Power System Model Reduction

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Model Reduction of Large Power Systems

1. Simulation of power system dynamics for stability analysis on a digital computer: needs the most comprehensive power system model so that
   i. the relevant dynamics can be accurately simulated given the computing resources
   ii. the simulation can be completed in a reasonable amount of time.

2. One of the decisions is the geographic extent of the power system data set: disturbance always in the study region, but external system(s) also needed.
Model Reduction of Large Power Systems

Internal and external systems

Disturbance travelling through the external system

Gen + Controls
\[ \tilde{V}_{b1} \]
\[ \vdots \]
Gen + Controls
\[ \tilde{V}_{b2} \]

Gen + Controls
\[ \tilde{I}_1 \]
\[ \tilde{I}_2 \]
\[ \vdots \]
Gen + Controls
\[ \tilde{I}_k \]

Gen + Controls
\[ \Delta P_1 \]
\[ \Delta P_2 \]

Gen + Controls
\[ \tilde{V}_{b1} \]
\[ \vdots \]
Gen + Controls
\[ \tilde{V}_{b2} \]

Gen + Controls
\[ \frac{A}{X} \]
\[ B \]

Gen + Controls

Study System

External System

Study System

External System
Model Reduction Approaches

1. Coherency – some machines swinging together after a disturbance
   i. Time simulation (R. Podmore)
   ii. Modal coherency (R. Schleuter)
   iii. Slow coherency (J. Chow, et. al) – coherency with respect to the slow interarea modes
   iv. Weak-link method – slow coherent areas are weakly connected (N. Rao, J. Zaborzsky)

2. Aggregation – obtaining an equivalent power system model
   i. Generator aggregation (R. Podmore)
   ii. Singular perturbations – provides improvements to equivalent generator reactances (J. Chow, et. al)
Model Reduction Approaches

3. Linear model analysis of external systems

i. Modal truncation (J. Undrill, W. W. Price)

ii. Selective modal analysis (SMA) (G. Verghese, I. Peres-Arriaga, L. Rouco)

iii. Krylov method – moment matching (D. Chaniotis, M.A. Pai)

iv. Balanced truncation – “best” reachibility (controllability) - observability (Shanshan Liu)

4. Artificial Neural Networks (F. Ma, V. Vittal)

5. Synchrophasor data (A. Chakrabortty, Chow)
Monographs

1982

Lecture Notes in Control and Information Sciences
Edited by A.V. Balakrishnan and M. Thoma

Time-Scale Modeling of Dynamic Networks with Applications to Power Systems
Edited by J.H. Chow

2013

Power System Coherency and Model Reduction
Joe H. Chow Editor
Springer
Topics

1. Slow coherency
2. Reduced order modeling of dominant transfer paths in large power systems using synchrophasor data (work with Dr. Aranya Chakrabortty)
A large power system usually consists of tightly connected control regions with few interarea ties for power exchange and reserve sharing.

The oscillations between these groups of strongly connected machines are the interarea modes.

These interarea modes are lower in frequency than local modes and intra-plant modes.

Singular perturbations method can be used to show this time-scale separation.
Coherency in 2-Area System

Disturbance: 3-phase fault at Bus 3, cleared by removal of 1 line between Buses 3 and 101
Power System Model

An $n$-machine, $N$-bus power system with classical electromechanical model and constant impedance loads:

$$m_i \ddot{\delta}_i = P_{mi} - P_{ei} = P_{mi} - \frac{E_i V_j \sin(\delta_i - \theta_j)}{x'_{di}} = f_i(\delta, V) \quad (1)$$

where

- machine $i$ modeled as a constant voltage $E_i$ behind a transient reactance $x'_{di}$
- $m_i = 2H_i/\Omega$, $H_i =$ inertia of machine $i$
- $\Omega = 2\pi f_o =$ nominal system frequency in rad/s
- damping $D = 0$
- $\delta = n$-vector of machine angles
- $P_{mi} =$ input mechanical power, $P_{ei} =$ output electrical power

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the bus voltage

$$V_j = \sqrt{V_{jre}^2 + V_{jim}^2}, \quad \theta_j = \tan^{-1} \left( \frac{V_{jim}}{V_{jre}} \right) \quad (2)$$

- $V_{jre}$ and $V_{jim}$ are the real and imaginary parts of the bus voltage phasor at Bus $j$, the terminal bus of Machine $i$
- $V = 2N$-vector of real and imaginary parts of load bus voltages
Power Flow Equations

