Erasure Coding Research for Reliable Distributed and Cluster Computing

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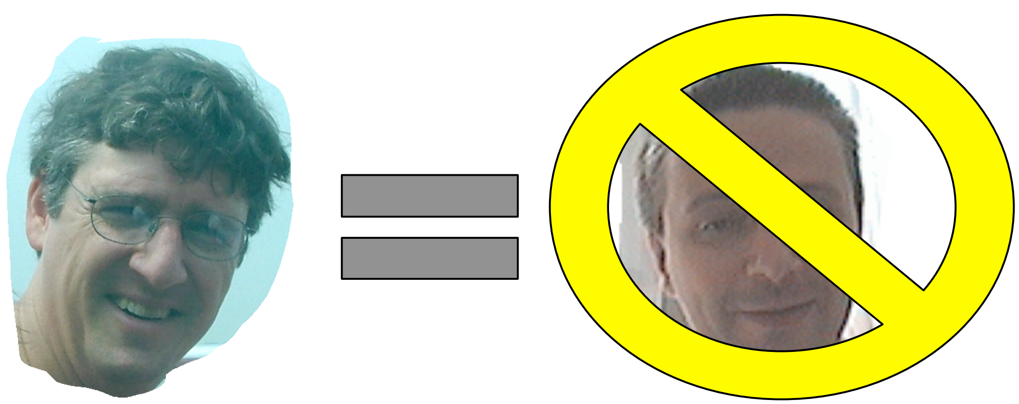
CCGSC History

- In 1998, I talked about checkpointing
- In 2000, I talked about economic models for scheduling.
- In 2002, I talked about logistical networking.
- In 2004, I was silent.
- In 2006, I'll talk about erasure codes.



Talk Outline

- What is an erasure code & what are the main issues?
- Who cares about erasure codes?
- Overview of current state of the art
- My research

