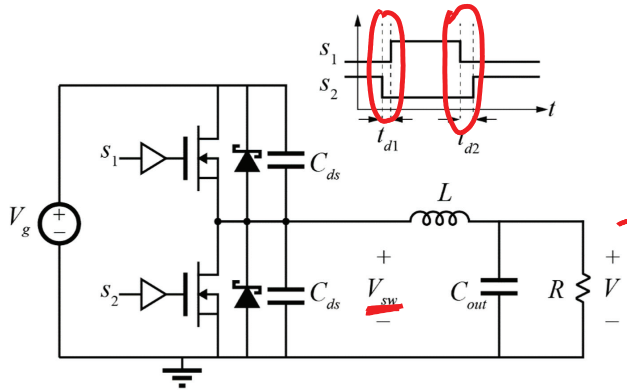
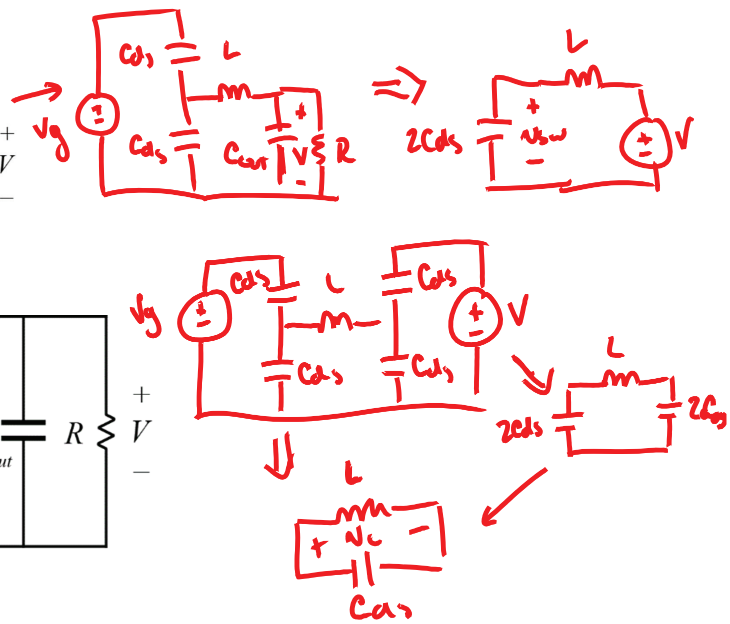
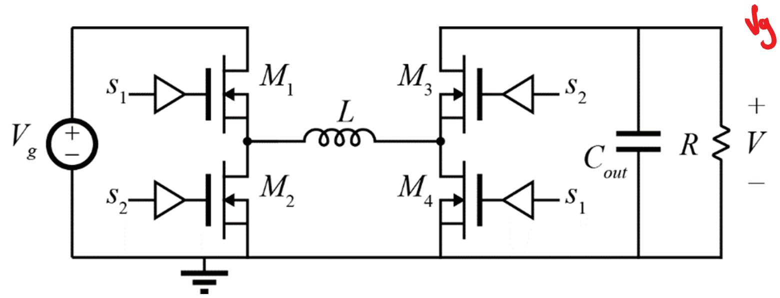
Time-Domain Analysis of Switching Transitions

(1) Assume C_{ds} is linear (e.g. by equivalent)

(2) Assume $C_{out} >> C_{ds}$

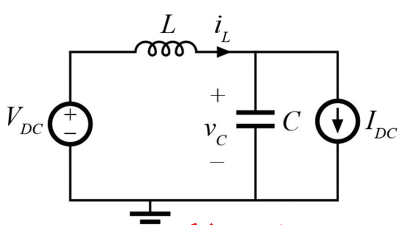






Resonant Circuit Solution

General L-C resonant circuit w/ bias



Initial conditions:

$v_c(t=0) = V_0$

$i_L(t=0) = I_0$

① $C\frac{dv_c}{dt} = i_L - I_{DC}$

② $L\frac{di_L}{dt} = V_{DC} - v_c$

$$V_{DC} - v_c = L\frac{d}{dt}\left(C\frac{dv_c}{dt} + \cancel{I_{DC}}\right) \quad \phi$$

$$V_{DC} - v_c = LC\frac{d^2v_c}{dt^2}$$

$$LC\frac{d^2v_c}{dt^2} + v_c - V_{DC} = \phi$$

$$\begin{cases} v_c = \sqrt{\frac{L}{C}}(I_0 - I_{DC})\sin\left(\frac{t}{\sqrt{LC}}\right) + (V_0 - V_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + V_{DC} \\ i_L = \sqrt{\frac{C}{L}}(V_{DC} - V_0)\sin\left(\frac{t}{\sqrt{LC}}\right) + (I_0 - I_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + I_{DC} \end{cases}$$

