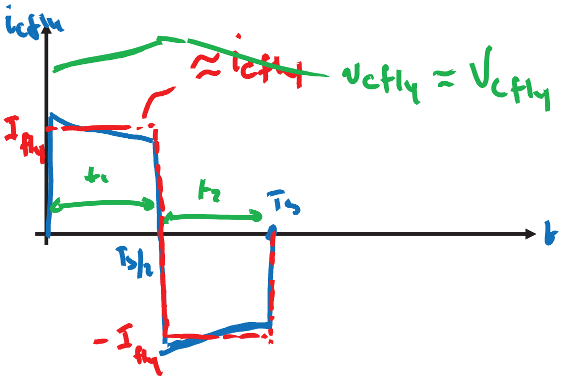
# 2:1 SC – FSL Model

$$P_{out} = V\left[\frac{V_g - V_{cfly} - V}{R}\cdot\frac{t_1}{T_s} + \frac{V_{cfly} - V}{R}\frac{t_2}{T_s}\right]$$

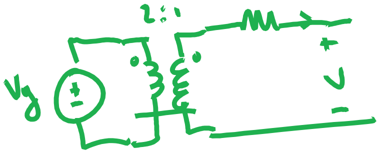


$$t_1 = t_2 = \frac{T_s}{2}$$

$$P_{out} = \frac{V}{R}\left[\frac{V_g}{2} - V\right]$$

$$= V\,\frac{\left(\frac{V_g}{2} - V\right)}{R}$$

$R = R_o$ wherever resistance is in the charging path



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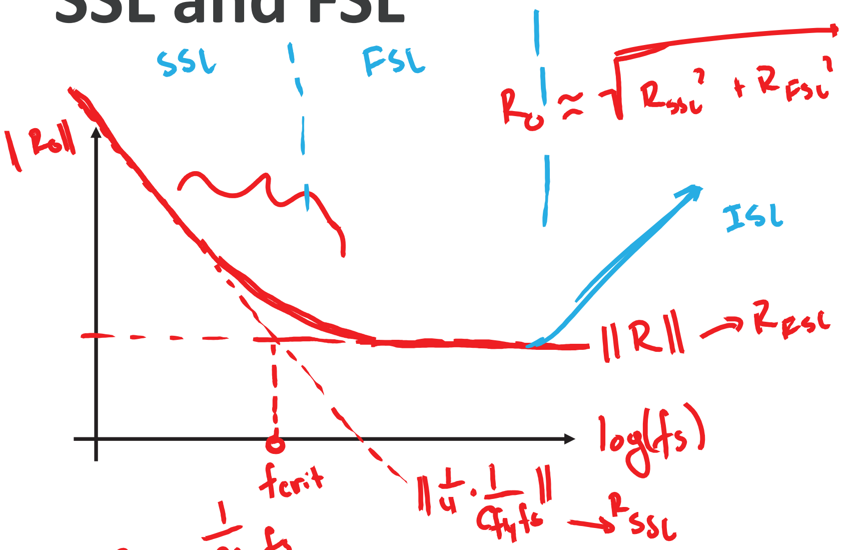
# SSL and FSL