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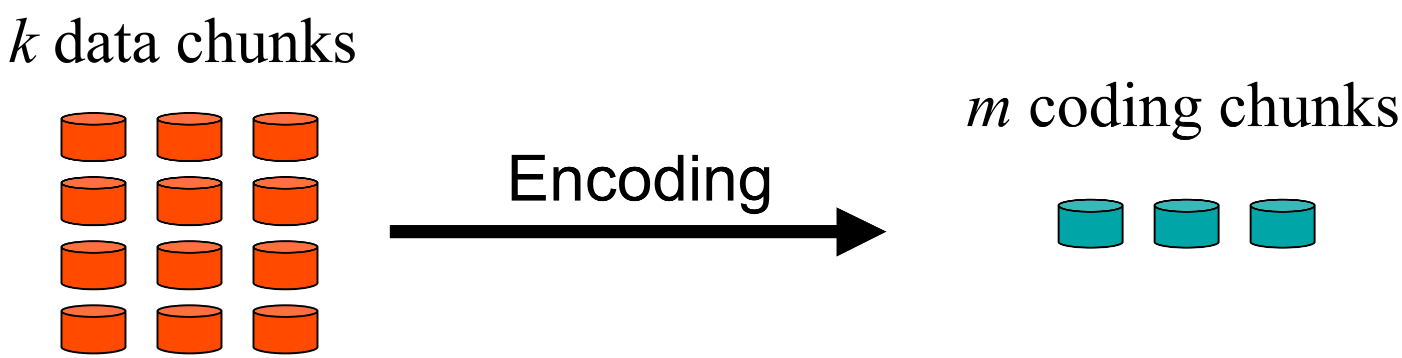
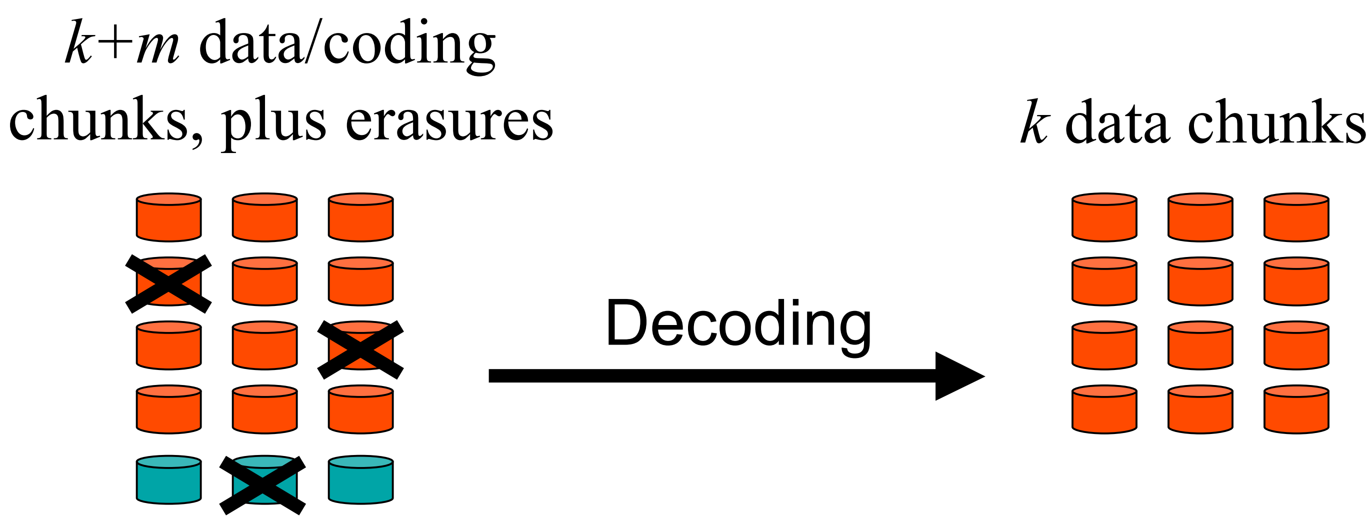
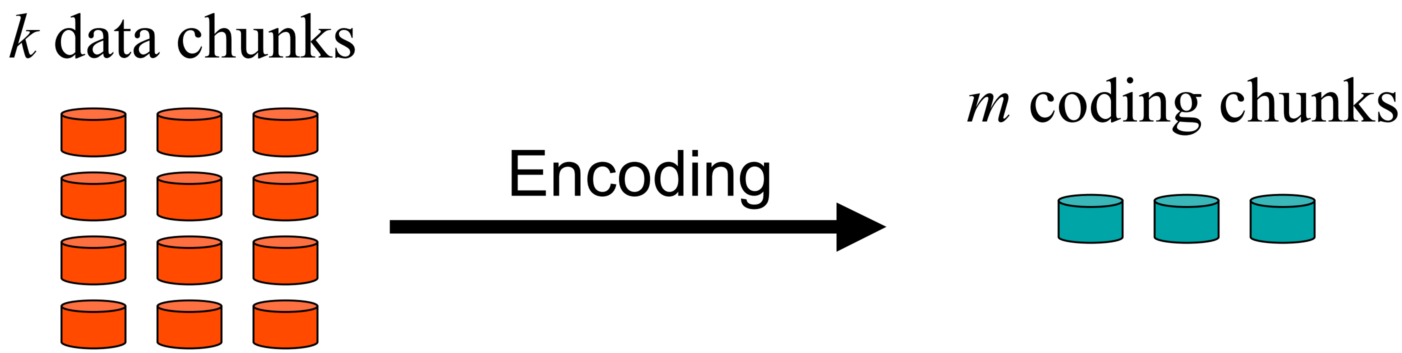
What is Erasure Coding?