For each load bus \( j \), the active power flow balance

\[
P_{ej} - \text{Real} \left\{ \sum_{k=1, k \neq j}^{N} (V_{jre} + jV_{jim} - V_{kre} - jV_{kim}) \left( \frac{V_{jre} + jV_{jim}}{R_{Ljk} + jX_{Ljk}} \right)^* \right\} - V_j^2 G_j = g_{2j} - 1 = 0 \quad (3)
\]

and the reactive power flow balance

\[
Q_{ej} - \text{Imag} \left\{ \sum_{k=1, k \neq j}^{N} (V_{jre} + jV_{jim} - V_{kre} - jV_{kim}) \left( \frac{V_{jre} + jV_{jim}}{R_{Ljk} + jX_{Ljk}} \right)^* \right\} - V_j^2 B_j + V_j^2 \frac{B_{Ljk}}{2} = g_{2j} = 0 \quad (4)
\]

- \( R_{Ljk}, X_{Ljk}, \) and \( B_{Ljk} \) are the resistance, reactance, and line charging, respectively, of the line \( j-k \)
- \( P_{ej} \) and \( Q_{ej} \) are generator active and reactive electrical output power, respectively, if bus \( j \) is a generator bus
- \( G_j \) and \( B_j \) are the load conductance and susceptance at bus \( j \)

Note that \( j \) denotes the imaginary number if it is not used as an index.
Electromechanical Model

\[ M \ddot{\delta} = f(\delta, V), \quad 0 = g(\delta, V) \quad (5) \]

- \( M \) = diagonal machine inertia matrix
- \( f \) = vector of acceleration torques
- \( g \) = power flow equation
Linearized Model

Linearize (5) about a nominal power flow equilibrium \((\delta_o, V_o)\) to obtain

\[
M \Delta \ddot{\delta} = \left. \frac{\partial f(\delta, V)}{\partial \delta} \right|_{\delta_o, V_o} + \left. \frac{\partial f(\delta, V)}{\partial V} \right|_{\delta_o, V_o} = K_1 \Delta \delta + K_2 \Delta V \quad (6)
\]

\[
0 = \left. \frac{\partial g(\delta, V)}{\partial \delta} \right|_{\delta_o, V_o} + \left. \frac{\partial g(\delta, V)}{\partial V} \right|_{\delta_o, V_o} = K_3 \Delta \delta + K_4 \Delta V \quad (7)
\]

- \(\Delta \delta = n\)-vector of machine angle deviations from \(\delta_o\)
- \(\Delta V = 2N\)-vector of the real and imaginary parts of the load bus voltage deviations from \(V_o\)
- \(K_1, K_2, \text{ and } K_3\) are partial derivatives of the power transfer between machines and terminal buses, \(K_1\) is diagonal
- \(K_4 = \) network admittance matrix and nonsingular.
- the sensitivity matrices \(K_i\) can be derived analytically or from numerical perturbations using the Power System Toolbox
Linearized Model

Solve (6) for

\[ \Delta V = -K_4^{-1} K_3 \Delta \delta \] (8)

to obtain

\[ M \Delta \ddot{\delta} = K_1 - K_2 K_4^{-1} K_3 \Delta \delta = K \Delta \delta \] (9)

where

\[ K_{ij} = E_i E_j (B_{ij} \cos(\delta_i - \delta_j) - G_{ij} \sin(\delta_i - \delta_j)) \big|_{\delta_o, V_o} , \quad i \neq j \] (10)

and \( G_{ij} + j B_{ij} \) is the equivalent admittance between machine \( i \) and \( j \). Furthermore,

\[ K_{ii} = - \sum_{j=1, j \neq i}^{n} K_{ij} \] (11)

Thus the row sum of \( K \) equal to zero. The entries \( K_{ij} \) are known as the synchronizing torque coefficients.
Slow Coherent Areas

Assume a power system has \( r \) slow coherent areas of machines and the load buses that interconnect these machines.

Define

\[ \Delta \delta_i^\alpha = \text{deviation of rotor angle of machine } i \text{ in area } \alpha \text{ from its equilibrium value} \]

\[ m_i^\alpha = \text{inertia of machine } i \text{ in area } \alpha \]

Order the machines such that \( \Delta \delta_i^\alpha \) from the same coherent areas appears consecutively in \( \Delta \delta \).
Weakly Coupled Areas

We attribute the slow coherency phenomenon to be primarily due to the connections between the machines in the same coherent areas being stiffer than those between different areas, which can be due to two reasons:

- The admittances of the external connections $B_{ij}^E$ much smaller than the admittances of the internal connections $B_{pq}^I$