$$R_o \approx \sqrt{R_{SSL}^2 + R_{FSL}^2}$$

$$R = \frac{1}{4C_{fly} f_s}$$

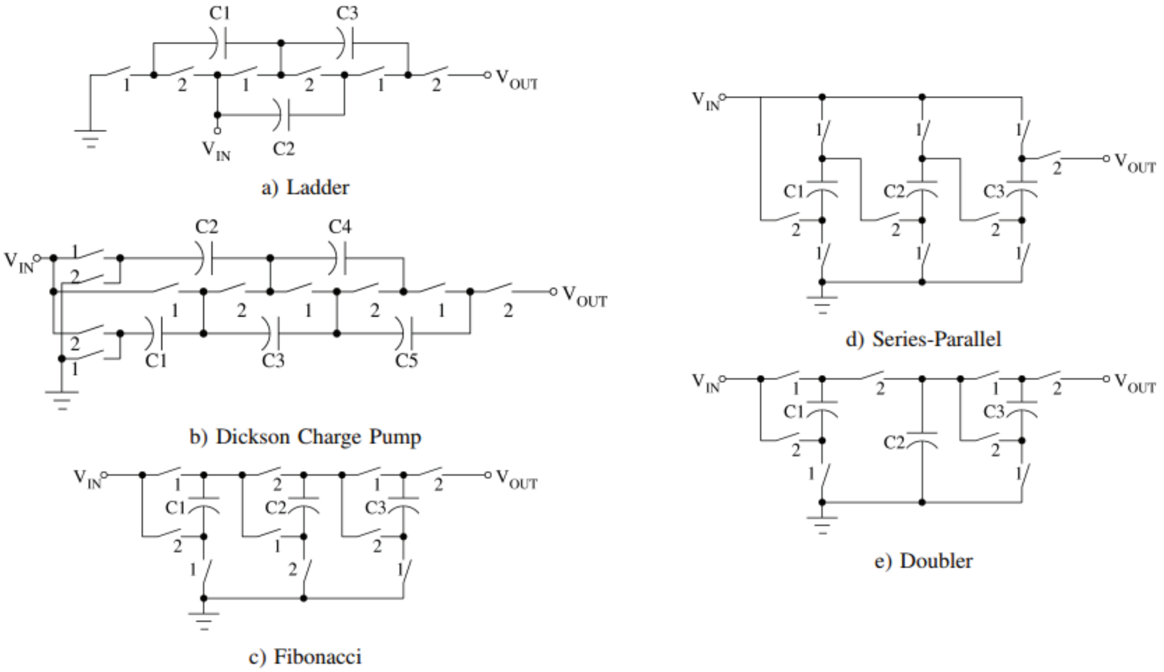
$$f_s = \frac{1}{4C_{fly} R}$$

$$\frac{1}{T_s} = \frac{1}{2t_c} = \frac{1}{4C_{fly} R}$$

$$t_c = 2C_{fly} R = 2\tau$$

@ $f_{crit}$

# SC Converter Topologies



a) Ladder

b) Dickson Charge Pump

c) Fibonacci

d) Series-Parallel

e) Doubler

M Seeman and S. Sanders, “Analysis and Optimization of Switched-Capacitor DC-DC Converters”

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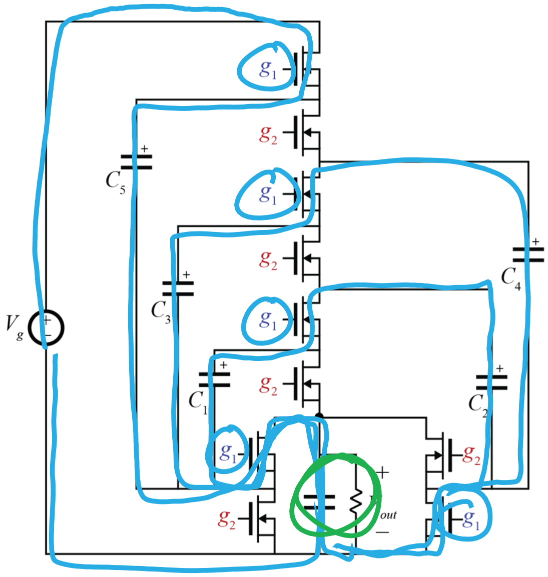
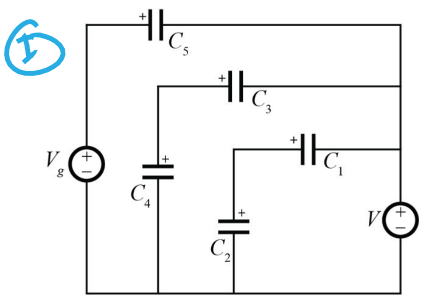
# Dickson Converter



Y. Lei, R. May, and R. Pilawa-Podgurski, "Split-Phase Control: Achieving Complete Soft-Charging Operation of a Dickson Switched- Capacitor Converter," 2016

# Dickson Subintervals

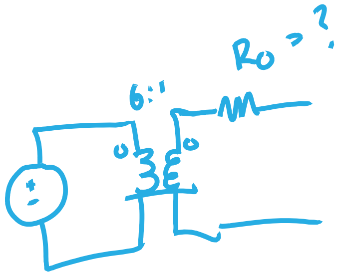
Approximate: all caps have very small ripple

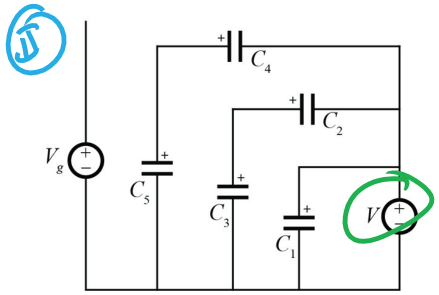


$V_g = 6V$ → This is a 6:1 SC converter

$V_{C4} = 4V$

$V_{C2} = V + V_{C1} = 2V$





$V_{C5} = 5V$

$V_{C3} = 3V$

$V_{C1} = V$

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# Charge Vector Analysis: Notation