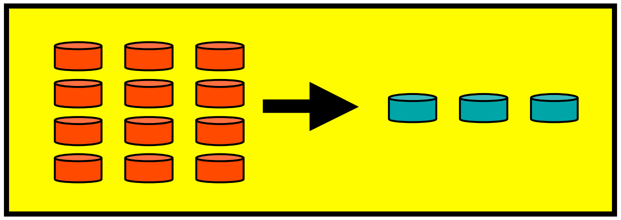
Specifically



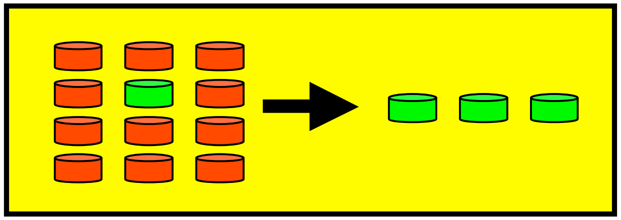
or perhaps

Issues with Erasure Coding

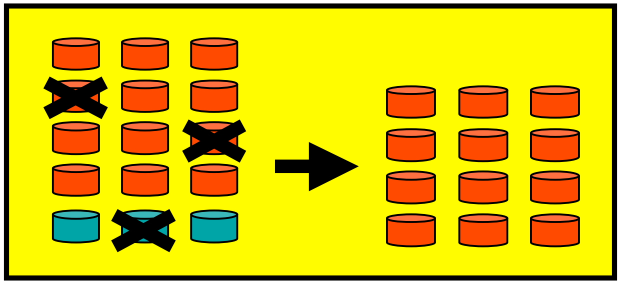
- Performance
  - <u>Encoding</u>
    - Typically *O(mk)*, but not always.



  - <u>Update</u>
    - Typically *O(m)*, but not always.



  - <u>Decoding</u>
    - Typically *O(mk)*, but not always.



Issues with Erasure Coding

- Space Usage
  - Quantified by two of four:
    - Data Pieces: k
    - Coding Pieces: m
    - Total Pieces: $n = (k+m)$
    - Rate: $R = k/n$
  - Higher rates are more space efficient, but less fault-tolerant / flexible.

Issues with Erasure Coding

- Failure Coverage - Four ways to specify
  - Specified by a threshold:
    - (e.g. 3 erasures always tolerated).
  - Specified by an average:
    - (e.g. can recover from an average of 11.84 erasures).
  - Specified as MDS (Maximum Distance Separable):
    - MDS: Threshold = average = m.
    - Space optimal.
  - Specified by Overhead Factor f:
    - f = factor from MDS = m/average.
    - f is always >= 1
    - f = 1 is MDS.

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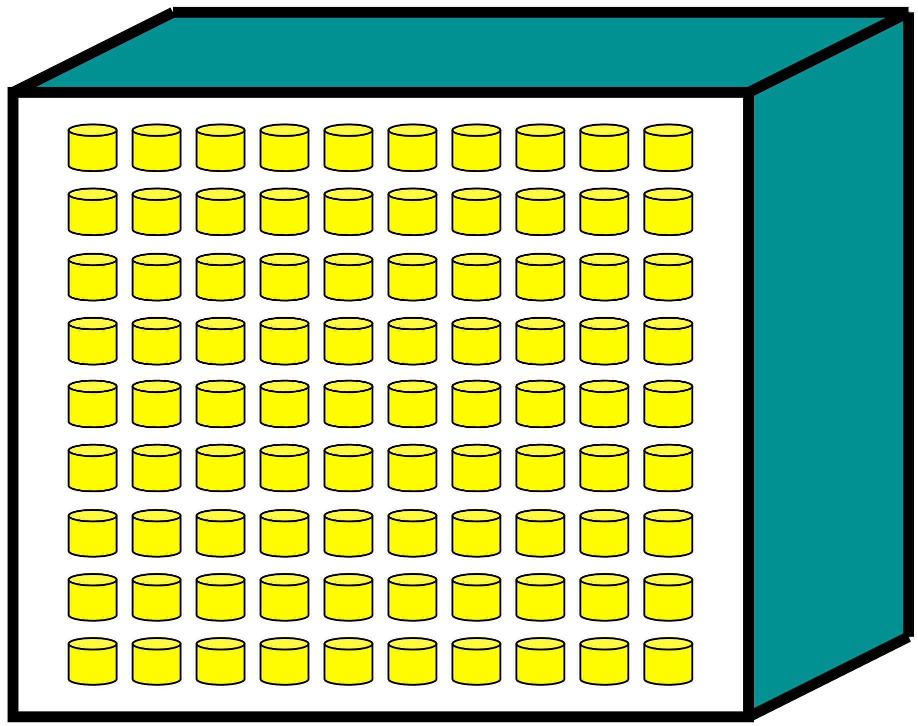
Who cares about erasure codes?