Normalization and Notation

Define $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$, $R_0 = \sqrt{\frac{L}{C}}$

$$v_c = R_0(I_0 - I_{DC})\sin(\omega_0 t) + (V_0 - V_{DC})\cos(\omega_0 t) + V_{DC}$$

$$i_L = -\frac{1}{R_0}(V_0 - V_{DC})\sin(\omega_0 t) + (I_0 - I_{DC})\cos(\omega_0 t) + I_{DC}$$

Normalize:

V_{base} is arbitrary DC voltage

$$m_c = \frac{v_c}{V_{base}} \qquad I_{base} = \frac{V_{base}}{R_0}$$

$$j_L = \frac{i_L}{I_{base}}$$

$$\theta_x = \omega_0 t_x$$

Pick $V_{base} = V_{DC}$

$I_{base} = \frac{V_{DC}}{R_0}$

$$v_c(t) = V_{DC} + (V_0 - V_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + (I_0 - I_{DC})\overbrace{\sqrt{\frac{L}{C}}}^{R_0}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$i_L(t) = I_{DC} + (I_0 - I_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + (V_{DC} - V_0)\overbrace{\sqrt{\frac{C}{L}}}^{1/R_0}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$\begin{cases} m_c = \frac{v_c}{V_{DC}} = 1 + \left(\frac{V_0}{V_{DC}} - 1\right)\cos(\omega_0 t) + \frac{I_0 - I_{DC}}{V_{DC}}R_0 \sin(\omega_0 t) \\ j_L = \frac{I_{DC}R_0}{V_{DC}} + \frac{R_0}{V_{DC}}(I_0 - I_{DC})\cos(\omega_0 t) + \cancel{R_0}\frac{1}{R_0}\left(1 - \frac{V_0}{V_{DC}}\right)\sin(\omega_0 t) \end{cases}$$

$$\begin{cases} m_c = 1 + A\cos\omega_0 t + B\sin\omega_0 t \\ j_L = \frac{I_{DC}R_0}{V_{DC}} + B\cos\omega_0 t - A\sin\omega_0 t \end{cases}$$

Try looking at $(m_c - 1)^2 + \left(j_L - \frac{I_{DC}R_0}{V_{DC}}\right)^2 =$

$$A^2\cos^2\omega_0 t + B^2\sin^2\omega_0 t + \cancel{2AB\cos\omega_0 t\sin\omega_0 t} + B^2\cos^2\omega_0 t + A^2\sin^2\omega_0 t$$

$$- \cancel{2AB\cos\omega_0 t\sin\omega_0 t}$$

$$A^2\underbrace{(\cos^2\omega_0 t + \sin^2\omega_0 t)}_{1} + B^2\underbrace{(\cos^2\omega_0 t + \sin^2\omega_0 t)}_{1}$$

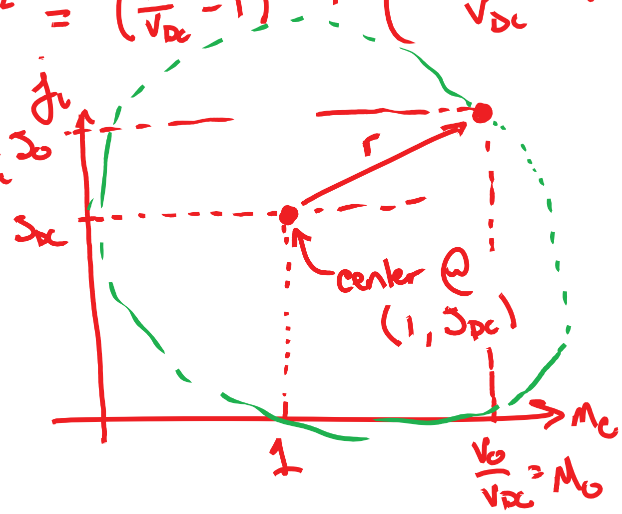
$$= A^2 + B^2 = \left(\frac{V_0}{V_{DC}} - 1\right)^2 + \left(\frac{R_0}{V_{DC}}(I_0 - I_{DC})\right)^2$$

$$(m_c(t)-1)^2 + \left(j_L(t) - \frac{I_{DC}R_o}{V_{DC}}\right)^2 = \left(\frac{V_o}{V_{DC}}-1\right)^2 + \left(\frac{R_o(I_o-I_{DC})}{V_{DC}}\right)^2$$

This is the equation for a circle!

