4 \frac{1}{C\omega_0}} = \frac{RC\pi}{4} \left(\frac{\pi}{t_c} \right) = \frac{\pi^2}{4} \frac{RC}{t_c}$

$E_R = \underline{\frac{\pi^2}{4} \frac{RC}{t_c}} \left[\frac{1}{2} C (V_2 - V_1)^2 \right]$

$\frac{\pi^2}{4} \approx 2.5$

Comparison of Capacitor Charging

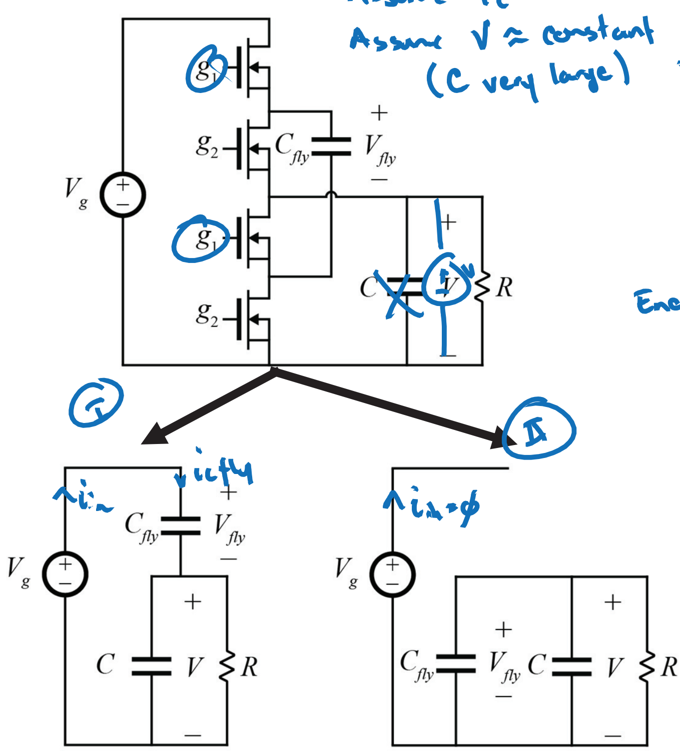
Cap charged from V_1 to V_2 in time t_c

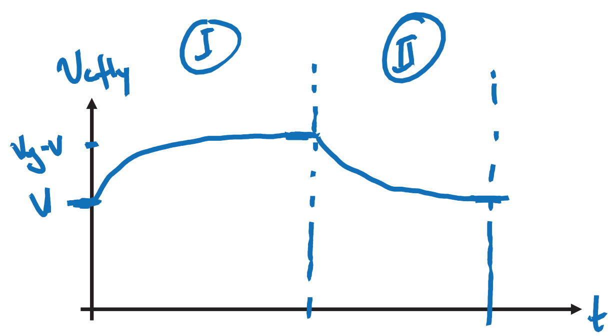
	E_{loss}	Assumptions
Voltage	$E_{cv} = \frac{1}{2}C(V_2 - V_1)^2$	$t_c \gg RC = \tau$
Current	$2\frac{RC}{t_c} E_{cv}$	$\times$
Resonant	$\frac{\pi^2}{4}\frac{RC}{t_c} E_{cv}$	High-Q resonance

2:1 SC Revisited

Assume $t_c \gg RC$

Assume $V \approx$ constant (C very large)





Energy loss

$$\begin{cases} \text{I} & \frac{1}{2}C_{fly}(2V - V_g)^2 \\ \text{II} & \frac{1}{2}C_{fly}(V_g - 2V)^2 \end{cases}$$

$$P_{loss} : C_{fly}(V_g - 2V)^2 f_s$$

$$P_{in} = V_g \, \Delta Q_{Cfly,I} \, f_s$$

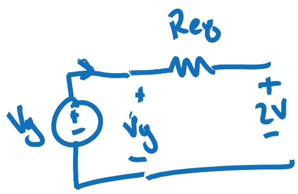
$$= V_g C_{fly}(V_g - 2V) f_s$$

Equivalent Circuit Model

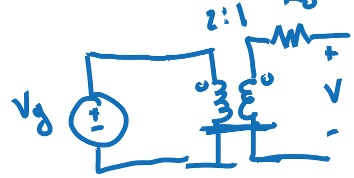
$$P_{loss} = \underbrace{C_{fly} f_s}(V_g - 2V)^2 \qquad P_{in} = V_g \underbrace{C_{fly} f_s}(V_g - 2V)$$

$$R_{eq} = \frac{1}{C_{fly} f_s}$$

$$R_o = \frac{1}{4 C_{fly} f_s}$$



$\Rightarrow$



for high η, want small R_o

$\rightarrow$ Large C_{fly}, Large f_s