Steady-State Large-Signal Analysis

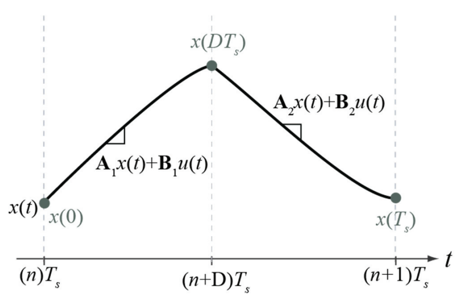
$$\boldsymbol{x}(T_s) = \left(\prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)\boldsymbol{x}(0) + \sum_{i=1}^{n_{sw}} \left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I})\boldsymbol{B}_i\right\} U$$

In steady-state, $\boldsymbol{x}(T_s) = \boldsymbol{x}(0)$

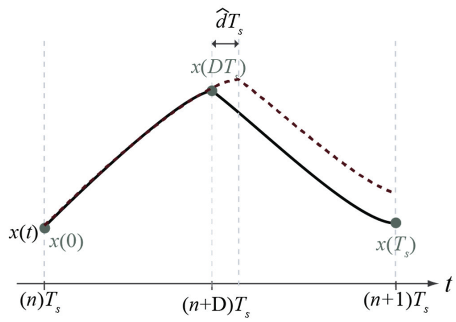
$$\boldsymbol{x}(T_s) = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I})\boldsymbol{B}_i\right\} U$$

Gives explicit solution for steady-state operation of any switching circuit

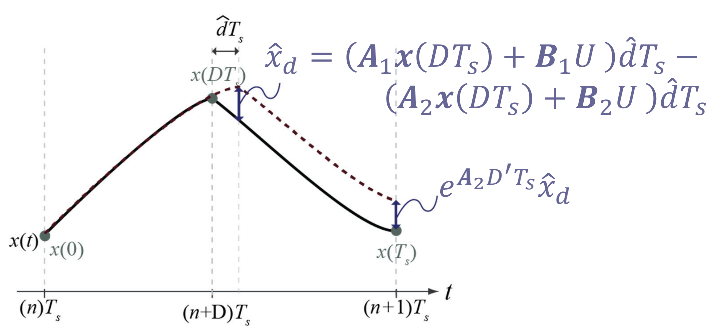
Small Signal Modeling



Small Signal Modeling



Small Signal Modeling



Complete Small Signal Model

This completes the small-signal model

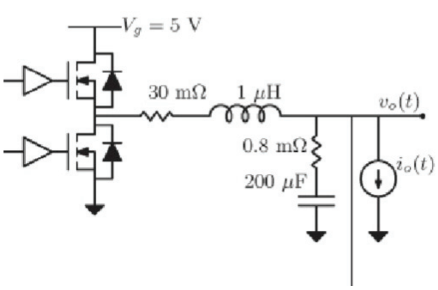
$$\widehat{\boldsymbol{x}}[n+1] = \boldsymbol{\Phi}\widehat{\boldsymbol{x}}[n] + \boldsymbol{\Psi}\hat{u}[n] + \boldsymbol{\Gamma}\hat{d}[n]$$

where

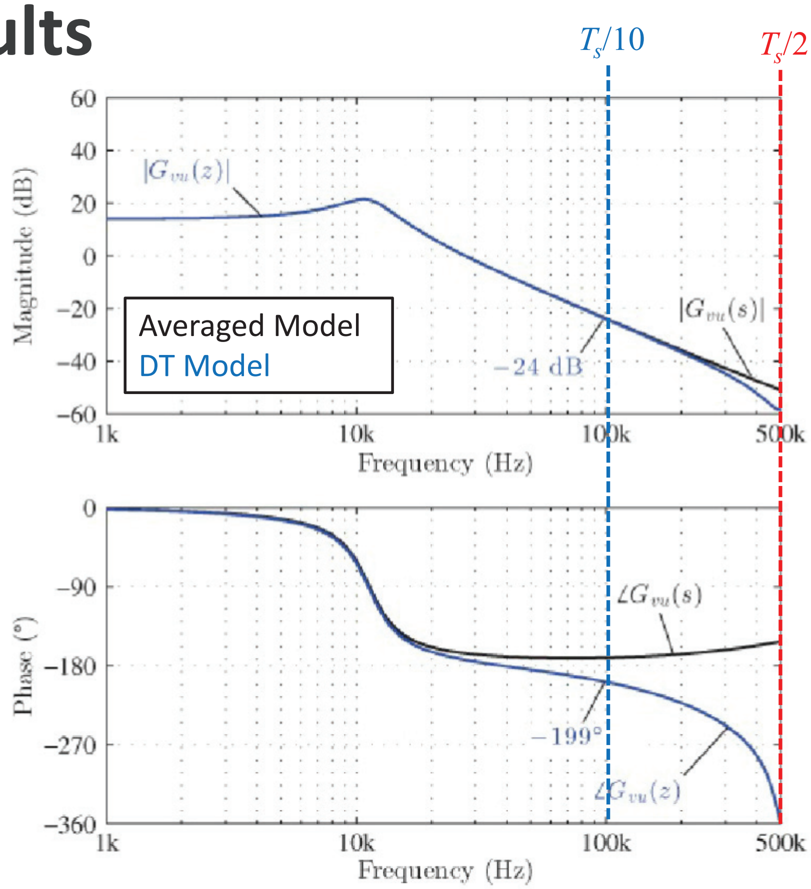
$$\boldsymbol{\Gamma} = e^{A_2 D' T_s}\big((\boldsymbol{A}_1 - \boldsymbol{A}_2)\boldsymbol{X}_D + (\boldsymbol{B}_1 - \boldsymbol{B}_2\,)U\big)T_s$$

with $\boldsymbol{X}_D = \boldsymbol{x}(DT_s)$ in steady-state

Example Results



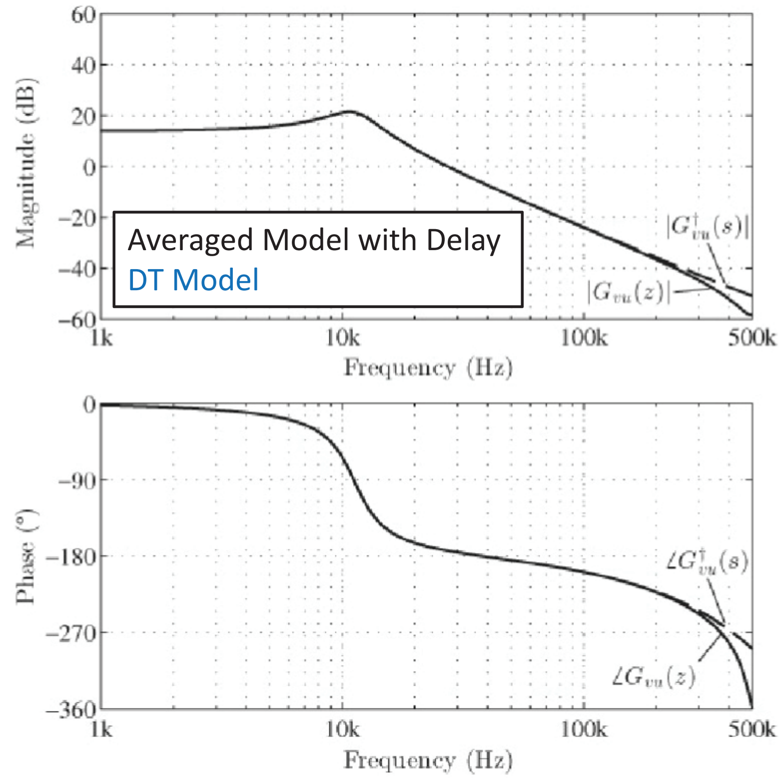
* Includes t_d=760ns of delay in feedback loop



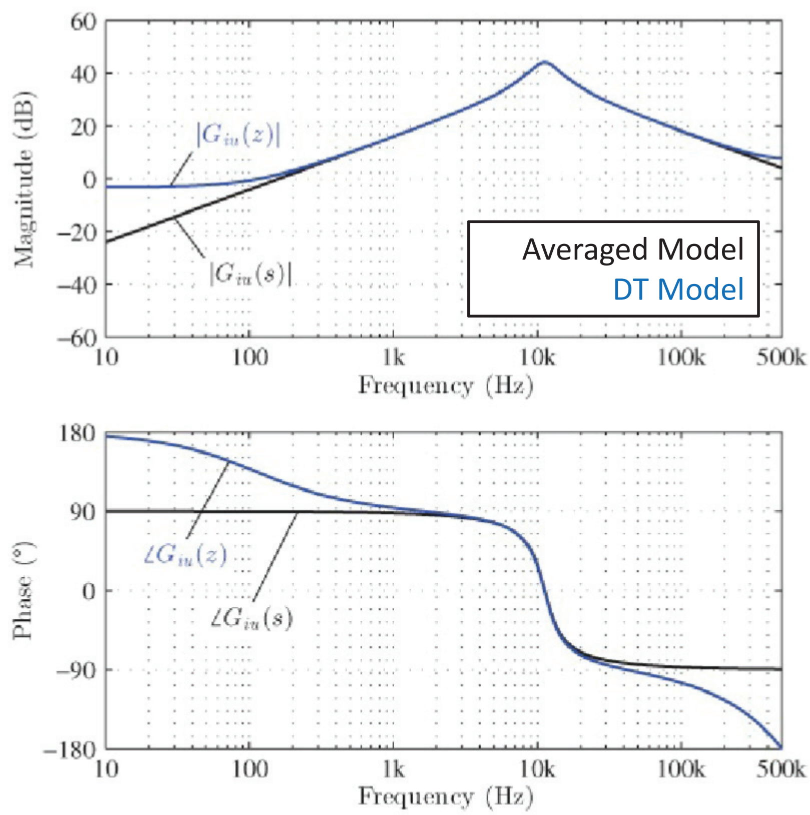
L. Corradini et. al. *Digital Control of High Frequency Switched-Mode Power Converters,* Section 3.2

Inclusion of Delay

$$G_{vu}^{\dagger}(s) = G_{vu}(s)e^{-st_d}$$



Current Control

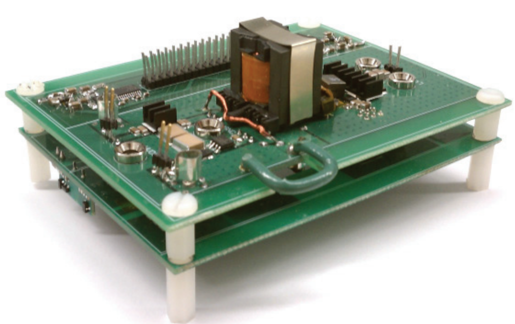


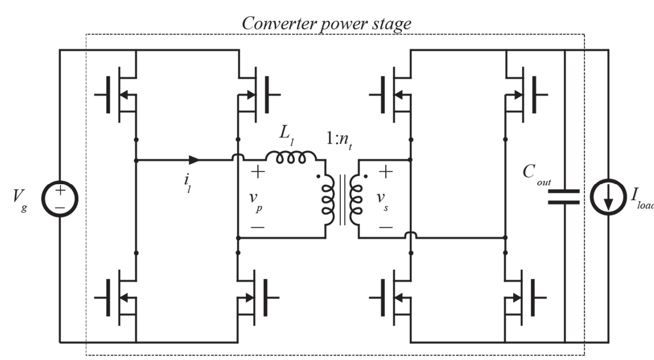
Discrete Time Analysis: Results

$$\boldsymbol{X}_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i} \right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k} \right) \boldsymbol{A}_i^{-1} \left(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I} \right) \boldsymbol{B}_i \right\} U$$

- Valid for any switched circuit, as long as
  1. Inputs, U, are constant or slowly varying
  2. All times t_i are known
  3. Every subinterval can be described by a linear circuit
- Requires no dedicated analysis other than finding $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$
- Decisively **not** a design-oriented equation

Example: DAB Design Using Dedicated Analysis





- Design of a high step-down DAB for Data Centers
- 150-to-12V, 120 W, 1MHz, design
- Prototype achieved 98.4% peak efficiency

D Costinett, D. Maksimovic, and R Zane, “Design and Control for High fficiency in High Step-Down Dual Active Bridge Converters Operating at High Switching Frequency,” *IEEE Trans. On Pwr. Elec.*, 2013