$q_x^{I}$ = charge in/out of element x during subinterval I

$a_x^{I} = \frac{q_x^{I}}{q_{out}}$, normalized with respect to $q_{out} = q_{out}^{I} + q_{out}^{II} + \dots$

$$\bar{a}^{I} = [\, a_{in}^{I},\ \overbrace{a_{c1}^{I},\ a_{c2}^{I},\ \dots,\ a_{cN}^{I}}^{\bar{a}_c^{I}},\ a_{out}^{I} \,]$$

$$\bar{v}^{I} = [\, V_g,\ V_{c1}^{I},\ V_{c2}^{I},\ \dots,\ V_{cN}^{I},\ V_{out} \,]$$

$V_{c1}^{I}$ = Voltage on $C_1$ at the end of the subinterval when operated in SSL

M. Makowski and D. Maksimovic, "Performance Limits of Switched-Capacitor DC-DC Converters," 1995
M Seeman and S. Sanders, "Analysis and Optimization of Switched-Capacitor DC-DC Converters," 2008

# Charge Vector Analysis: Rules

- KVL & KCL must be satisfied
- for all caps $\langle i_{c_i} \rangle \big|_{T_s} = 0$ in steady state

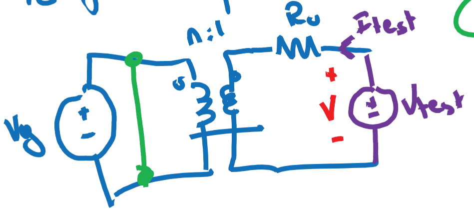
$$\int_0^{T_s} i_{c_i}\,dt = 0 = q_{ci}^{I} + q_{ci}^{II} + \dots + q_{ci}^{N} = 0$$

↳ looking @ our vectors

$$a_{ci}^{I} + a_{ci}^{II} + \dots = 0$$

for two switching subintervals, $a_{ci}^{I} = -a_{ci}^{II}$

To find output resistance



① Set Vg to zero

② Place source @ output,

$$R_o = \frac{V_{test}}{I_{test}} = \frac{V}{-I_o}$$

# Tellegen's Theorem

For any circuit which satisfies KVL & KCL

$$\sum_{i \in \text{elements}} V_i I_i = 0$$

(energy conservation)

voltage across

current through

Now, for our SC converter:

$$\bar{a}^{I}\bar{V}^{I} = 0 \quad \& \quad \bar{a}^{II}\bar{V}^{II} = 0$$

$$\bar{a}^{I}\bar{V}^{I} + \bar{a}^{II}\bar{V}^{II} = 0$$

$$V_{out}\left(\cancelto{1}{a_{out}^{I} + a_{out}^{II}}\right) + V_{in}\left(\cancelto{0}{a_{in}^{I} + a_{in}^{II}}\right) + \bar{a}_c^{I}\bar{V}_c^{I} + \bar{a}_c^{II}\bar{V}_c^{II} = 0$$

1 by normalization

0 when finding Ro

$$V_{out} + \bar{a}_c^{I}\left(\bar{V}_c^{I} - \bar{V}_c^{II}\right) = 0$$

$$V_{out} + \bar{a}_c^{I}\left(\Delta \bar{V}_c\right) = 0$$

# Tellegen's Theorem

$$V_{out} + \bar{a}_c^{I}\left(\Delta \bar{V}_c\right) = 0$$

$$\Delta V_{ci} = \frac{q_i}{C_i}$$

$$V_{out} + \bar{a}_c^{I} \cdot \left(\frac{\tilde{q}_c^{II}}{\bar{C}}\right) = 0$$

$$\frac{1}{q_{out} f_s}\left(V_{out} + \sum_{caps} \frac{q_i}{q_{out}} \frac{q_{ci}}{C_i}\right) = 0$$

$$\frac{V_{out}}{q_{out} f_s} = \frac{V_{out}}{I_{out}} = -R_o = -\sum_{caps}\left(\frac{q_{ci}}{q_{out}}\right)^2 \frac{1}{C_i f_s}$$

$$\boxed{R_{o,ssl} = \sum_{caps} \frac{(a_{ci})^2}{C_i f_s}}$$