  $\varepsilon_1 = \frac{B_{ij}^E}{B_{pq}^I}$  \hspace{1cm} (12)

where $E$ denotes external, $I$ denotes internal, and $i, j, p, q$ are bus indices. This situation also includes heavily loaded high-voltage, long transmission lines between two coherent areas.
The number of external connections is much less than the number of internal connections

\[ \varepsilon_2 = \frac{\bar{\gamma}^E}{\gamma^I} \tag{13} \]

where

\[ \bar{\gamma}^E = \max_{\alpha} \{ \gamma^E_\alpha \}, \quad \gamma^I = \min_{\alpha} \{ \gamma^I_\alpha \}, \quad \alpha = 1, \ldots, r \tag{14} \]

\[ \gamma^E_\alpha = \text{(the number of external connections of area } \alpha)/N^\alpha \]
\[ \gamma^I_\alpha = \text{(the number of internal connections of area } \alpha)/N^\alpha \]

where \( N^\alpha \) is the number of buses in area \( \alpha \).
Internal and External Connections

For a large power system, the weak connections between coherent areas can be represented by the small parameter

\[ \varepsilon = \varepsilon_1 \varepsilon_2 \] (15)

Separate the network admittance matrix into

\[ K_4 = K_4^I + \epsilon K_4^E \] (16)

where \( K_4^I = \) internal connections and \( K_4^E = \) external connections. The synchronizing torque or connection matrix \( K \) is

\[
K = K_1 - K_2(K_4^I + \varepsilon(K_4^I))^{-1}K_3 \\
= K_1 - K_2(K_4^I(I + \varepsilon(K_4^I)^{-1}K_4^E))^{-1}K_3 = K^I + \epsilon K^E
\] (17)

where

\[
K^I = K_1 - K_2(K_4^I)^{-1}K_3, \quad K^E = -K_2K_4^E\varepsilon K_3
\] (18)

In the separation (17), the property that each row of \( K^I \) sums to zero is preserved.
Define for each area an inertia weighted \textit{aggregate variable}

\[ y^\alpha = \sum_{i=1}^{n^\alpha} m_i^\alpha \Delta \delta_i^\alpha / m^\alpha, \quad \alpha = 1, 2, \ldots, r \]  

where \( m_i^\alpha = \) inertia of machine \( i \) in area \( \alpha \), \( n^\alpha = \) number of machines in area \( \alpha \), and

\[ m^\alpha = \sum_{i=1}^{n^\alpha} m_i^\alpha, \quad \alpha = 1, 2, \ldots, r \]

is the aggregate inertia of area \( \alpha \).
Denote by $y$ the $r$-vector whose $\alpha$th entry is $y^\alpha$. The matrix form of (19) is

$$y = C \Delta \delta = M_a^{-1} U^T M \Delta \delta$$

(21)

where

$$U = \text{blockdiag}(u_1, u_2, \ldots, u_r)$$

(22)

is the grouping matrix with $n_\alpha \times 1$ column vectors

$$u_\alpha = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}^T, \quad \alpha = 1, 2, \ldots, r$$

(23)

$$M_a = \text{diag}(m^1, m^2, \ldots, m^r) = U^T M U$$

(24)

is the $r \times r$ diagonal aggregate inertia matrix.
Fast Variables

Select in each area a reference machine, say the first machine, and define the motions of the other machines in the same area relative to this reference machine by the local variables

\[ z_{i-1}^\alpha = \Delta \delta_i^\alpha - \Delta \delta_1^\alpha, \quad i = 2, 3, \ldots, n_\alpha, \quad \alpha = 1, 2, \ldots, r \]  

(25)

Denote by \( z^\alpha \) the \((n_\alpha - 1)\)-vector of \( z_i^\alpha \) and consider \( z^\alpha \) as the \( \alpha \)th subvector of the \((n - r)\)-vector \( z \). Eqn. (25) in matrix form is

\[ z = G \Delta \delta = \text{blockdiag}(G_1, G_2, \ldots, G_r) \Delta \delta \]  

(26)

where \( G_\alpha \) is the \((n_\alpha - 1) \times n_\alpha \) matrix

\[
G_\alpha = \begin{bmatrix}
-1 & 1 & 0 & \ldots & 0 \\
-1 & 0 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-1 & 0 & 0 & \ldots & 1
\end{bmatrix}
\]  

(27)
Slow and Fast Variable Transformation

A transformation of the original state $\Delta\delta$ into the aggregate variable $y$ and the local variable $z$

\[
\begin{bmatrix}
y \\
z
\end{bmatrix} = \begin{bmatrix}
C \\
G
\end{bmatrix} \Delta\delta
\]  

(28)

The inverse of this transformation is

\[
\Delta\delta = \begin{pmatrix}
U & G^+
\end{pmatrix} \begin{bmatrix}
y \\
z
\end{bmatrix}
\]  

(29)

where

\[
G^+ = G^T(GG^T)^{-1}
\]  

