Three-Phase Voltage-Source Converters

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Outline
• Basic Operation & Applications
• Pulse-Width Modulation
• AC-Side Current Control
• DC-Link Voltage Regulation

Three-Phase VSC Circuit
• A Versatile Interface between DC and Three-Phase AC
• Middle of DC Link is Usually Used as Reference Point
• Inductive Elements at AC Terminals Required
• Bidirectional Power Flow Capabilities

Applications in Power Systems
• Renewable Energy Integration into the Grid
  – Including Energy Storage
• High-Voltage DC Transmission
• Reactive Power Compensation
  – Including Harmonics as Active Power Filter
• Dynamic Voltage Restorer (DVR)
• Unified Power Flow Controller (UPFC)
• Other FACTS Devices

Carrier-Based PWM
$V_{c}(t) = M_{d} V_{o} \cos(\theta_{0} + \phi_{c})$

Phase and Line-Line Voltages
Phase Voltages have 2 Levels
Line-Line Voltages have 3 Levels
### PWM Spectrum

\[ v_m(t) = V \cos(\omega_f t + \theta) \]

\[ v_m(t) = V \cos(\omega_f t + \theta) - \sum_{n=1}^{\infty} \frac{4}{\pi n} J_n(\frac{m \pi}{2}) \sin(\frac{m \pi}{2}) \cos(n \omega_f t + \theta) \]

\[ M = V_i/V_o; \text{ Modulation Index} \]
\[ \theta; \text{ Initial Phase Angle of Reference} \]
\[ \omega_f; \text{ Reference Frequency} \]
\[ \phi; \text{ Initial Phase Angle of Carrier} \]
\[ \omega_c; \text{ Carrier Frequency} \]
\[ J_n(x); \text{ Bessel Function of the 1st Kind} \]

### Spectral Composition

**Fundamental (f_0)**

\[ v_m(t) = M \cos(\omega_f t + \theta) + \frac{4}{\pi n} J_n(\frac{m \pi}{2}) \sin(\frac{m \pi}{2}) \cos(n \omega_f t + \theta) \]

**Sideband Components:**

\[ mf_{c1}, mf_{c2}, mf_{c3}, \ldots \]

**Alternative Form:**

\[ + \sum_{n=1}^{\infty} \frac{4}{\pi n} J_n(\frac{m \pi}{2}) \sin(\frac{m \pi}{2}) \cos(n \omega_f t + \theta) \]

### Phase Voltage Spectra

\[ v_r(t) = \frac{E}{2} \cos(\omega_f t + \theta) + \frac{2V}{ \pi} \sum_{n=1}^{\infty} \frac{1}{n} J_n(\frac{m \pi}{2}) \sin(\frac{m \pi}{2}) \cos(n \omega_f t + \theta) \]

Carrier and its harmonics are the same in all three phases; Sideband components are phase shifted by multiple times of that between the fundamental components.

### Line-Line Voltage Spectra

\[ v_f(t) = \frac{E}{2} \cos(\omega_f t + \theta) + \frac{2V}{ \pi} \sum_{n=1}^{\infty} \frac{1}{n} J_n(\frac{m \pi}{2}) \sin(\frac{m \pi}{2}) \cos(n \omega_f t + \theta) \]

### AC Neutral Voltage

Assume Balanced AC with a Floating Neutral Point, N:

\[ I_f(t) = I_f(t) + I_f(t) + I_f(t) = 0 \]
\[ v_r(t) = v_r(t) + v_r(t) + v_r(t) \]

\[ P_v = P_v + P_v + P_v \]

\[ V_f(t) = V_f(t) + V_f(t) + V_f(t) \]

### Common-Mode Voltage

- **Defined as the Common Component of Output Voltages**
- Also called Zero-Sequence Voltage

\[ v_{cm}(t) = \frac{1}{3} \left( v_r(t) + v_f(t) + v_n(t) \right) \]

Carrier and Its Harmonics

\[ v_{cm}(t) = \frac{1}{3} \left( \frac{E}{2} \cos(\omega_f t + \theta) + \frac{2V}{ \pi} \sum_{n=1}^{\infty} \frac{1}{n} J_n(\frac{m \pi}{2}) \sin(\frac{m \pi}{2}) \cos(n \omega_f t + \theta) \right) \]

Sideband Components Centered around the Carrier Frequency and Its Harmonics

\[ v_{cm}(t) = \frac{1}{3} \left( \frac{E}{2} \cos(\omega_f t + \theta) + \frac{2V}{ \pi} \sum_{n=1}^{\infty} \frac{1}{n} J_n(\frac{m \pi}{2}) \sin(\frac{m \pi}{2}) \cos(n \omega_f t + \theta) \right) \]
Effects of CM Voltage