Anyone who deals with distributed data, where failures are a reality.

Who Cares?

#1: Disk array systems.

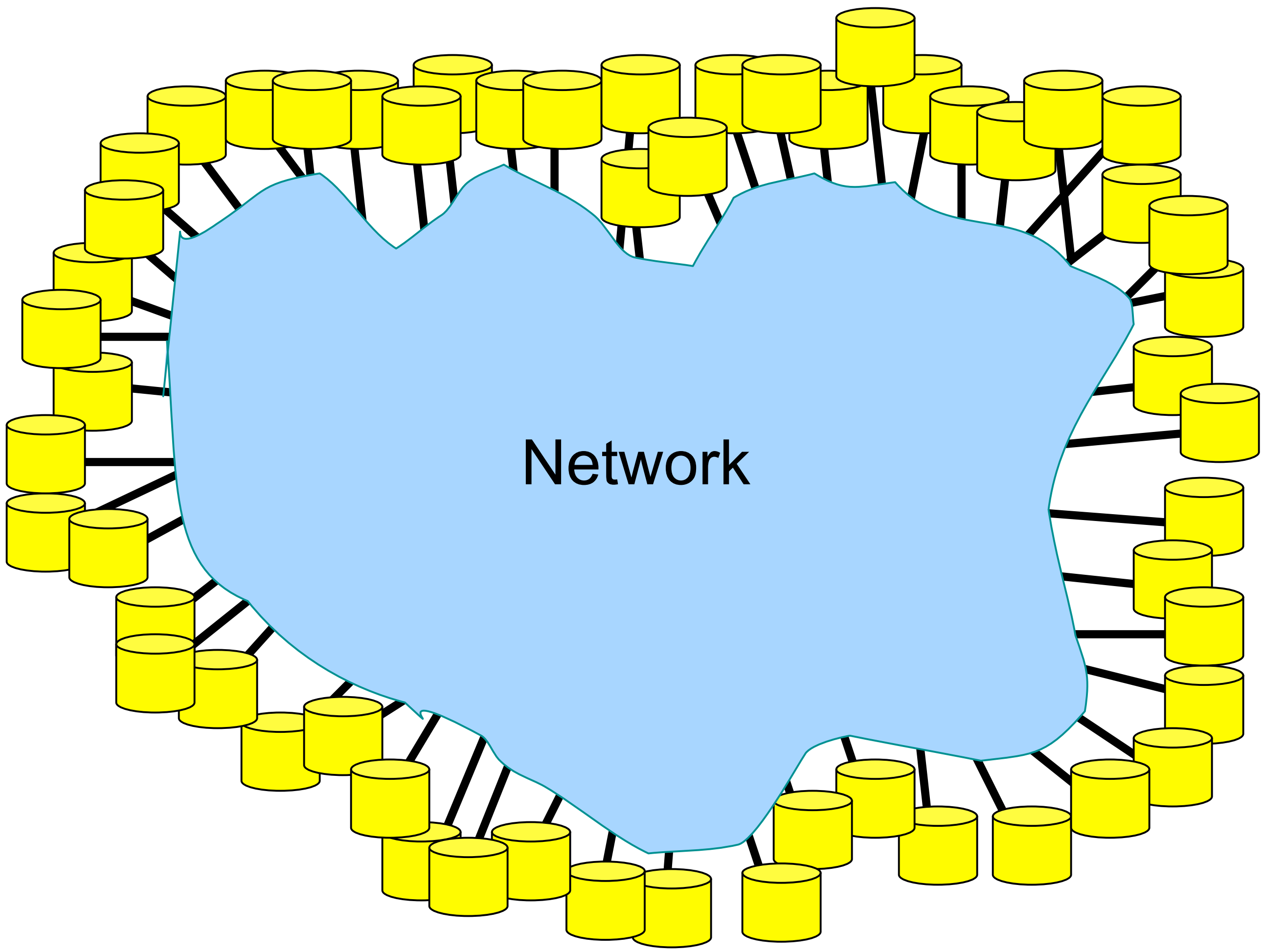
- *k* large, *m* small (< 4)
- Minimum baseline is a requirement.
- Performance is critical.
- Implemented in controllers usually.
- RAID is the norm.