(30)

is block-diagonal.
Slow Subsystem

Apply the transformation (28) to the model (9), (17)

\[ M_a \ddot{y} = \epsilon K_a y + \epsilon K_{ad} z \]
\[ M_d \ddot{z} = \epsilon K_{da} y + (K_d + \epsilon K_{dd}) z \]  \hspace{1cm} (31)

where

\[ M_d = (GM^{-1}G^T)^{-1}, \quad K_a = U^T K^E U \]
\[ K_{da} = U^T K^E M^{-1} G^T M_d, \quad K_{da} = M_d G M^{-1} K^E U \]
\[ K_d = M_d G M^{-1} K^I M^{-1} G^T M_d, \quad K_{dd} = M_d G M^{-1} K^E M^{-1} G^T M_d \]  \hspace{1cm} (32)

- \( K_a, K_{ad}, \) and \( K_{da} \) are independent of \( K^I \)
- System (31) is in the standard singularly perturbed form
- \( \epsilon \) is both the weak connection parameter and the singular perturbation parameter

Neglecting the fast dynamics, the slow subsystem is

\[ M_a \ddot{y} = \epsilon K_a y \]  \hspace{1cm} (33)
Formulation including Power Network

Apply (28) to the model (6)-(7)

\[
\begin{align*}
M_a \ddot{y} &= K_{11} y + K_{12} z + K_{13} \Delta V \\
M_d \ddot{z} &= K_{21} y + K_{22} z + K_{23} \Delta V \\
0 &= K_{31} y + K_{32} z + (K_4^I + \varepsilon K_4^E) \Delta V 
\end{align*}
\]

(34)

where

\[
\begin{align*}
K_{11} &= U^T K_1 U, & K_{12} &= U^T K_1 G^+, & K_{13} &= U^T K_2, & K_{21} &= (G^+)^T K_1 U \\
K_{22} &= (G^+)^T K_1 G^+, & K_{23} &= (G^+)^T K_2, & K_{31} &= K_3 U, & K_{32} &= K_3 G^+(35)
\end{align*}
\]

Eliminating the fast variables, the slow subsystem is

\[
\begin{align*}
M_a \ddot{y} &= K_{11} y + K_{13} \Delta V \\
0 &= K_{31} y + K_4 \Delta V 
\end{align*}
\]

(36)

This is the inertial aggregate model which is equivalent to linking the internal nodes of the coherent machines by infinite admittances.
2-Area System Example

Connection matrix

\[
K = \begin{bmatrix}
-9.4574 & 8.0159 & 0.5063 & 0.9351 \\
8.7238 & -11.3978 & 0.9268 & 1.7472 \\
0.6739 & 0.9520 & -9.6175 & 7.9917 \\
1.3644 & 1.9325 & 8.1747 & -11.4716
\end{bmatrix}
\]  

(37)

Decompose \( K \) into internal and external connections

\[
K^I = \begin{bmatrix}
-8.0159 & 8.0159 & 0 & 0 \\
8.7238 & -8.7238 & 0 & 0 \\
0 & 0 & -7.9917 & 7.9917 \\
0 & 0 & 8.1747 & -8.1747
\end{bmatrix}
\]  

(38)

\[
\varepsilon K^E = \begin{bmatrix}
-1.4414 & 0 & 0.5063 & 0.9351 \\
0 & -2.6739 & 0.9268 & 1.7472 \\
0.6739 & 0.9520 & -1.6258 & 0 \\
1.3644 & 1.9325 & 0 & -3.2969
\end{bmatrix}
\]  

(39)
EM Modes in 2-Area System

$$\lambda(M^{-1}K) = 0, -14.2787, -60.7554, -62.2531$$ (40)

with the corresponding eigenvector vectors

$$v_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0.4878 \\ 0.4031 \\ -0.5672 \\ -0.5271 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0.6333 \\ -0.7446 \\ 0.1924 \\ -0.0863 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0.1102 \\ -0.1494 \\ -0.8098 \\ 0.5566 \end{bmatrix}$$ (41)

- Interarea mode: $\sqrt{-14.279} = \pm j3.779$ rad/s
- Local modes: $\sqrt{-60.755} = \pm j7.795$ rad/s and $\sqrt{-62.253} = \pm j7.890$ rad/s
Slow and Fast Subsystems of 2-Area System

Slow dynamics:

\[
M_a = \frac{1}{2\pi \times 60} \begin{bmatrix} 234 & 0 \\ 0 & 234 \end{bmatrix}, \quad \varepsilon K_a = \begin{bmatrix} -4.1154 & 4.1154 \\ 4.9227 & -4.9227 \end{bmatrix}
\]  

(42)

The eigenvalues of \( M_a^{-1}K_a \) are 0 and \(-14.561 \Rightarrow \) an interarea mode frequency of \( \sqrt{-14.561} = \pm j3.816 \) rad/s.