- Common-Mode (Ground) Current – EMI Problem
- Motor Bearing Current – Motor Reliability

Possible Solutions:
- Common-Mode Voltage Filtering
- CM Voltage Reduction by Circuit Topology and PWM Techniques
- Ceramic Bearing (High Performance Systems e.g. Aerospace)

3rd Harmonic Injection

- 3rd Harmonics (of the Fundamental) are of Zero-Sequence
  - Identical for All Three Phases: 3 x 120° = 360°
- Adding the Same 3rd-Order Harmonic to the References
  - Doesn’t Affect Load Phase Voltages
  - Allows the Fundamental Amplitude to be Higher than the Carrier

Limits for Harmonics Injection

\[ v_{ref}(t) = F_v \sin(\omega t) - v_{ref}(t), \quad F_v > 0.5F_c \]

v_{ref}(t) must fall in the shadowed area in order to avoid distortion

Triplen Harmonics Injection

All triplen harmonics are zero-sequence components, hence can be injected without affecting the load phase voltage. A special case is a triangular wave.

Maximal Output Voltage

\[ V_{max} = \frac{2}{\sqrt{3}} V_{dc} \]

\[ V_{ref} = 1.05 V_{tri} \]

\[ V_{ref} = 1.05 V_{tri} \]

Overmodulation

Linear Operation
Maximal Output Voltage

\[ V_{\text{dc}} \] (Phase Voltage Amplitude)
\[ 2\pi V_{\text{pk}} \]
\[ \sqrt{3}V_{\text{dc}} \]
\[ 0.5V_{\text{dc}} \]

Linear Region
Harmonic Injection
Overmodulation

1
\[ \frac{2\pi}{\sqrt{3}} \]

Different Injection Methods

\[ V_1 = V_{pk}/\sqrt{3} \]

Effects on Phase Voltages

- Phase Voltages Contain Additional Harmonics under Harmonic Injection
  - Injected Harmonics and Sideband Components
  - Identical in All Three Phases
- These Harmonics Become Part of Common-Mode Voltage
  - Relatively Low Frequency and Magnitude Compared to the CM Voltage Generated by PWM
- Phase Voltage Spectra can also be Obtained by Double Fourier Series Analysis
  - Closed-Form Results have been Reported in the Literature
- Optimal Harmonic Injection

AC Current Control

- DC-AC Converter (Inverter) Current Control
  - Torque Control in Motor Drives
  - Active and Reactive Power Control in Grid-Connected DG
  - Current-Mode Control in UPS and Standalone DG
- AC-DC Converter (Rectifier) Current Control
  - Unity Input Power Factor
  - Regulation of DC Voltage

Example Current Waveforms

Active Power Generation (DG)
Reactive Power Compensation
DG with Reactive Compensation
Rectification (PFC)
Active Power Filtering
UPS (with Nonlinear Loads)
Current Control Principles

Three-Phase VSC 25

- Single Phase
- Three Phase

va vas

\[ \frac{V_a - V_{as}}{2} \]

\[ \frac{V_a}{\tau} = \frac{V_{as}}{\tau} \]

Current Control Principles

Averaged Modeling

Three-Phase VSC 26

- \( d_a, d_b, \) and \( d_c \) is the Duty Ratio of the Upper Switch in Phase a, b, and c, Respectively
- Averaging Removes Switching Ripple
  - Resulting Model is in General Valid at Frequencies Lower than Half the Switching (Carrier) Frequency

Basic Control Structure

Three-Phase VSC 27

- Objective: Control Phase Currents \( i_a, i_b, \) and \( i_c \) to Follow Given References \( \{i_{a,\text{ref}}, i_{b,\text{ref}}, i_{c,\text{ref}}\} \)
- Assumption: Terminal Voltages \( v_{sa}, v_{sb}, \) and \( v_{sc} \) are Known
- Current Responses Governed by

\[ \begin{align*}
L \frac{di_a}{dt} + (v_{sa} - v_{as}) &= i_{a,\text{ref}} - i_a \\
L \frac{di_b}{dt} + (v_{sb} - v_{as}) &= i_{b,\text{ref}} - i_b \\
L \frac{di_c}{dt} + (v_{sc} - v_{as}) &= i_{c,\text{ref}} - i_c
\end{align*} \]