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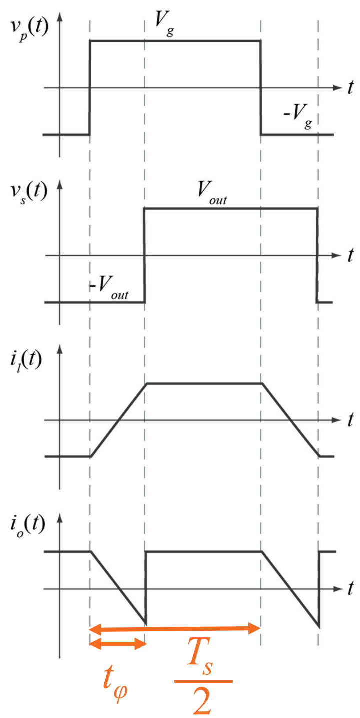
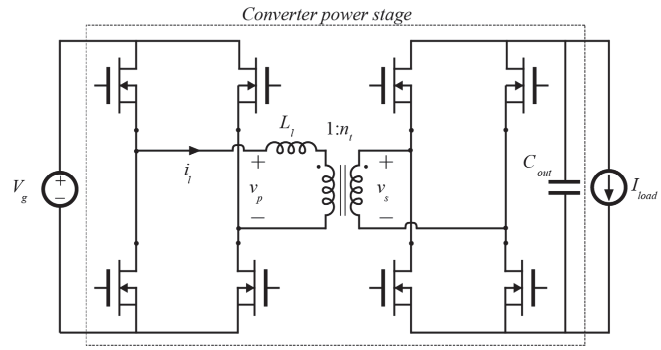
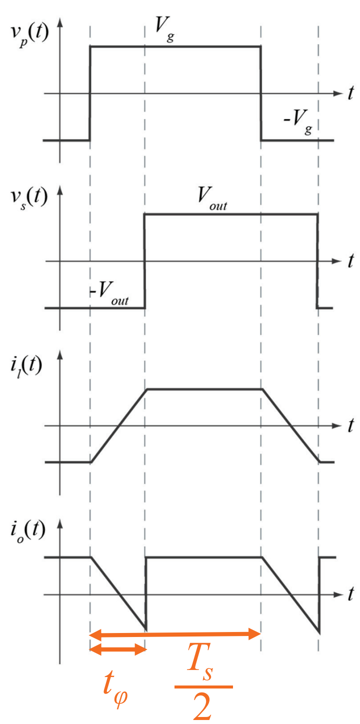
DAB Topology

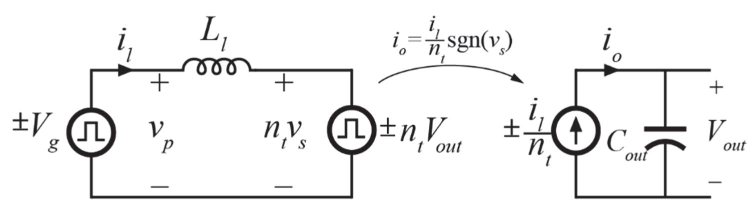




- Near-DCX Operation
- Phase-shift modulation to control power flow
- Zero-voltage switching of all devices at high power

Linear Averaged Modeling of DAB

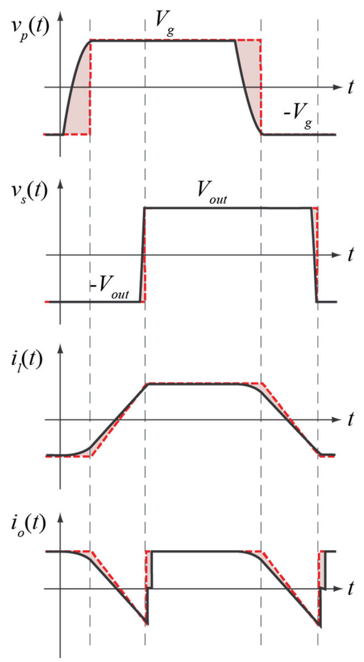


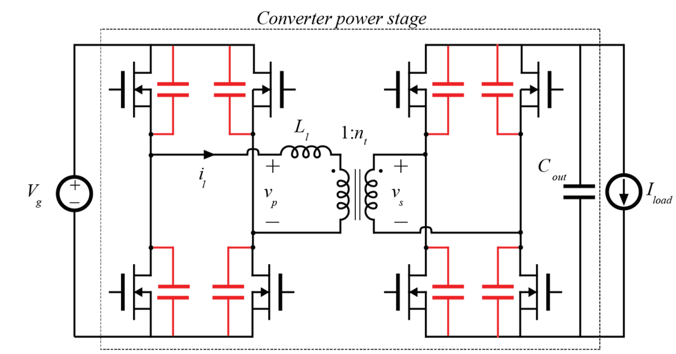


- Modeling $\dot{\boldsymbol{x}}_i$ as constant within any subinterval, waveforms are PWL
- Can solve low-frequency behavior from waveforms

$$\langle i_o \rangle \big|_{T_s} \approx \int_0^{T_s} \frac{i_o(t)dt}{T_s} = \frac{v_g(t)}{n_t L_l T_s}\left(T_s t_\varphi - 2{t_\varphi}^2\right)$$

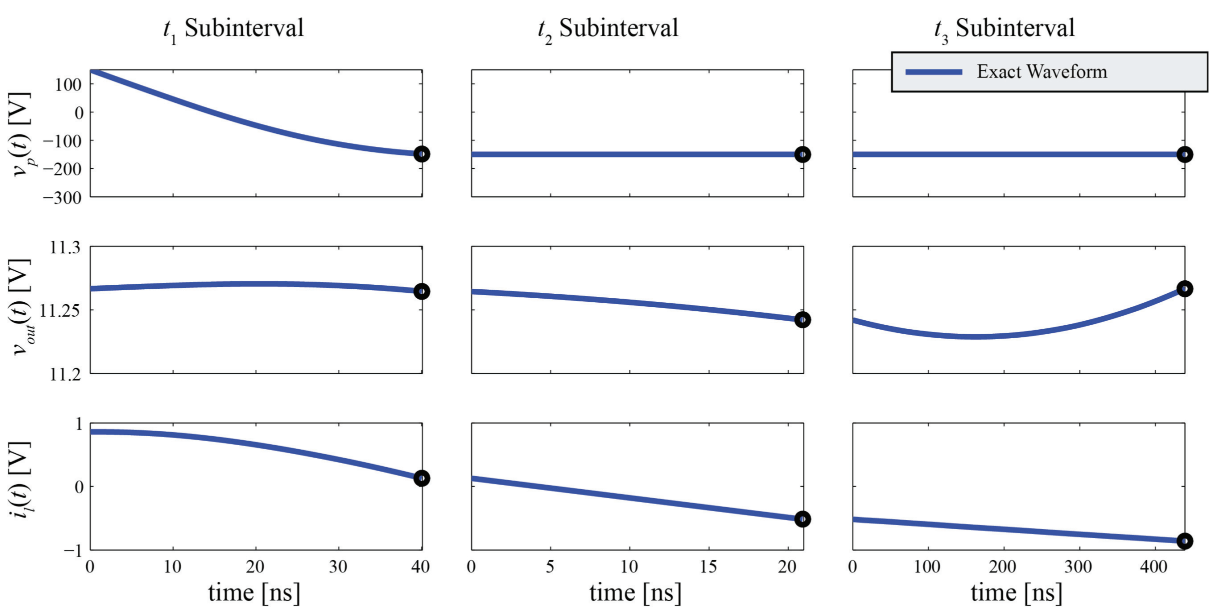
DAB Operated at High Frequency





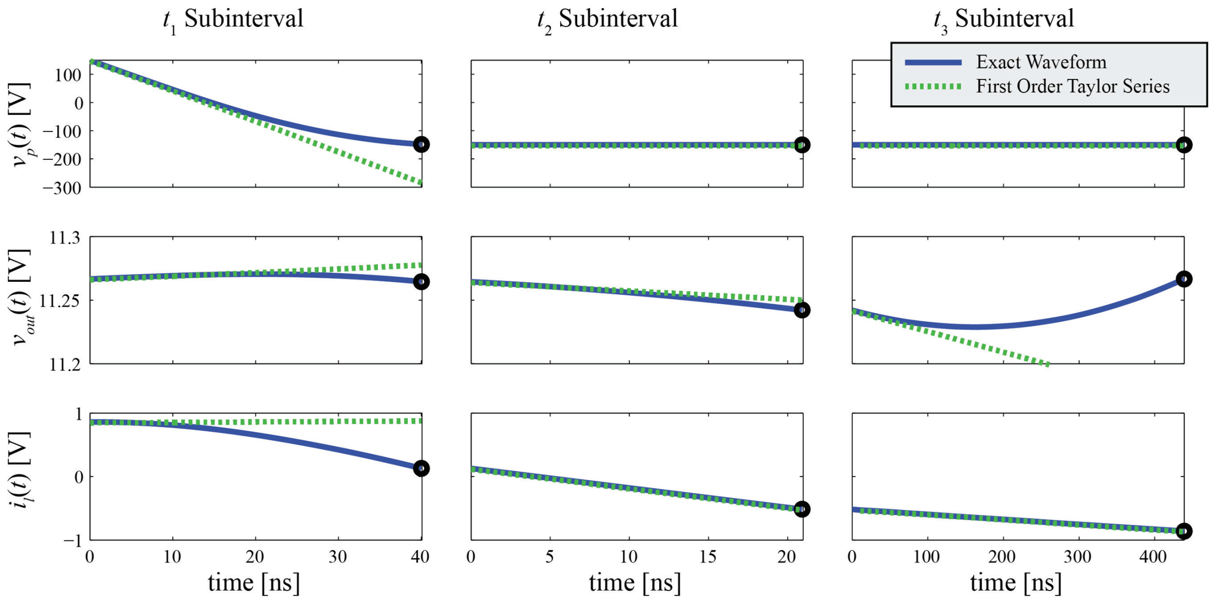
- Resonance between L_l and transistor capacitance distorts waveforms
- Resonance *may* need to be modeled when operating at high frequency