Fast local dynamics:

\[
M_d = \frac{1}{2\pi \times 60} \begin{bmatrix} 58.500 & 0 \\ 0 & 55.611 \end{bmatrix}, \quad K_d = \begin{bmatrix} -8.3699 & 0 \\ 0 & -8.0628 \end{bmatrix}
\]  

(43)

The eigenvalues of \( M_d^{-1}K_d \) are \(-53.939 \) and \(-54.660 \Rightarrow \) local modes of \( \pm j7.3443 \) and \( \pm j7.3932 \) rad/s.
Finding Coherent Groups of Machines

1. Compute the electromechanical modes of an $N$-machine power system.
2. Select the (interarea) modes with frequencies less than 1 Hz.
3. Compute the eigenvectors (mode shapes) of these slower modes.
4. Group the machines with similar mode shapes into slow coherent groups.
5. These slow coherent groups have weak or sparse connections between them.
Grouping Algorithm

Plot rows of the slow eigenvector $V_s$ of the interarea modes

Use Gaussian elimination to select the $r$ most separated rows as reference vectors and group them into $V_{s1}$ and reorder $V_s$ as

$$V_s = \begin{bmatrix} V_{s1} \\ V_{s2} \end{bmatrix} \Rightarrow \begin{bmatrix} V_{s1} \\ V_{s2} \end{bmatrix} V_{s1}^{-1} = \begin{bmatrix} I \\ L \end{bmatrix} = \begin{bmatrix} I \\ L_g \end{bmatrix} + \begin{bmatrix} 0 \\ O(\varepsilon) \end{bmatrix}$$

(44)

Use $V_{s1}$ to form a new coordinate system
Coherent Groups in 2-Area System

\[
V_s = \begin{bmatrix}
0.5 & 0.4878 \\
0.5 & 0.4031 \\
0.5 & -0.5672 \\
0.5 & -0.5271
\end{bmatrix}
\begin{array}{c}
\text{Gen 1} \\
\text{Gen 2} \\
\text{Gen 11} \\
\text{Gen 12}
\end{array},
V_{s1} = \begin{bmatrix}
0.5 & 0.4878 \\
0.5 & -0.5672
\end{bmatrix}
\begin{array}{c}
\text{Gen 1} \\
\text{Gen 11}
\end{array}
\]

Then

\[
V_s'V_{s1}^{-1} = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0.9198 & 0.0802 \\
0.0380 & 0.9620
\end{bmatrix}
\begin{array}{c}
\text{Gen 1} \\
\text{Gen 11} \\
\text{Gen 2} \\
\text{Gen 12}
\end{array}
\]

(45)
Coherency for Load Buses

\[
V_\theta = \begin{bmatrix}
0.5 & 0.4283 \\
0.5 & 0.3535 \\
0.5 & 0.2556 \\
0.5 & 0.3844 \\
0.5 & -0.5018 \\
0.5 & -0.4667 \\
0.5 & -0.3556 \\
0.5 & 0.3128 \\
0.5 & -0.0523 \\
0.5 & -0.4671 \\
0.5 & -0.4125
\end{bmatrix}
\]

Bus 1

Bus 2

Bus 3

Bus 10

Bus 11

Bus 12

Bus 13

Bus 20

Bus 101

Bus 110

Bus 120

\[
V_\theta V_{s1}^{-1} = \begin{bmatrix}
0.9436 & 0.0564 \\
0.8727 & 0.1273 \\
0.7800 & 0.2200 \\
0.9020 & 0.0980 \\
0.0620 & 0.9380 \\
0.0953 & 0.9047 \\
0.2006 & 0.7994 \\
0.8342 & 0.1658 \\
0.4880 & 0.5120 \\
0.0949 & 0.9051 \\
0.1466 & 0.8534
\end{bmatrix}
\] (47)

- Bus 101

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17-Area Partition of NPCC 48-Machine System
EQUIV and AGGREG Functions for Power System Toolbox

- L_group: grouping algorithm
- coh-map, ex_group: tolerance-based grouping algorithm
- Podmore: R. Pormore’s algorithm of aggregating generators at terminal buses
- i_agg: inertial aggregation at generator internal buses
- slow_coh: slow-coherency aggregation, with additional impedance corrections

These functions use the same power system loadflow input data files as the Power System Toolbox.
Power Transfer Paths/Interfaces
Power Transfer Paths/Interfaces

1. The integrity of the power transfer interfaces is crucial for a large interconnected system to function properly. A disruption of a major transfer path may have a “spillover” effect into parallel power transfer paths in neighboring regions.