- center at $m_c = 1$, $j_L = \frac{I_{DC}R_o}{V_{DC}}$
- radius is $r^2 = \left(\frac{V_o}{V_{DC}}-1\right)^2 + \left(\frac{R_o(I_o-I_{DC})}{V_{DC}}\right)^2$

$J_{DC} = \frac{I_{DC}R_o}{V_{DC}} = \frac{I_{DC}}{I_{base}}$ $\quad \frac{I_o}{I_{base}} = J_o$



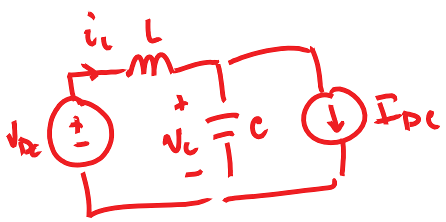
Take Diff EQs →

Geometric / trig analysis

State Plane Analysis

Direction: look at 1st derivatives

@ t=0 : $C\frac{dv_c}{dt} = I_o - I_{DC}$



$m_c = \frac{v_c}{V_{base}}$ $\quad V_{base} =$ ____

$j_L = \frac{i_L}{I_{base}}$ $\quad I_{base} = \frac{V_{base}}{R_o}$

Circle center = DC solution

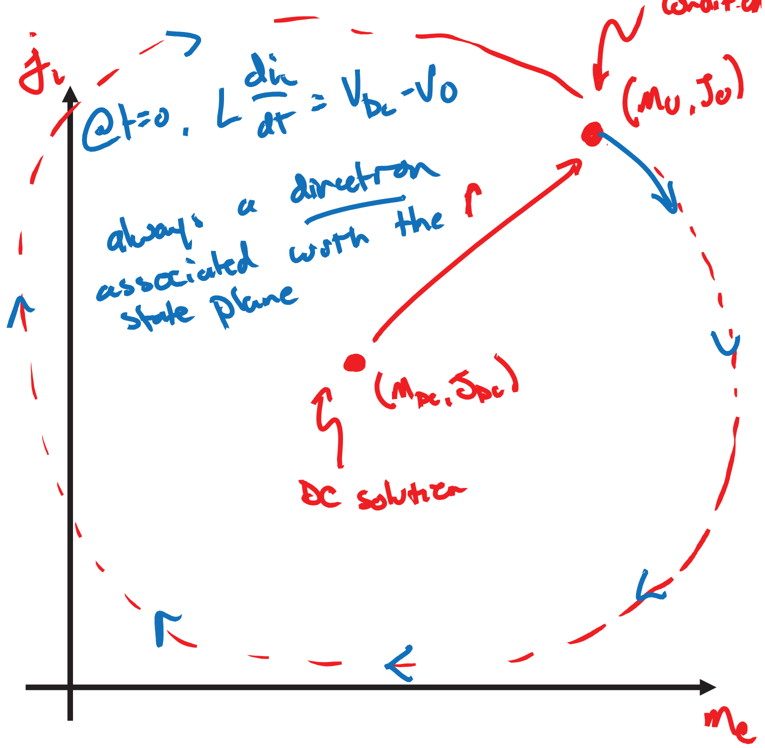
$v_c = V_{DC}$ $\quad m_c = M_{DC}$

$i_L = I_{DC}$ $\quad j_L = J_{DC}$

Initial condition:

$v_c(t=0) = V_o \rightarrow M_o$

$i_L(t=0) = I_o \rightarrow J_o$



[1] R. Oruganti and F. C. Lee, "Resonant Power Processors, Part I – State Plane Analysis", Industry Applications, IEEE Tran. on, vol. 21, no. 6, nov 1985.

[2] D. P. Atherton, Nonlinear Control Engineering. London: Van Nostrand Reinhold, 1982, Ch. 2.