Who Cares?

#2: Peer-to-peer Systems

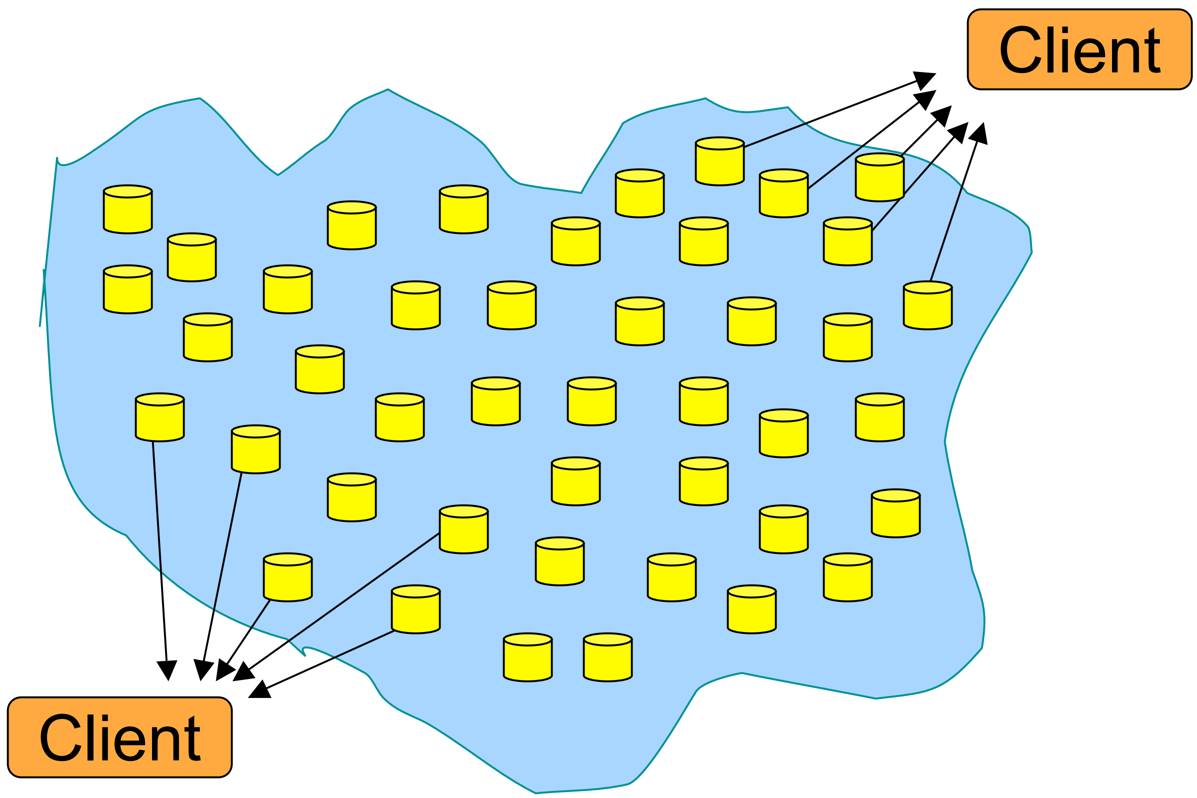
- *k* huge, *m* huge.
- Resources highly faulty, but plentiful (typically).
- Replication the norm.



Who Cares?