2. Questions:
   a. How can PMU data be used to monitor the integrity of the power transfer interfaces, for both internal and external visibility?
   b. How many PMUs do we need and where should they be located?
   c. Are there analytical concepts and approaches to extract stability margin information from PMU data?
Power Transfer Paths/Interfaces

Power transfer between two areas with multiple lines and PMU monitoring

Some US Interconnection interfaces may consist of

- a limited number of transmission lines (CA, NY, NE)
- many transmission lines
Inter-area Model Estimation

Problem: Given synchronized voltage phasors (magnitude and phase) at Buses 3 and 13 and current phasor between the two buses in the 2-area power system, develop an equivalent 2-machine model to represent the power transfer between the two areas.

Variations:
- Voltage phasors at Bus 101 also available.
- Voltage phasor at Bus 3, the current phasor going from Bus 3 to Bus 101, and the impedance between Buses 3 and 13.

Difficulty is how to look outward into the importing and exporting areas.

Need dynamic data, i.e., can’t do it with State Estimator solution. In particular, need interarea mode oscillations in the voltage and current phasors.
Inter-area Model Estimation

• Measurement-based model reduction approach
• Use bus voltage and line current phasors
• Develop new concepts and calculation methods:
  ◆ Exact extrapolation equations can be developed for an ideal 2-machine system – Dynamic Model Estimation (DME) algorithm
  ◆ Extend DME to 2-area system – Interarea Model Estimation (IME) algorithm: verify with disturbances
• Additional research and customization needed: transfer paths with intermediate voltage support, transfer interfaces with multiple transmission lines, impact of disturbances, ...
Dynamic Model Estimation

DME Problem Formulation for a 2-machine system

Two machine power system

Classical model representation

- Two generators $G_1$ and $G_2$ with inertias $H_1$ and $H_2$ are connected to Buses 1 and 2
- $G_1$ supplies power to $G_2$ (load)
- Dynamic model – swing equations

$$\dot{\delta} = \Omega \omega$$
$$2H \dot{\omega} = P_m - P_e - P_{\text{loss}} \quad (i = 1, 2)$$

where

$$P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_s}$$
$$P_{\text{loss}} = \frac{r}{z_m} \frac{H_2 E_1^2 - H_1 E_2^2}{H_s}$$
$$P_e = \frac{-E_1 E_2}{z_m} \frac{H_2 \cos(\delta + \alpha) - H_1 \cos(\delta - \alpha)}{H_s}$$

$z_{T_i}$ = transformer impedance
$z_i$ = transformer impedance + direct-axis transient reactance
Dynamic Model Estimation

DME Problem Formulation for a 2-machine system (Contd.)

\[ \tilde{E}_1 = E_1 \angle \delta_1 \]
\[ \tilde{E}_2 = E_2 \angle \delta_2 \]

- Assume PMUs are located at Buses 1 and 2
- Available Phasor Variables – \( V_1, \theta_1, V_2, \theta_2, I, \theta_I \)

**DME Problem**: Given the available phasor variables, that exhibit a few cycles of oscillations, and assuming \( E_1 \) and \( E_2 \) to be constant, compute

\[ E_1, \delta_1, E_2, \delta_2, z_1, z_2, z_e, H_1, H_2 \]

to completely characterize the dynamic behavior of the 2-machine system.

- Because \( z_e = (\tilde{V}_1 - \tilde{V}_2) / \tilde{I} \) and \( \tilde{E}_1 = \tilde{V}_1 + jz_1 \tilde{I}, \tilde{E}_2 = \tilde{V}_2 - jz_2 \tilde{I} \)
the problem reduces to the estimation of \( z_1, z_2, H_1, H_2 \)
Dynamic Model Estimation

**Reactance Extrapolation**

- **Key idea**: Amplitude of voltage oscillation at any point on the transfer path is a function of its electrical distance from the two fixed voltage sources.

\[
E_1, \delta_1 = \delta \quad \tilde{I} \quad \tilde{V}(x) \quad \tilde{E}_2, \delta_2 = 0
\]

\[
\tilde{V}(x, r) = [ E_2 (1 - a) + E_1 (a \cos(\delta) - b \sin(\delta))] + j [ E_1 (b \cos(\delta) + a \sin(\delta)) - bE_2 ]
\]

where

\[
a = \frac{rr_e' + xx_e'}{r_e'^2 + x_e'^2} \quad b = \frac{rx_e' - rx_e'}{r_e'^2 + x_e'^2}
\]