DAB Waveforms



$e^{A_1 t_1}$ $e^{A_2 t_2}$ $e^{A_3 t_3}$

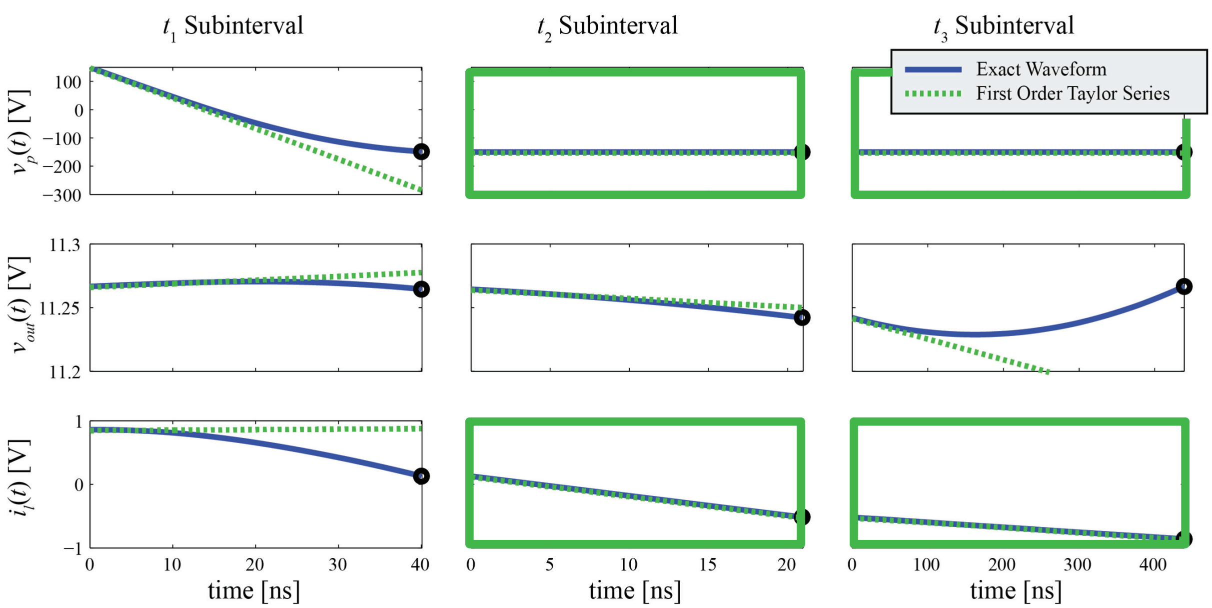
Linear Waveform Approximation



$e^{A_1 t_1}$ $e^{A_2 t_2}$ $e^{A_3 t_3}$

$I + A_1 t_1$ $I + A_2 t_2$ $I + A_3 t_3$

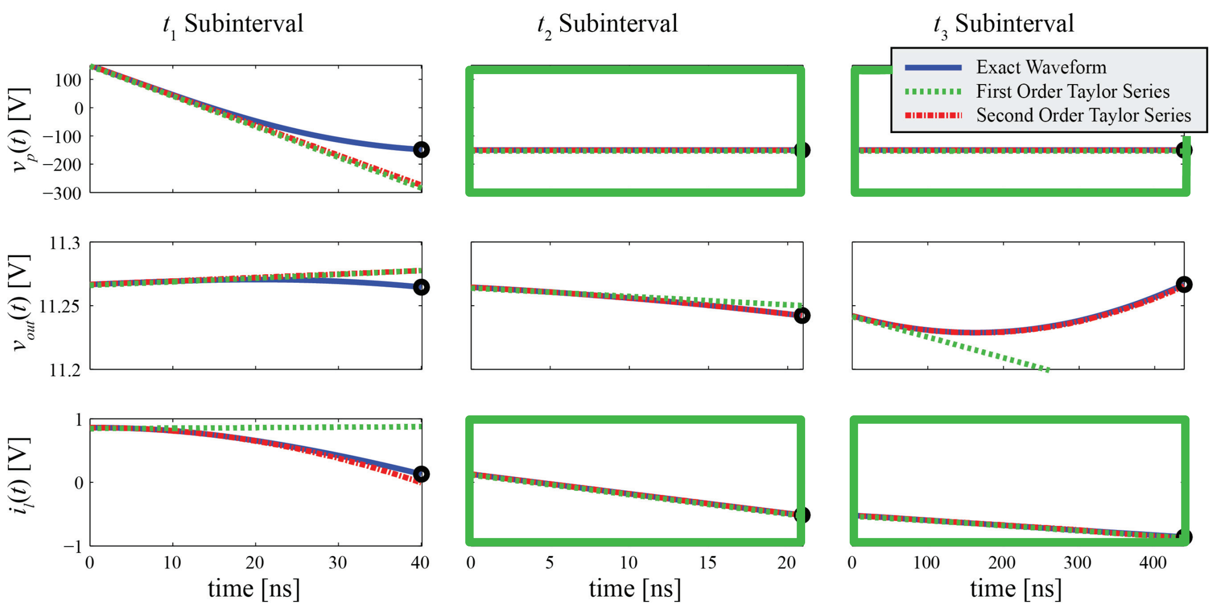
Linear Waveform Approximation



$e^{A_1 t_1}$ $e^{A_2 t_2}$ $e^{A_3 t_3}$

$I + A_1 t_1$ $I + A_2 t_2$ $I + A_3 t_3$

Second Order Approximation

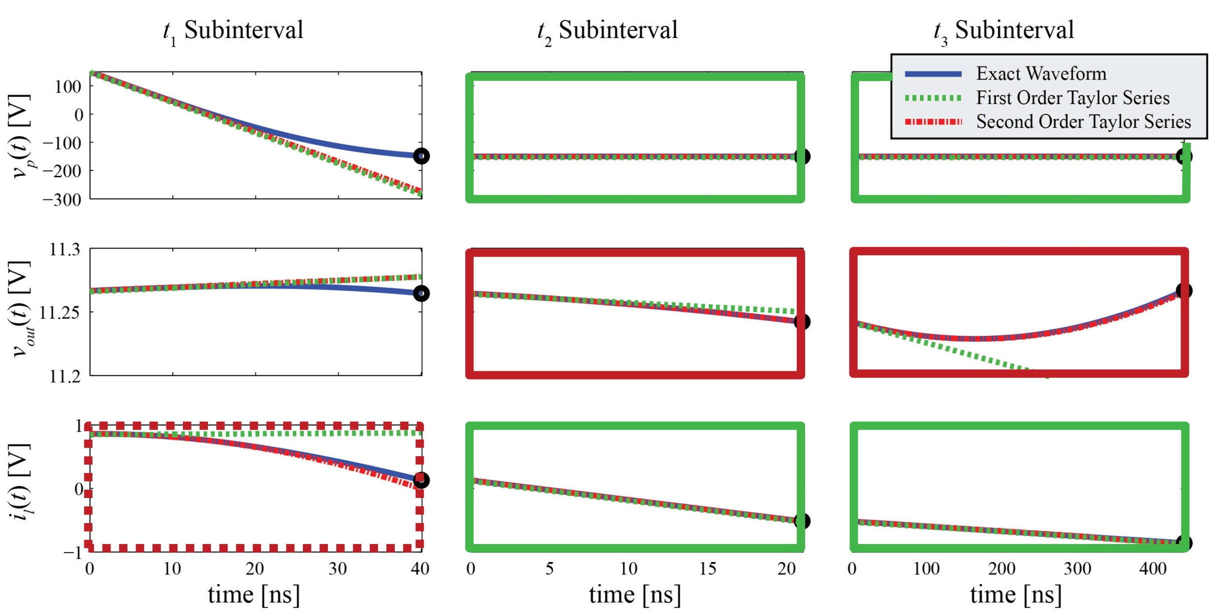


$$e^{A_1 t_1} \qquad e^{A_2 t_2} \qquad e^{A_3 t_3}$$

$$I + A_1 t_1 \qquad I + A_2 t_2 \qquad I + A_3 t_3$$

$$I + A_1 t_1 + \tfrac{1}{2}(A_1 t_1)^2 \qquad I + A_2 t_2 + \tfrac{1}{2}(A_2 t_2)^2 \qquad I + A_3 t_3 + \tfrac{1}{2}(A_3 t_3)^2$$

Second Order Approximation

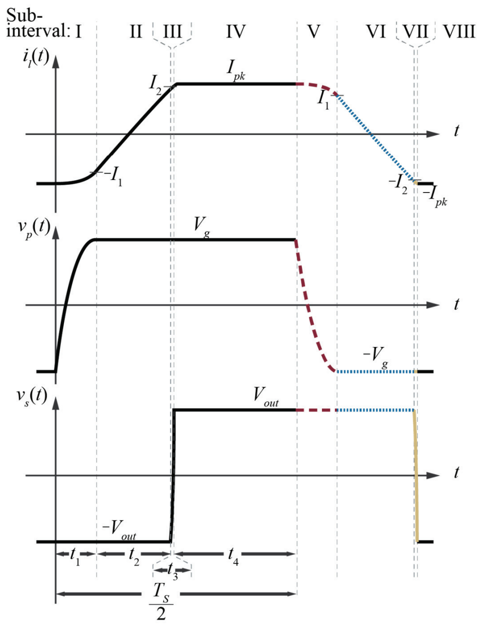


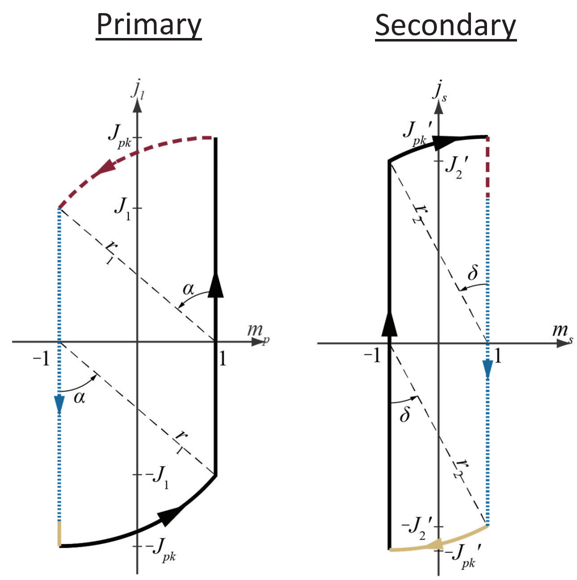
$$e^{A_1 t_1} \qquad e^{A_2 t_2} \qquad e^{A_3 t_3}$$

$$I + A_1 t_1 \qquad I + A_2 t_2 \qquad I + A_3 t_3$$

$$I + A_1 t_1 + \tfrac{1}{2}(A_1 t_1)^2 \qquad I + A_2 t_2 + \tfrac{1}{2}(A_2 t_2)^2 \qquad I + A_3 t_3 + \tfrac{1}{2}(A_3 t_3)^2$$

State Plane Analysis of DAB Converter





$$I_{base} = V_g \sqrt{\frac{C_p}{L_l}} \qquad I_{base} = V_g \sqrt{\frac{C_s}{n_t^2 L_l}}$$

State Plane Solution

Solution “Read off” state plane

$$\alpha = \sin^{-1}\frac{2}{J_p}$$

Primary Dead Time

$$\beta = \frac{1}{2}(J_2 + J_1)$$

Phase Shift

$$\delta = n_t \sin^{-1}\frac{2n_t^2}{\frac{R_0'}{R_0}J_p}\sqrt{\frac{C_s}{C_p}}$$

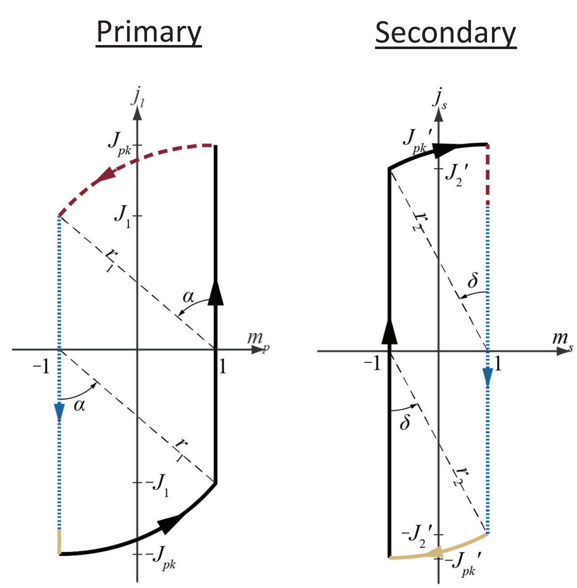
Secondary Dead Time

$$\zeta = \frac{F}{\pi} - \alpha - \beta - \delta$$

$$J_1 = \sqrt{J_p^2 - 4}$$

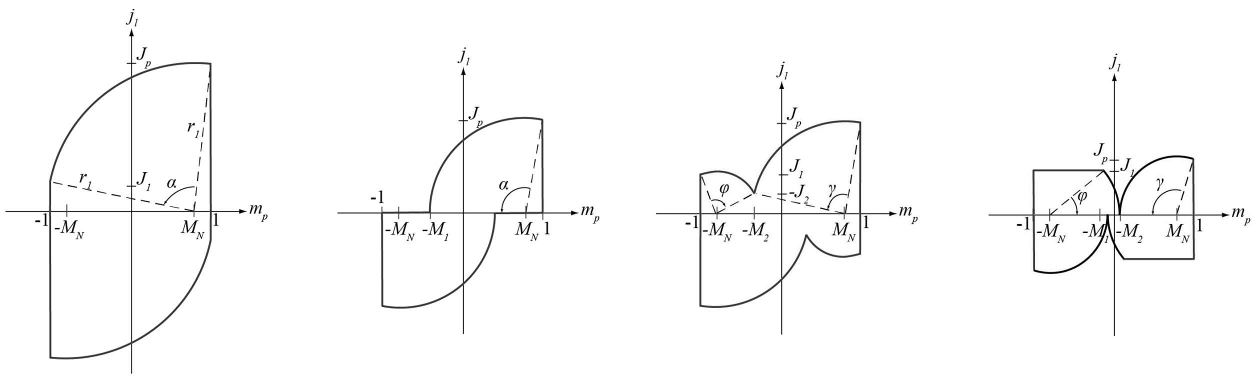
$$J_2 = \frac{R_0}{R_0'}\sqrt{\left(J_p\frac{R_0'}{R_0}\right)^2 - (2n_t)^2}$$

$$J = \frac{F}{\pi}\left[2 + \frac{1}{4}\left(J_1^2 - J_2^2\right) + J_p\left(\frac{\pi}{F} - \alpha - \beta - \delta\right)\right]$$