The modulator and the averaged model can be modeled together by a constant gain \( \frac{V_d}{2V_{tr}} \)

Feedback Control

Three-Phase VSC 28

- Vector Representation
- Problem: Performance is Sensitive to Parameter Variations and Uncertainty

Linear Feedback Control

Three-Phase VSC 29

- One Linear (PI) Regulator per Phase
- Simple, Robust
- Limited Performance
  - Existence of Control Error

\[ F(s) - \left[ \frac{V_d}{L_s} \right] \left[ K_p + \frac{K_v}{s} \right] \frac{V_d}{2V_{tr}} \]

Feedback + Feedforward

Three-Phase VSC 30

- Control Structure
- PI Regulator:
  \[ v_{ref} = K_p \Delta i_a + K_v \Delta i_d \]

\[ \int (v_{ref} - v_{ref}) \, dt \]

\[ \frac{1}{L_s} \]

\[ L_s \]

\[ V_d(s) \]

\[ K_p + \frac{K_v}{s} \]
Nonlinear (Hysteretic) Control

- Features
  - Direct Generation of Switch Gate Signals
  - No Need for PWM
  - Simple, Robust
  - Stable Operation, Fast Response
  - Variable Switching Frequencies
  - Not Suitable for Digital Control

- Possible Improvement
  - Variable Hysteretic Bands to Reduce Switching Frequency Variation
  - Space Vector-Based Hysteretic Control

Ripple Current and L Selection

- Inductor Currents Contain Ripple Components at the Switching Frequency
- Amplitude \( \propto (L_f)^{-1} \)
  - Varies over a Line Cycle
- Effects of Current Ripple
  - Contribute to System Harmonics
  - Increase Converter Losses
- Inductor Design Considerations
  - Performance vs. Cost & Size

DC-Link Voltage Control

- Applications Requiring Control of the DC-Link Voltage
  - Power Factor Corrected Rectifiers
  - Active Power Filters
  - Static VAR Compensators
  - Grid Interface for Fuel Cells, Solar, and Wind Power

V\(_d\)c Response to Phase Currents

- Assume Balanced Sinusoidal Source Voltages
- Only Active Component of Phase Currents Affects Average of V\(_d\)c
- Reactive Component doesn’t Affect V\(_d\)c
- Unbalanced Fundamental Components (Active or Reactive) Lead to 2nd Harmonic in V\(_d\)c
- Balanced (6±1)th Harmonic Currents Generates 68th Harmonic in V\(_d\)c

\[
\begin{align*}
V_{dc}(t) &= V_i \cos(\alpha t), \\
V_{dc}(t) &= V_i \cos(\alpha t - 2\pi/3), \\
V_{dc}(t) &= V_i \cos(\alpha t - 4\pi/3)
\end{align*}
\]

\[
\begin{align*}
\cos x + \cos y &= 2 \cos x \cos y, \\
\sin x + \sin y &= 2 \sin \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)
\end{align*}
\]

\[
\begin{align*}
\sin x \cos x + \sin y \cos y &= \frac{1}{2} \left[ \cos y - \cos (x+y) + \sin x \cos y + \sin y \sin x \right], \\
\sin x \cos x + \sin y \cos y &= \frac{1}{2} \left[ \cos y - \cos (x+y) + \sin x \cos y + \sin y \sin x \right]
\end{align*}
\]
Voltage Dynamics and Control

Assume Active Component of Phase Current $i_g = g$

$\frac{dV}{dt} = \sqrt{3}V_{rms} - \frac{V_i}{R_s} - I_i - 3gV_{rms} - I_g$

$V_{rms} = \text{Input Phase RMS Voltage}$

- $g$ is the Control Variable
  - Represents Voltage Compensator Output
- Dynamics Described by a Nonlinear Model
  - Linearization about a Steady-State Operation Point

Reading & HW Assignments

- Holmes and Lipo, Pulse Width Modulation for Power Converters, Chapters 5&6

- Consider a Three-Phase Solar Inverter Switched at 20 kHz and with a 350 V DC Input. The Inverter is Connected to a 120 V (Phase RMS) Grid through a 1 mH (per Phase) Inductor, and Supplies 10 kW Active Power to the Grid. Calculate
  - The Modulation Index $M$
  - The Amplitude of Phase Harmonic Current at 20 kHz, 20 kHz $\leq 60$ Hz, 20 kHz $\leq 120$ Hz, 20 kHz $\leq 180$ Hz, and 40 kHz $\leq 60$ Hz, 40 kHz $\leq 120$ Hz, 40 kHz $\leq 180$ Hz