- Voltage Magnitude at B

\[
|V(x, r)| = |\tilde{V}(x, r)| = \sqrt{c + 2E_1E_2 (a - a^2 - b^2) \cos(\delta) - b \sin(\delta)}
\]

where

\[
c = E_2^2 (b^2 + (1 - a)^2) + E_1^2 (b^2 + a^2)
\]
Dynamic Model Estimation

Reactance Extrapolation

- Linearize model about \((\delta_0, \omega_0 = 0, V_{ss})\) :

\[ \Delta V(x) = J(a, b, \delta_0) \Delta \delta \]

where the Jacobian is

\[ J(a, b, \delta_0) := \left. \frac{\partial V(a, b, \delta_0)}{\partial \delta} \right|_{\delta = \delta_0} = \frac{-2E_1E_2}{V(a, b, \delta_0)} [(a - a^2 - b^2) \sin(\delta_0) - b \cos(\delta_0)] \]

- Assume uniform impedance along transfer path \(a = \frac{x}{x_e} \) and \(b = 0\)

\[ J(a, 0, \delta_0) = \frac{-2E_1E_2 \sin(\delta_0)}{V(a, b, \delta_0)} a(1-a) \quad (*) \]

- **Note**: \(J(x, \delta_0)\) in (*) has a numerator varying with \(x\), and a denominator equal to steady state voltage magnitude at B.
Dynamic Model Estimation

Reactance Extrapolation

- $J(x, \delta_0)$ in (*) can be used to estimate $x_1$ and $x_2$ if voltage oscillations are measured or calculated at an additional intermediate bus between Bus 1 and 2.

\[ J(x, \delta_0) \]

\[ \tilde{V}_1 \]
\[ z_1 \]
\[ E_1, \delta_1 \]

\[ \tilde{V}_3 \]
\[ z_{e1} \]
1

\[ \tilde{V}_2 \]
\[ z_{e2} \]
2

\[ z_2 \]

- Following a perturbation, let

\[ V_{im} = \text{Amplitude of Voltage Swing at Bus i} \]

\[ V_{iss} = \text{Steady-state Voltage at Bus i} \]

\[ V_{in} = V_{im} V_{iss} = A (1 - a_i) a_i, \quad i = 1, 2, 3 \]

with

\[ A = 2E_1E_2 \sin(\delta_o) \]

- The reactances $x_1$ and $x_2$, and the unknown constant $A$ can be solved numerically (3 nonlinear equations with 3 unknowns).
Dynamic Model Estimation

Inertia Estimation

• From linearized model

\[ f_s = \frac{1}{2\pi} \sqrt{\frac{E_1 E_2 \cos(\delta_0) \Omega}{2H(x_e + x_1 + x_2)}} \quad (r_e = 0) \]

where \( f_s \) is the measured swing frequency and \( H = \frac{H_1 H_2}{H_1 + H_2} \) is the aggregate Inertia

• For a second equation in \( H_1 \) and \( H_2 \), use conservation of angular momentum

\[ 2H_1 \omega_1 + 2H_2 \omega_2 = 2 \int (H_1 \dot{\omega}_1 + H_2 \dot{\omega}_2) \, dt = \int (P_{m1} - P_{e1} + P_{m2} - P_{e2}) \, dt = 0 \]

\[ \implies \frac{H_1}{H_2} = -\frac{\omega_2}{\omega_1} \]

• However, \( \omega_1 \) and \( \omega_2 \) are not available from PMU data,

• Estimate \( \omega_1 \) and \( \omega_2 \) from the measured frequencies \( \xi_1 \) and \( \xi_2 \) at Buses 1 and 2
Dynamic Model Estimation

Inertia Estimation

• Again use linearization to derive bus frequency and machine speed expressions

\[
\xi_1 = \frac{a_1 \omega_1 + b_1 (\omega_1 + \omega_2) \cos(\delta_1 - \delta_2) + c_1 \omega_2}{a_1 + 2b_1 \cos(\delta_1 - \delta_2) + c_1}
\]

\[
\xi_2 = \frac{a_2 \omega_1 + b_2 (\omega_1 + \omega_2) \cos(\delta_1 - \delta_2) + c_2 \omega_2}{a_2 + 2b_2 \cos(\delta_1 - \delta_2) + c_2}
\]

where

\[
a_i = E_i^2 (1 - r_i)^2, \quad b_i = E_1 E_2 r_i (1 - r_i), \quad c_i = E_2^2 r_i^2 \quad (i=1, 2)
\]

and \( r_i = \frac{x_1}{x_1 + x_2 + x_e} \), \( r_2 = \frac{x_1 + x_e}{x_1 + x_2 + x_e} \)

• \( \xi_1 \) and \( \xi_2 \) are measured, and \( a_i, b_i, c_i \) are known from reactance extrapolation