Different Operating Modes

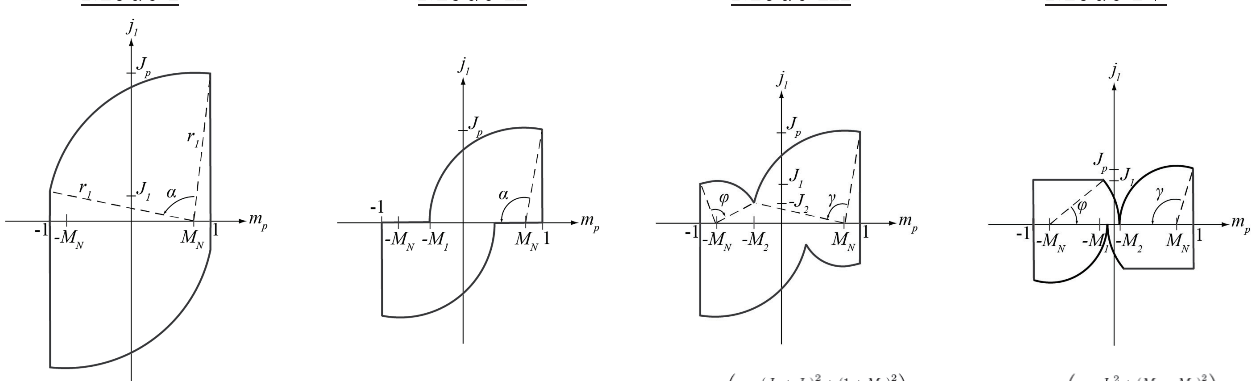
Mode I | Mode II | Mode III | Mode IV



- As control, input and load vary, operating mode changes
- In each mode, solution is a set of transcendental equations

Different Operating Modes

Mode I | Mode II | Mode III | Mode IV



Mode I

$$J_1 = \sqrt{{J_p}^2 - 4M_N}$$

$$\alpha = \cos^{-1}\left(1 - \frac{(J_p - J_1)^2 + 4}{2{J_p}^2 + 2(1 - M_n)^2}\right)$$

$$\beta = \frac{J_1 + J_2}{1 + M_N}$$

$$\zeta = \frac{J_p - J_2}{1 - M_N}$$

$$\frac{\pi}{F} = \alpha + \beta + \delta$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2 + \frac{J_p + J_2}{2}\zeta + \frac{J_1 - J_2}{2}\beta\ .\right)$$

Mode II

$$M_1 = \sqrt{{J_p}^2 + (1 - M_N)^2} - M_N$$

$$\alpha = \cos^{-1}\left(1 - \frac{{J_p}^2 + (1 + M_1)^2}{2(M_1 - M_n)^2}\right)$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(1 + M_1 + \frac{J_p + J_2}{2}\zeta - \frac{J_2}{2}\beta\right)$$

$$\beta = \frac{J_1 + J_2}{1 + M_N}$$

$$\zeta = \frac{J_p - J_2}{1 - M_N}$$

$$\frac{\pi}{F} = \alpha + \beta + \delta$$

Mode III

$$\gamma = \cos^{-1}\left(1 - \frac{(J_p + J_2)^2 + (1 + M_2)^2}{2(1 - M_n)^2 + 2{J_p}^2}\right)$$

$$\phi = \cos^{-1}\left(1 - \frac{(J_1 + J_2)^2 + (1 - M_2)^2}{2(M_n - M_2)^2 + 2{J_2}^2}\right)$$

$${J_2}^2 = (1 - M_N)^2 + {J_p}^2 - (M_2 + M_N)^2$$

$${J_1}^2 = -(1 - M_N)^2 + {J_2}^2 + (M_N - M_2)^2$$

$$\zeta = \frac{J_p + J_1}{1 - M_N}$$

$$\frac{\pi}{F} = \gamma + \phi + \zeta = \alpha + \zeta$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2M_2 + \frac{J_p - J_1}{2}\zeta\right)$$

Mode IV

$$\phi = \cos^{-1}\left(1 - \frac{{J_1}^2 + (M_1 - M_2)^2}{2(M_n - M_2)^2}\right)$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2M_2 + 1 - M_1 + \frac{J_p - J_1}{2}\zeta\right)$$

$${J_2}^2 = (1 - M_N)^2 + {J_p}^2 - (M_2 + M_N)^2$$

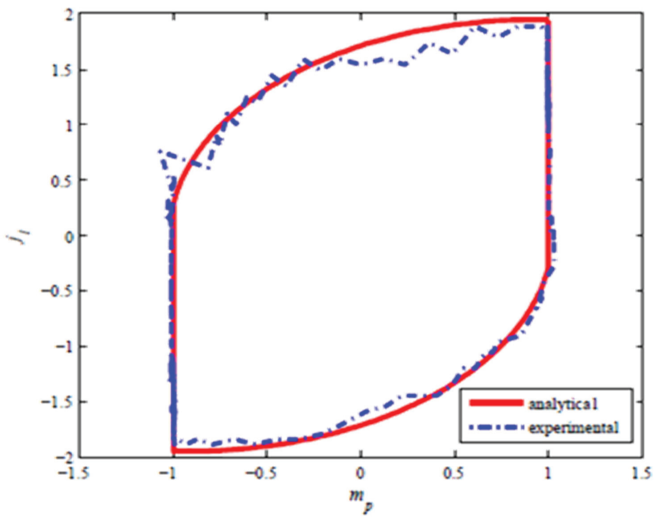
$${J_1}^2 = -(1 - M_N)^2 + {J_2}^2 + (M_N - M_2)^2$$

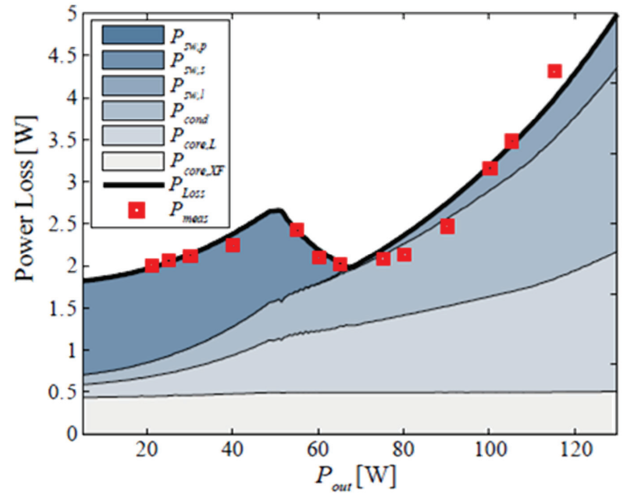
$$\zeta = \frac{J_p + J_1}{1 - M_N}$$

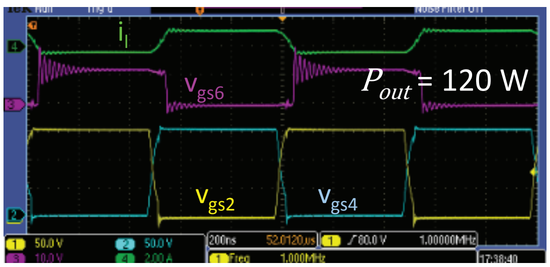
$$\frac{\pi}{F} = \gamma + \phi + \zeta = \alpha + \zeta$$

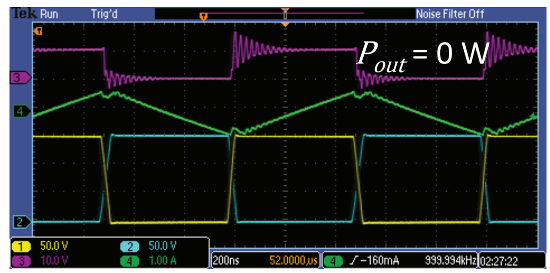
D Costinett, D. Maksimovic, and R Zane, "Design and Control for High Efficiency in High Step-Down Dual Active Bridge Converters Operating at High Switching Frequency," *Trans. On Pwr. Elec.* 2013

Model Validation



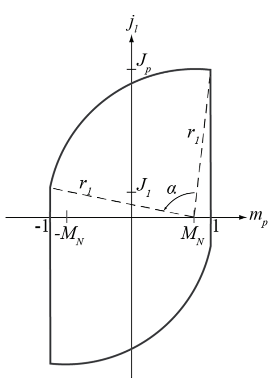






D Costinett, D. Maksimovic, and R Zane, "Design and Control for High fficiency in High Step-Down Dual Active Bridge Converters Operating at High Switching Frequency," *IEEE Trans. On Pwr. Elec.*, 2013

State Plane Analysis Characteristics



$$J_1 = \sqrt{J_p{}^2 - 4M_N}$$

$$\alpha = \cos^{-1}\left(1 - \frac{(J_p - J_1)^2 + 4}{2J_p{}^2 + 2(1 - M_n)^2}\right)$$

$$\beta = \frac{J_1 + J_2}{1 + M_N}$$

$$\zeta = \frac{J_p - J_2}{1 - M_N}$$

$$\frac{\pi}{F} = \alpha + \beta + \delta$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2 + \frac{J_p + J_2}{2}\zeta + \frac{J_1 - J_2}{2}\beta\,.\right)$$

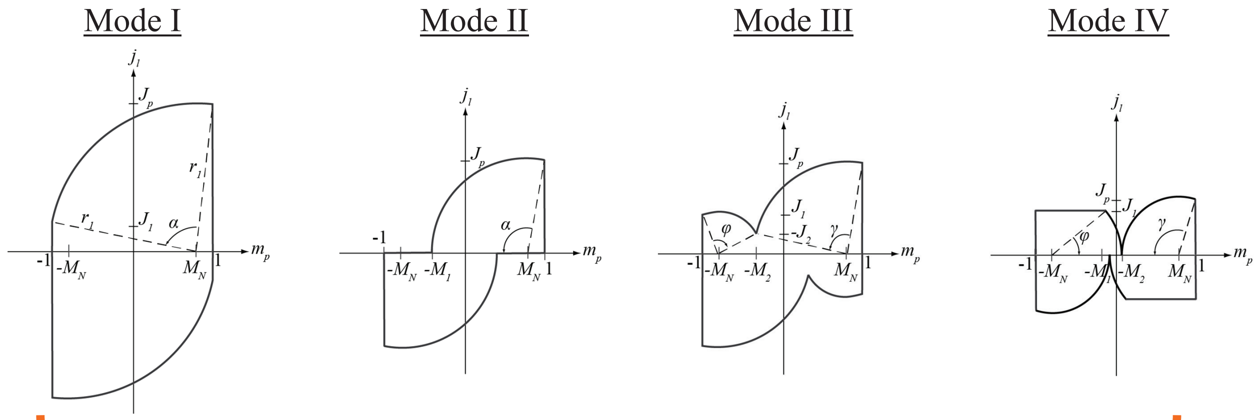
<u>For analysis</u>

- State plane analysis is simple and intuitive
- Plots give insight into converter operation
- Easy to write equations for single parameter of interest

<u>For Design</u>

- Complete solution is not solvable in closed-form
- No loss mechanisms included inherently
- Can only be computed, inverted numerically
- Design intuition comes only from repeated numerical evaluation

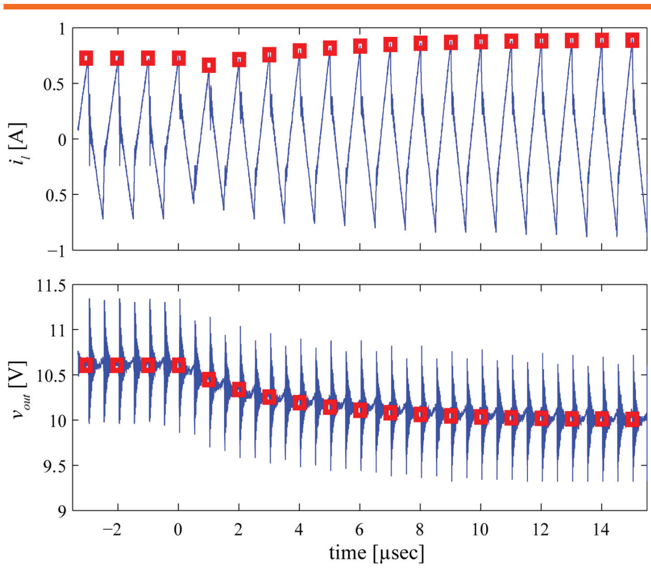
Different Operating Modes



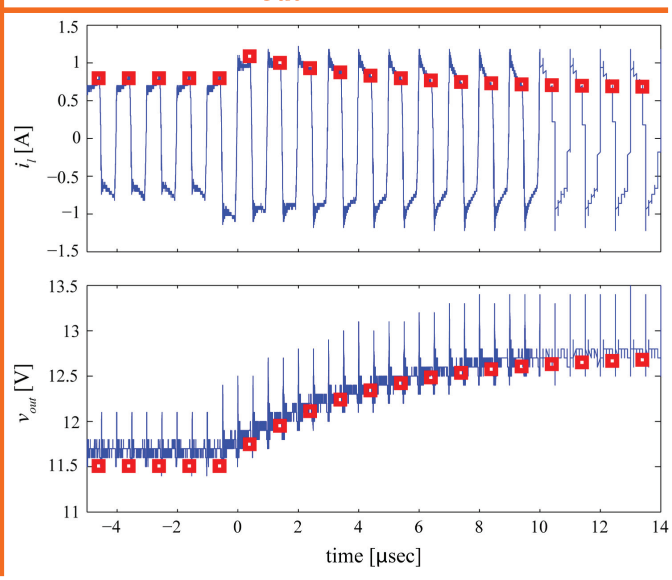
$$X_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{A_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{A_k t_k} \right) \boldsymbol{A}_i^{-1} \left(e^{A_i t_i} - \boldsymbol{I}\right) \boldsymbol{B}_i \right\} U$$

Discrete Time Model Validation

P_{out} = 30 W



P_{out} = 90 W



D. Costinett, R. Zane, and D. Maksimovic, "Discrete-time small-signal modeling of a 1 MHz efficiency-optimized dual active bridge converter with varying load," in Proc. IEEE Workshop Contr. Modl. (COMPEL), june 2012, pp. 1 –7.

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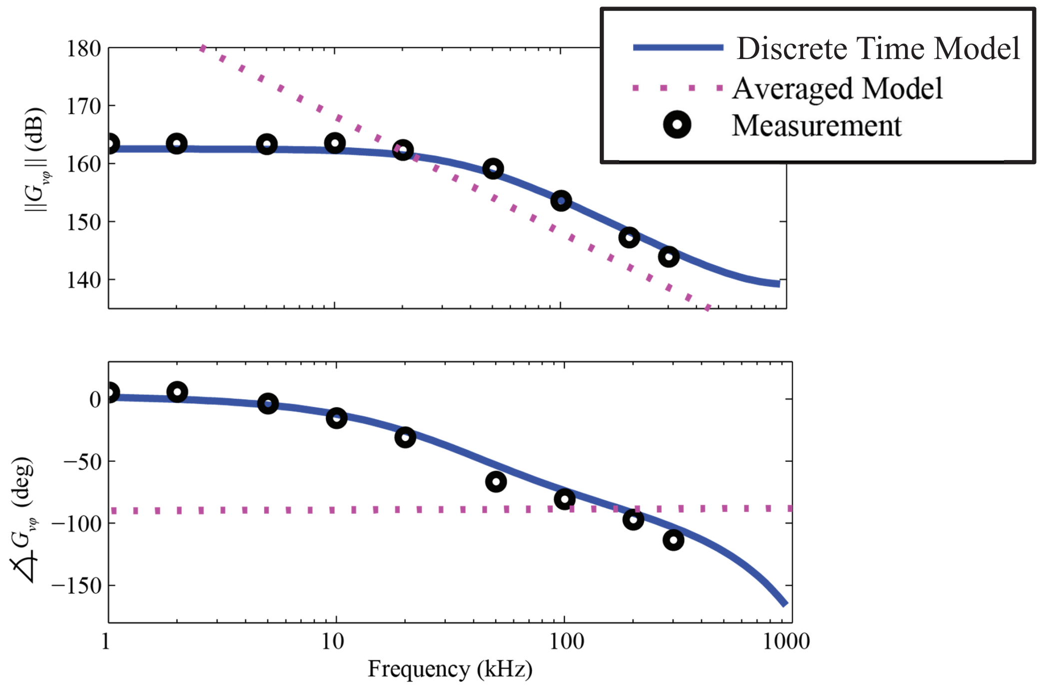
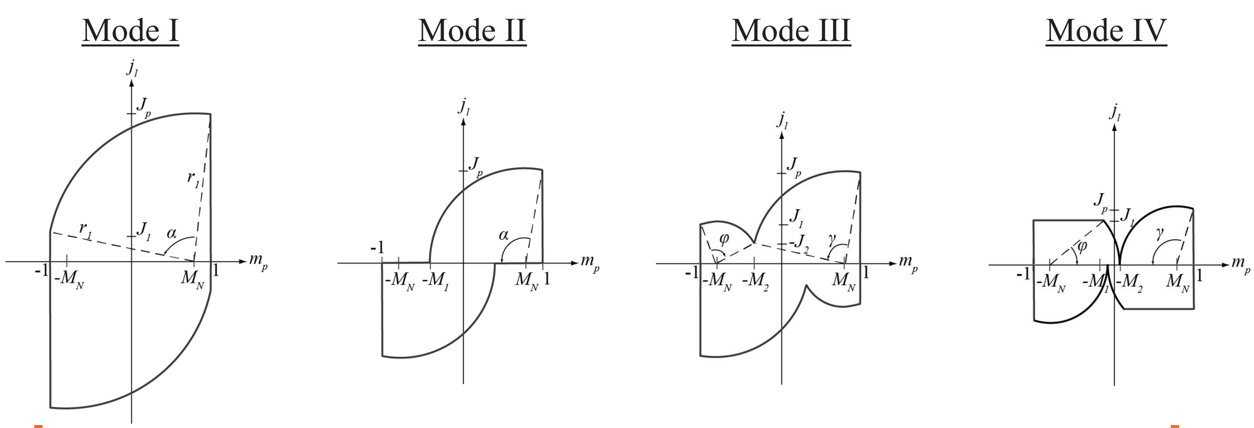
Discrete Time Dynamic Model Validation



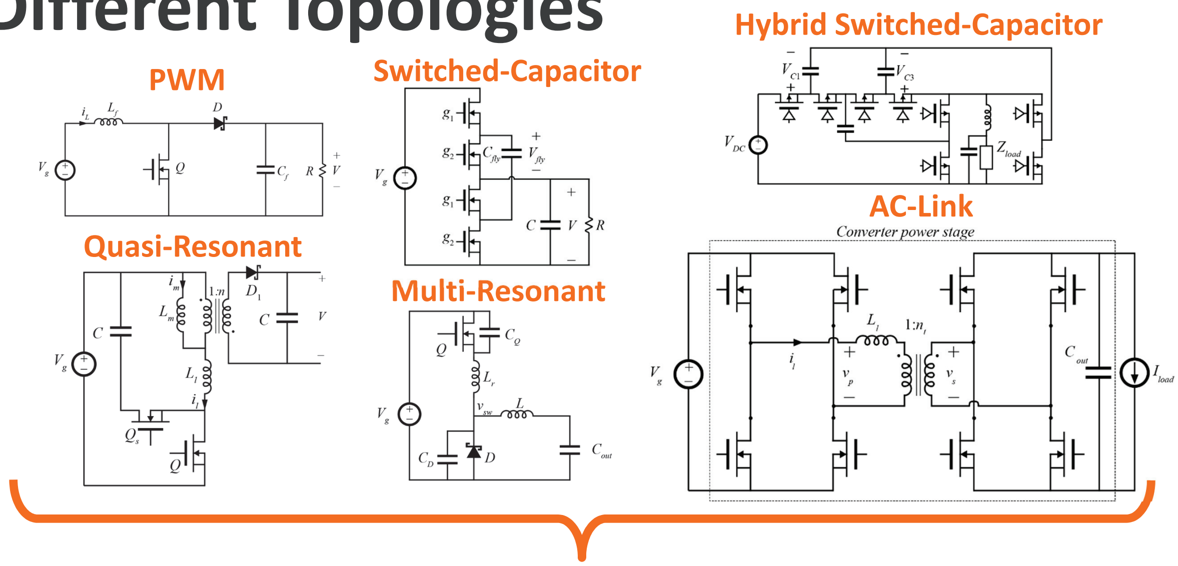
D. Costinett, R. Zane, and D. Maksimovic, "Discrete-time small-signal modeling of a 1 MHz efficiency-optimized dual active bridge converter with varying load," in Proc. IEEE Workshop Contr. Modl. (COMPEL), june 2012, pp. 1 –7.

Different Operating Modes



$$X_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{A_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{A_k t_k} \right) \boldsymbol{A}_i{}^{-1} (e^{A_i t_i} - \boldsymbol{I}) \boldsymbol{B}_i \right\} U$$

Different Topologies

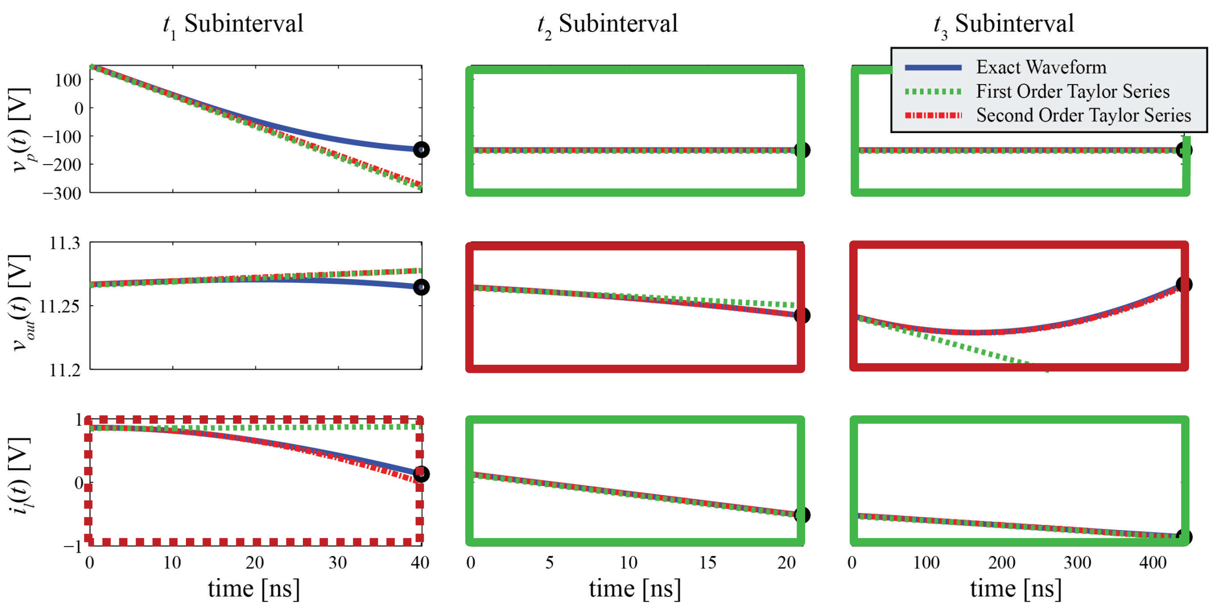


$$\boldsymbol{X}_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{A_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{A_k t_k} \right) \boldsymbol{A}_i^{-1} \left(e^{A_i t_i} - \boldsymbol{I}\right) \boldsymbol{B}_i \right\} U$$

Applications of Analysis

- Dedicated analysis (e.g. State Plane) useful to obtain detailed knowledge of circuit operation
  - Effort required precludes consideration of broad range of vastly different designs
  - In complex, high-performance circuits, result is often still not “invertible”
- Discrete time analysis is general, and requires no dedicated analysis
  - Can we get the same level of **design intuition**
    - By supplementing with dedicated analysis
    - By using computational design

Analytical Approach: Selective Linearization



$e^{A_1 t_1}$	$e^{A_2 t_2}$	$e^{A_3 t_3}$
$I + A_1 t_1$	$I + A_2 t_2$	$I + A_3 t_3$
$I + A_1 t_1 + \frac{1}{2}(A_1 t_1)^2$	$I + A_2 t_2 + \frac{1}{2}(A_2 t_2)^2$	$I + A_3 t_3 + \frac{1}{2}(A_3 t_3)^2$

Model ZVS as Disturbance

Consider how model takes transition into account:

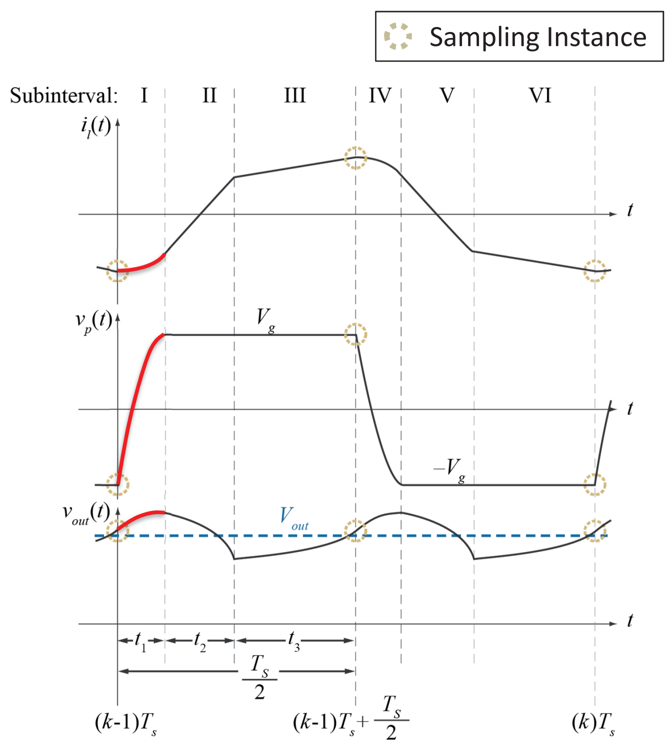
$$\boldsymbol{\Phi} = e^{\mathbf{A}_3 t_3} e^{\mathbf{A}_2 t_2} e^{\mathbf{A}_1 t_1} \boldsymbol{I}_{HC}$$

where

$$\hat{\boldsymbol{x}}(t_1) = e^{\mathbf{A}_1 t_1} \hat{\boldsymbol{x}}(t_0)$$

with $\boldsymbol{A}_1 \in \Re^{3x3}$

Using dedicated analysis, solve new matrix $\boldsymbol{A}_{res} \in \Re^{2x2}$ which models how ZVS transition affects states at t=t_1



Resonant Transition Matrix

Eliminating resonant interval reduces system to second order, $\boldsymbol{x} = [\ i_l \ \ v_{out}]^T$

New matrix takes the form

$$\boldsymbol{A}_{res} = \begin{bmatrix} \frac{\partial v_{out}(t_1)}{\partial v_{out}(0)} & \frac{\partial v_{out}(t_1)}{\partial i_l(0)} \\ \frac{\partial i_l(t_1)}{\partial v_{out}(0)} & \frac{\partial i_l(t_1)}{\partial i_l(0)} \end{bmatrix}$$

Linearized with respect to $\boldsymbol{x}(0)$, rather than time

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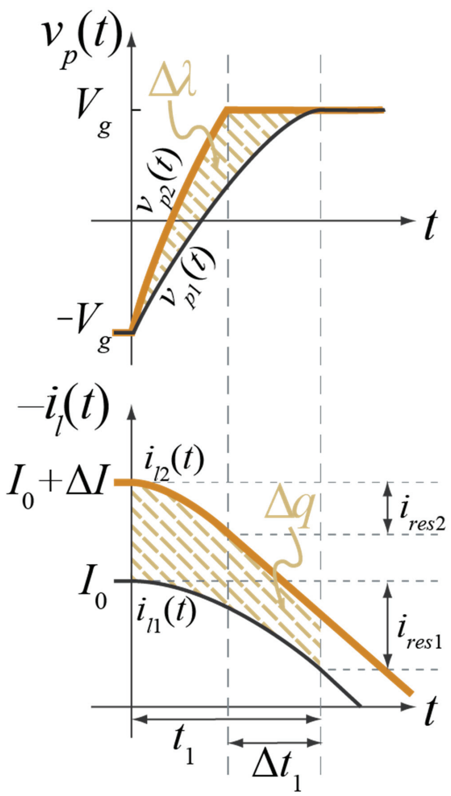
Resonant Interval Solution



Linearized relations solved using circuit analysis

$$\frac{\Delta\lambda}{\Delta I} = L_l - L_l\sqrt{1 - \left(\frac{2V_g}{R_0 I_0}\right)^2} \ ,$$

$$\frac{\Delta Q}{\Delta I} = \frac{2V_g C_p}{I_0} \ ,$$

Relationships vary depending on operating mode

D. Costinett, D. Seltzer, R. Zane, and D. Maksimovic, "Analysis of inherent volt-second balancing of magnetic devices in zero-voltage switched power converters," in Proc. Appl. Power Electron. Conf. (APEC), arch 2013, pp. 9–15.

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Linearized Result

Assuming small ripple on V_{out}

$$\boldsymbol{A}_{res} = \begin{bmatrix} \frac{\partial v_{out}(t_1)}{\partial v_{out}(0)} & \frac{\partial v_{out}(t_1)}{\partial i_l(0)} \\ \frac{\partial i_l(t_1)}{\partial v_{out}(0)} & \frac{\partial i_l(t_1)}{\partial i_l(0)} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\Delta Q}{\Delta I}\frac{1}{C_{out}} \\ 0 & 1 - \frac{\Delta\lambda}{\Delta I}\frac{1}{L_l} \end{bmatrix}$$

Resulting model is now

$$\boldsymbol{\Phi} = e^{\mathbf{A}_3 t_3} e^{\mathbf{A}_2 t_2} \mathbf{A}_{\text{res}} \boldsymbol{I}_{HC}$$

$$\boldsymbol{\Gamma} = e^{\mathbf{A}_3 t_3}(\mathbf{A}_2 - \mathbf{A}_3)\mathbf{X}_0$$

where $\boldsymbol{\Phi} \in \mathfrak{R}^{2x2}$ and $\boldsymbol{\Gamma} \in \mathfrak{R}^{2x1}$

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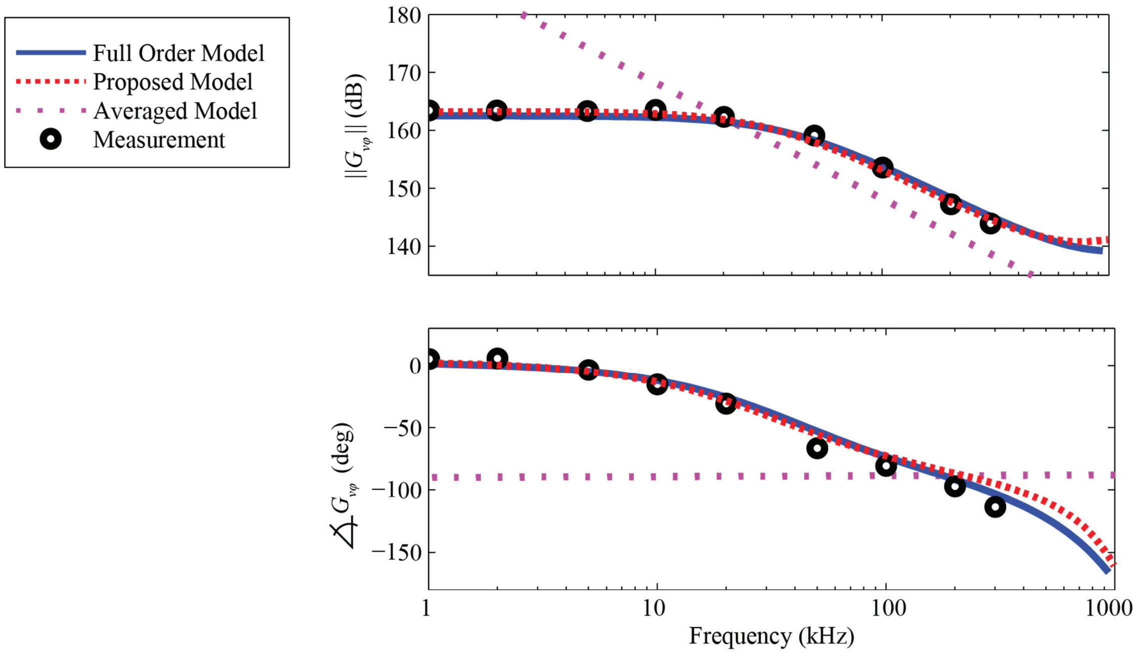
Resulting Model

$$\mathbf{\Phi} = \begin{bmatrix} 1 - \frac{{\omega_f}^2 {t_\zeta}^2}{2{n_t}^2} & \frac{\frac{\frac{\Delta Q}{\Delta I}(L_l n_t - \frac{{t_\zeta}^2}{2C_{out} n_t}) + t_\zeta(\frac{\Delta\lambda}{\Delta I} - L_l)}{n_t}}{{w_f}^2} \\ \frac{t_\zeta}{L_l n_t} & -1 + \frac{\frac{\frac{\Delta\lambda}{\Delta I}(C_{out} n_t - \frac{{t_\zeta}^2}{2L_l n_t}) - t_\zeta \frac{\Delta Q}{\Delta I} + \frac{{t_\zeta}^2}{2n_t}}{n_t}}{{w_f}^2} \end{bmatrix}$$

Previously, an accurate, closed-form expression for $\boldsymbol{\Phi}$ was intractable

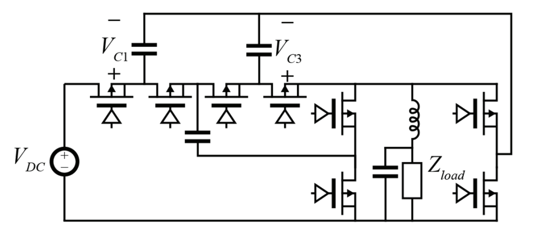
Model now of suitable complexity for design-oriented analysis

Model Accuracy

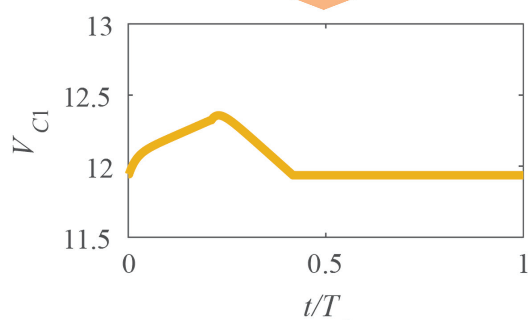


Numerical Approach: HDSC Example

- 4:1 Hybrid Dickson Switched-Capacitor Converter
- 48-to-5 V, 0-100 A output
- Including C_{oss}, 13 states, 3 subintervals

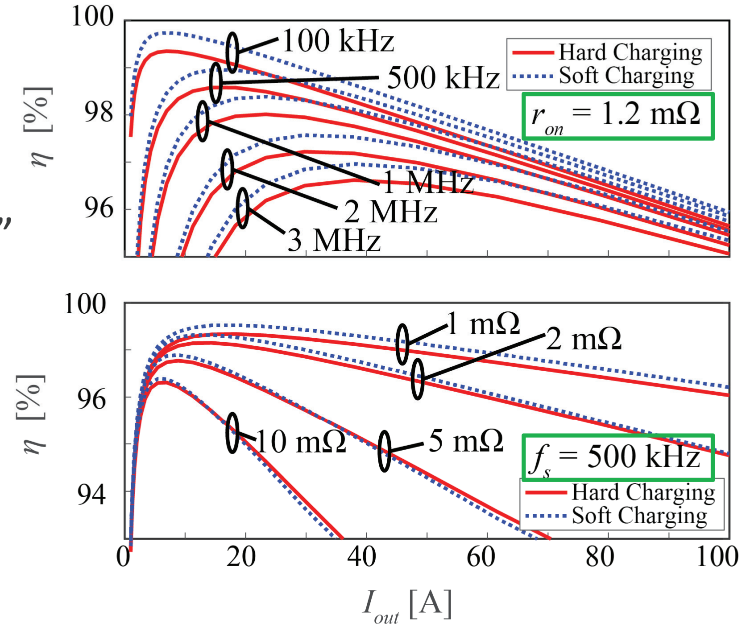


$$X_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{A_i t_i} \right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{A_k t_k} \right) A_i^{-1} (e^{A_i t_i} - I) B_i \right\} U$$