• Hence, we can calculate \( \omega_1/\omega_2 \) and solve for \( H_1 \) and \( H_2 \)
Dynamic Model Estimation

Example

• Illustrate DME on classical 2-machine model \((r_e = 0)\)
• Disturbance is applied to the system and the response simulated in MATLAB

Voltage Oscillations at 3 buses

\[
\begin{align*}
V_{1m} &= 0.0292 & V_{2m} &= 0.0316 & V_{3m} &= 0.0371 \\
V_{1ss} &= 1.0320 & V_{2ss} &= 1.0317 & V_{3ss} &= 1.0136 \\
V_{1n} &= 0.0301 & V_{2n} &= 0.0326 & V_{3n} &= 0.0376
\end{align*}
\]

DME Algorithm

\[
\begin{align*}
x_1 &= 0.3382 \text{ pu} \\
x_2 &= 0.3880 \text{ pu}
\end{align*}
\]

• Exact values: \(x_1 = 0.34 \text{ pu}, \ x_2 = 0.39 \text{ pu}\)
Dynamic Model Estimation

Example (cont.)

• Jacobian curve fit for reactance extrapolation
Dynamic Model Estimation

Example (cont.)

- Inertia estimation
- Find the bus frequencies by passing the bus voltage angles through a derivative filter with bandwidth higher than the inter-area mode frequency (0.912 Hz)

\[
G(s) = \frac{s}{0.02s + 1}
\]

DME Algorithm

\[
H_1 = 6.48 \text{ pu}, \quad H_2 = 9.49 \text{ pu}
\]

- Exact values: \(H_1 = 6.5 \text{ pu, } H_2 = 9.5 \text{ pu}\)
Dynamic Model Estimation

Example (cont.)

• Frequency curve fit for inertia estimation
Inter-area Model Estimation

• Consider inter-area dynamics across the transfer interface of a multi-machine power system

• Extend the DME algorithm to estimate an equivalent two-machine model

• Assumption: Interface exhibits a *single dominant* mode of inter-area oscillation

• A new approach to power system model reduction based on PMU measurements
Inter-area Model Estimation

IME Problem: Given the phasor measurements $V_3, \theta_3, V_{13}, \theta_{13}, I, \theta_I$ that exhibit a few cycles of inter-area oscillations, and assuming $E_1$ and $E_2$ to be constant, compute $E_1, \delta_1, E_2, \delta_2, z_1, z_2, z_e, H_{a1}, H_{a2}$ to completely characterize the dynamic behavior of the reduced 2-machine system.

- Similar derivations as DME – but in this case local modes are present in system response
- Apply a modal estimation method such as ERA to isolate inter-area mode
Inter-area Model Estimation

• Disturbance applied to a 2-area system

- Bus 3 Voltage
- Bus 13 Voltage
- Bus 101 Voltage

• With 300 MW power transfer the inter-area oscillation frequency is 0.574 Hz, and the local mode frequencies are 1.293 Hz and 1.308 Hz.

• IME estimates: $x_1 = 0.5069 \text{ pu}$, $x_2 = 0.0618 \text{ pu}$
  
  $H_{a1} = 18.98 \text{ pu}$, $H_{a2} = 12.55 \text{ pu}$
Inter-area Model Estimation

- Disturbance applied to a 2-area system

Frequency at Bus 3

Frequency at Bus 13
Inter-area Model Estimation

Reactance Curve Fit

Frequency Curve Fit
Inter-area Model Estimation

- Disturbance plots comparing full model to reduced models

Voltage Oscillations at Bus 3

Voltage Oscillations at Bus 13

Angular Difference

Speed Difference
Some important observations

• Inter-area model estimation is independent of the severity of the disturbance event resulting in the same post-fault condition (robustness)

• Inter-area model is dependent on the amount of inter-area power transfer. In the 2-area system example, larger amount of power transfers results in:
  i. Higher angular separation between the sending end and the receiving end
  ii. Slower frequency of oscillation
  iii. Smaller reactance at the sending end and larger reactance at the receiving area
  iv. Equivalent inertias of the two areas also change
Conclusions and Future Research

• New techniques in using synchronized phasor data to identify power transfer paths/interfaces and hence for PMU siting.
• Establish power-angle curves and energy function analysis on each power transfer path. Monitor the power-angle curves as the system evolves through a sequence of disturbances.
• Monitor energy levels in the transfer paths to determine stability (loss of synchronism or negative damping) and to determine which transfer path is under the most stress.
• Improve system transmission planning and operations planning studies
  ✷ Different types of studies
  ✷ Different ways to see and evaluate results
• Extend calculations to dynamic voltage stability.
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