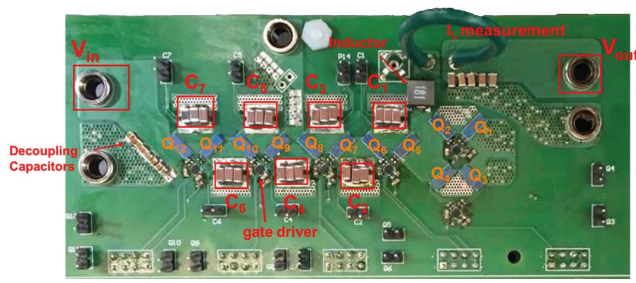
HDSC: Predicted Efficiency

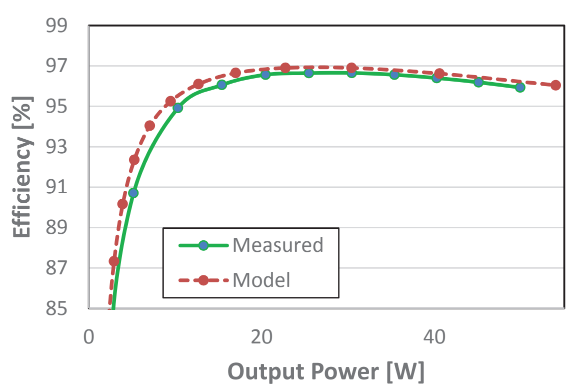
- Inherent efficiency prediction
  - Full, large-signal models
  - No "ideal waveform" assumption
- Total computation time ~10ms per operating point
- No converter-specific derivation



Model Validation

- Constructed 8:1 HDSC converter
  - Measured 96.7% peak efficiency at 30W
  - Model predicts 96.9% at 30.4W
- Model includes capacitor ESR





Limiting Assumptions of the Approach

1. Inputs are DC or slowly varying
2. All subinterval times are known
3. In each subinterval, converter reduces to a linear equivalent circuit

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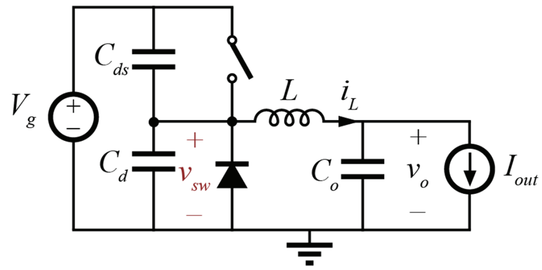
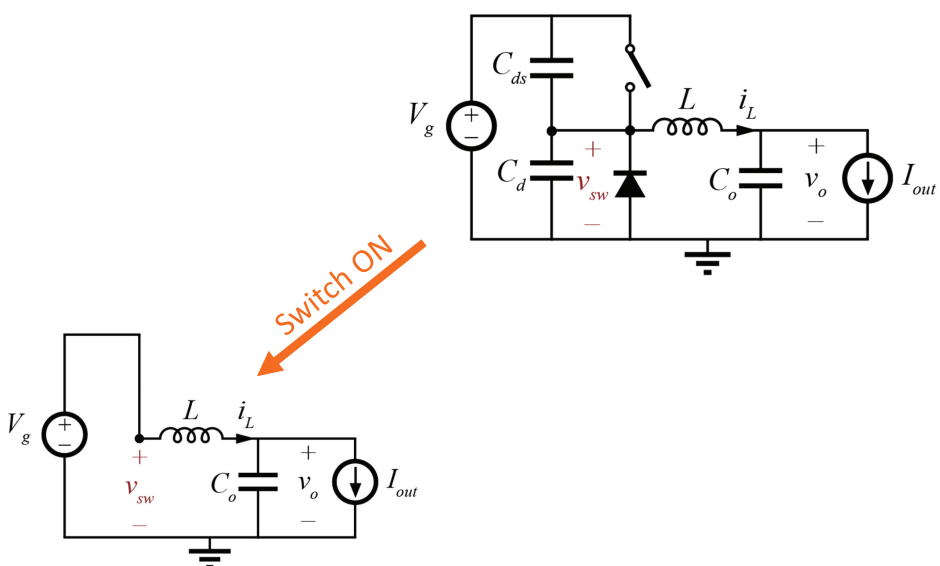
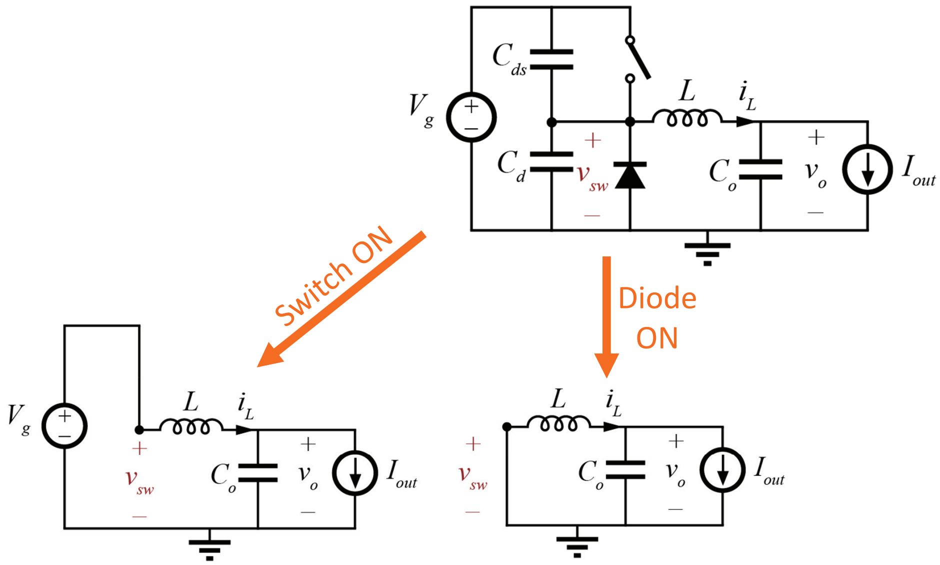
Non-Controlled Switching Times



Non-Controlled Switching Times



Non-Controlled Switching Times



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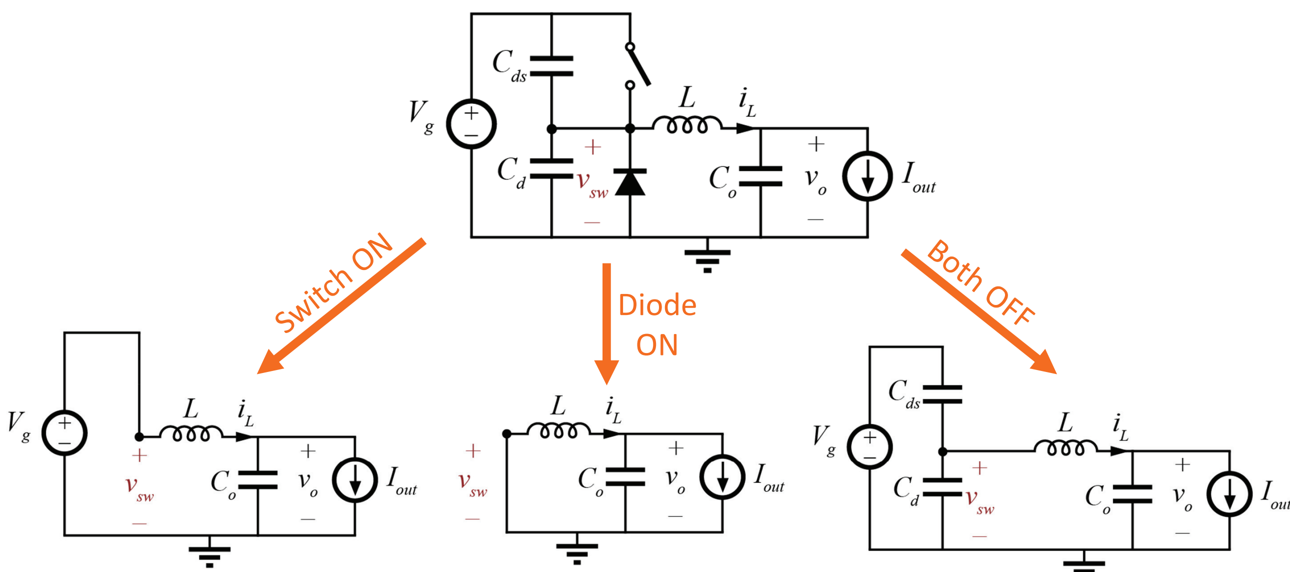

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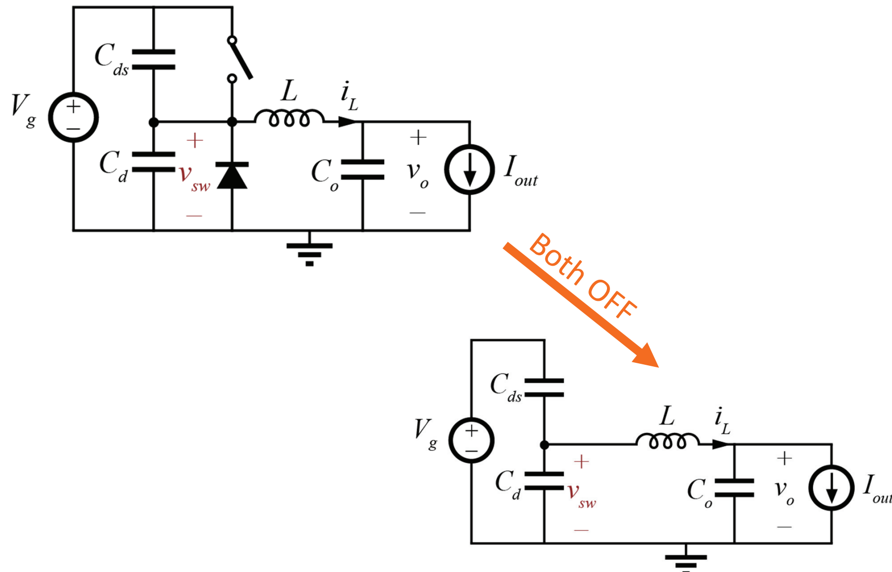

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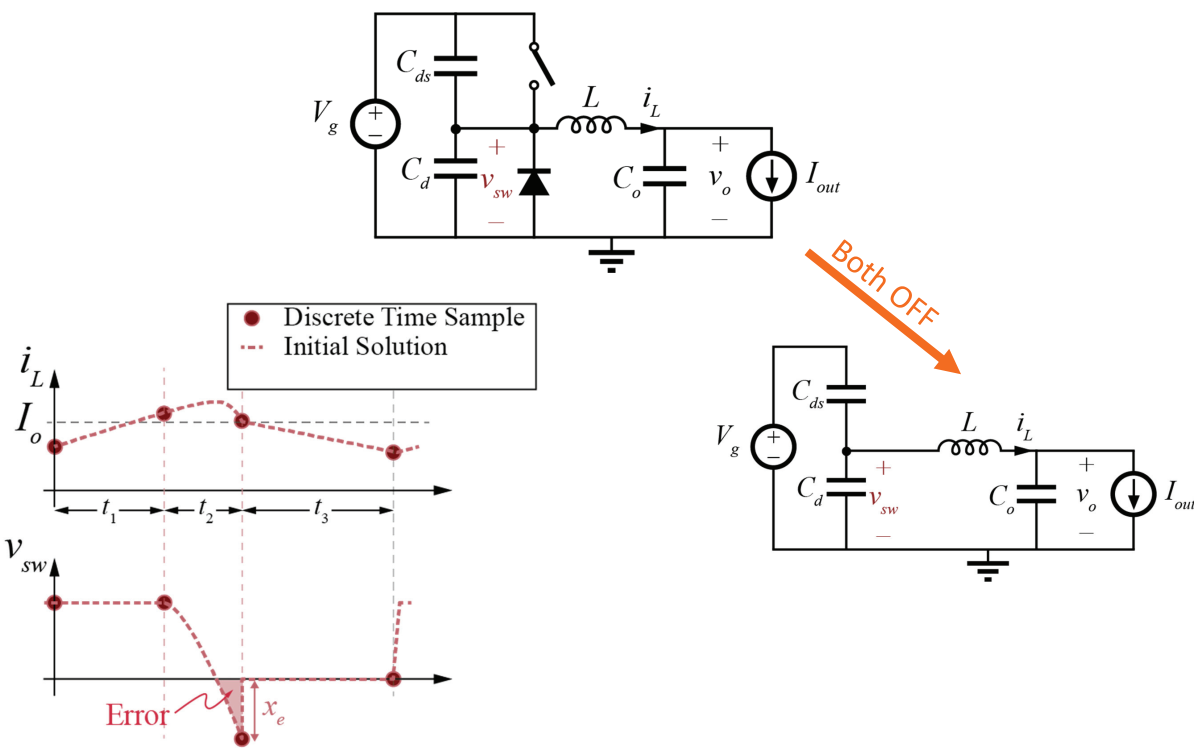

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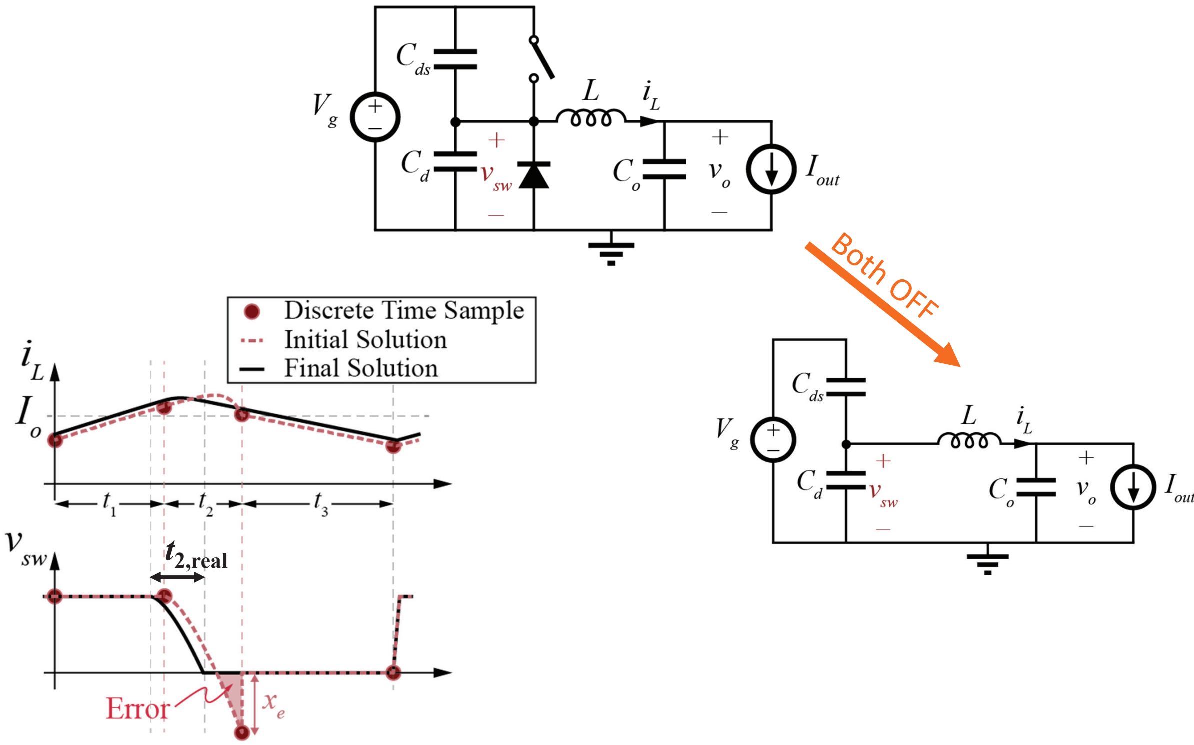

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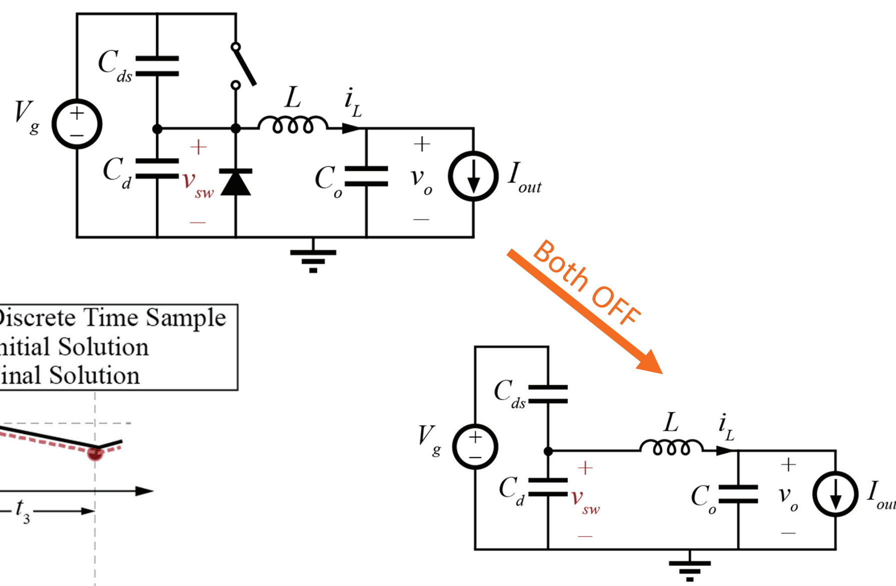
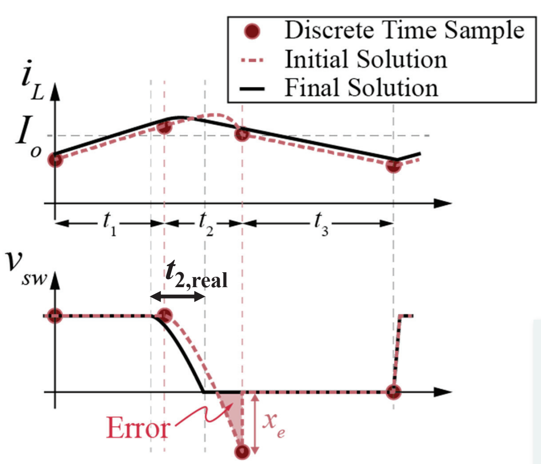
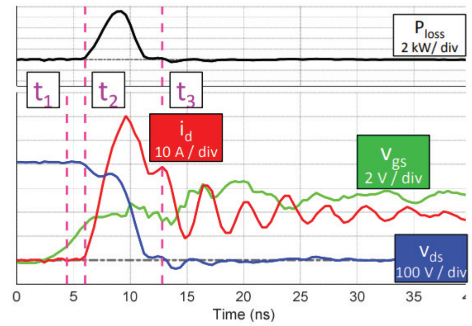




May require iteration and validation to account for non-controlled switching

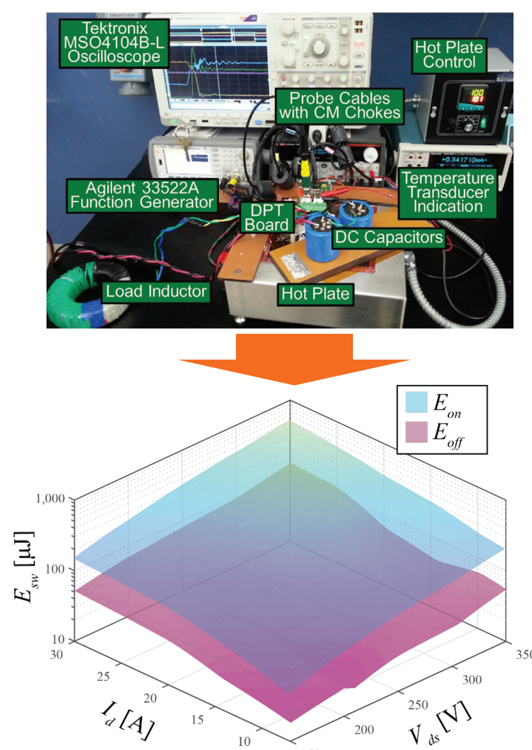
Nonlinear Losses: Switching Loss

- High frequency switching behaviors highly nonlinear



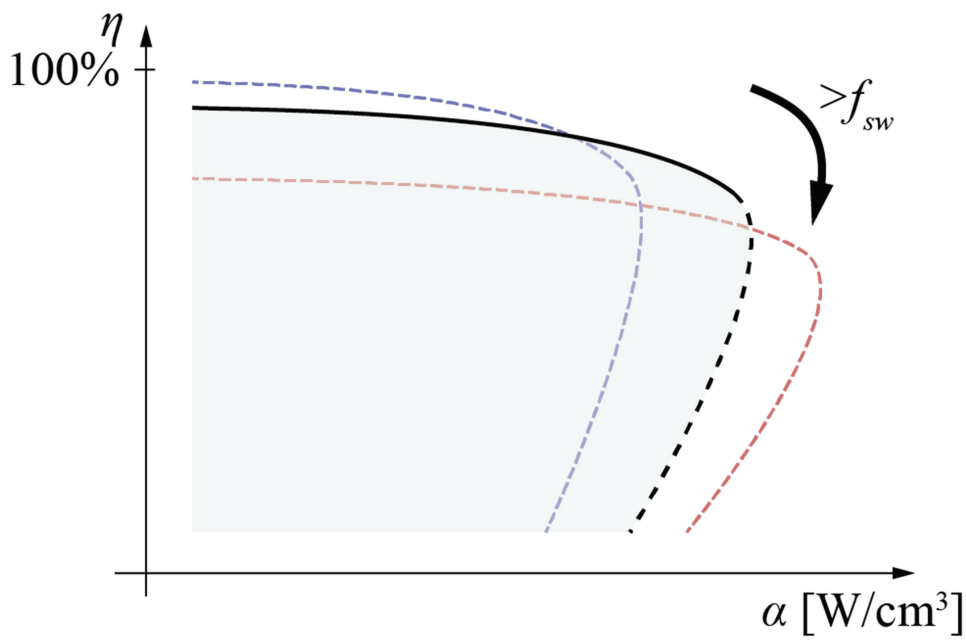
Switching Loss Modeling

- High frequency switching behaviors highly nonlinear
- Common approach: Empirical characterization
- Still reverts to "high-η" approximation

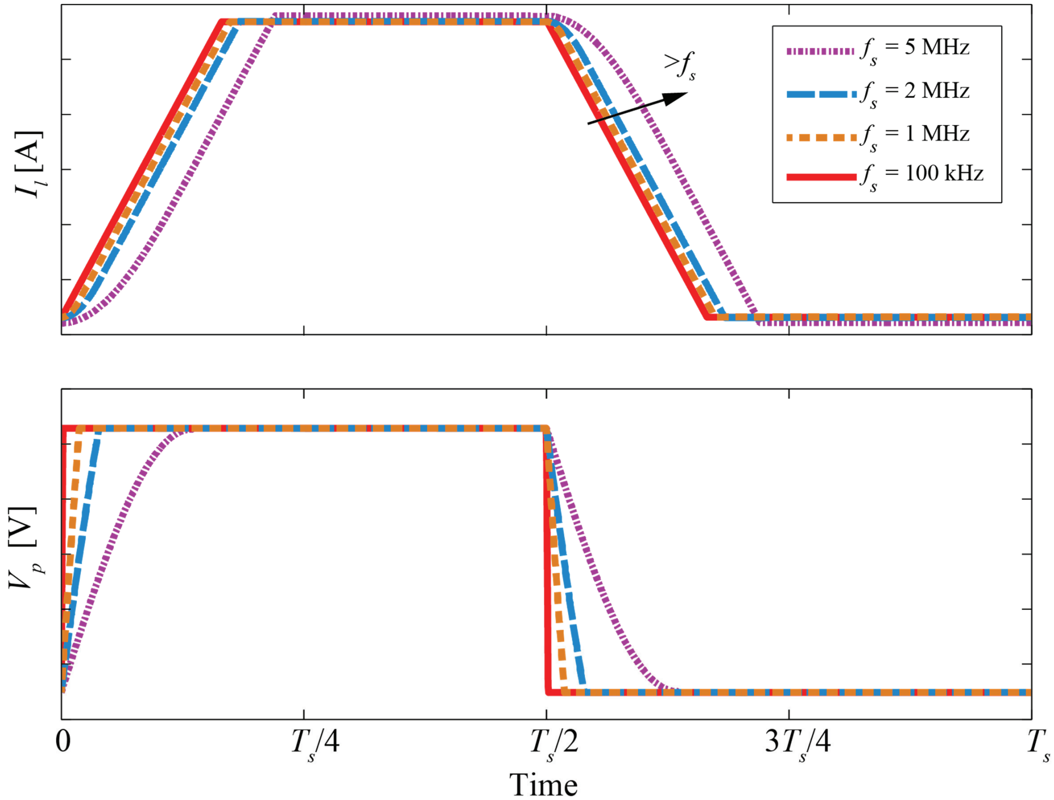


COURSE CONCLUSIONS

HF Power Electronics – When and Why



HF Power Electronics – When and Why



Thank you for all your hard work, and good luck with finals!

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