#3: Distributed (Logistical) Data/Object Stores

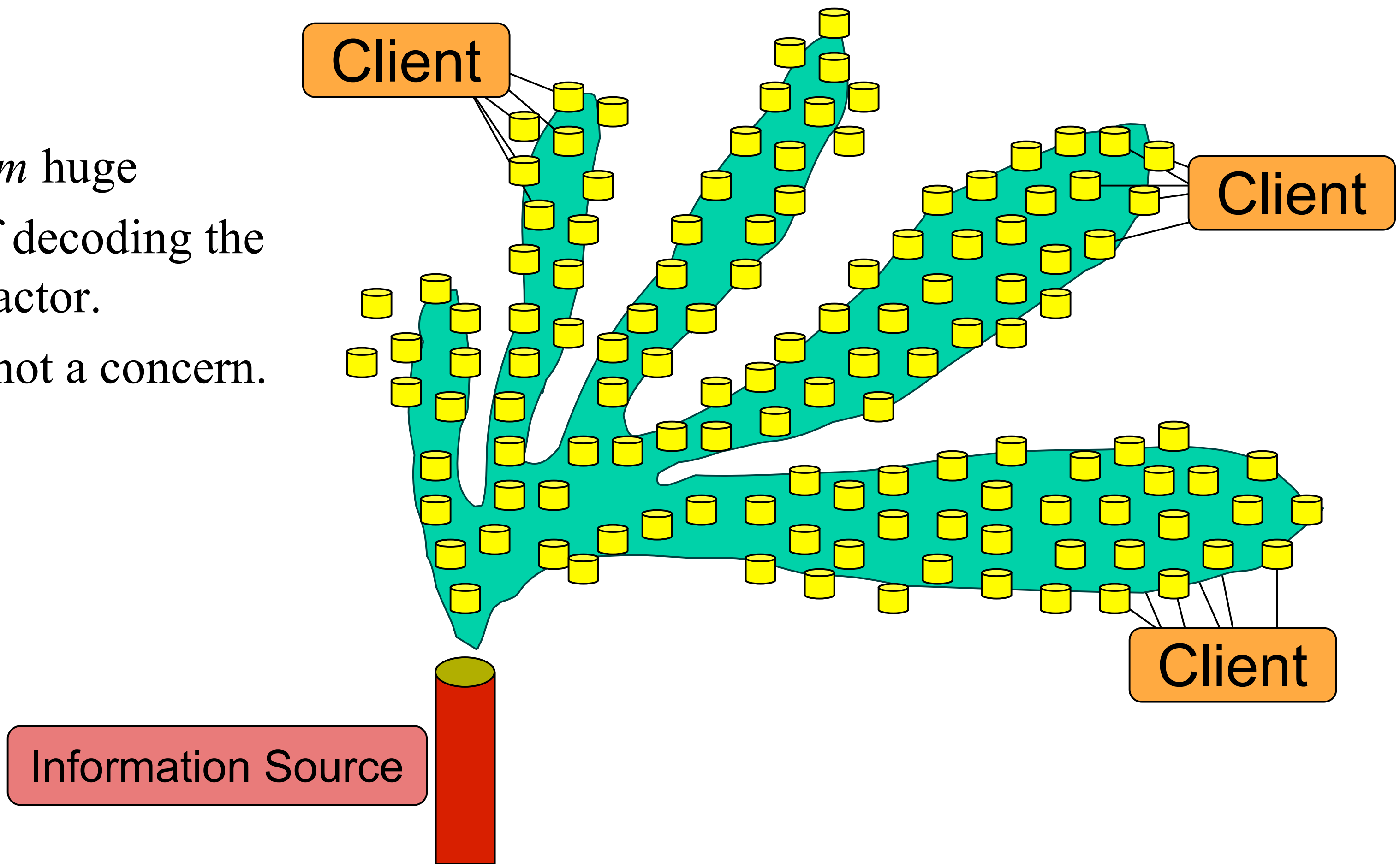
- *k* huge, *m* medium.
- Fluid environment.
- Speed of decoding the critical factor.
- MDS not a requirement.



Who Cares?

#4: Digital Fountains

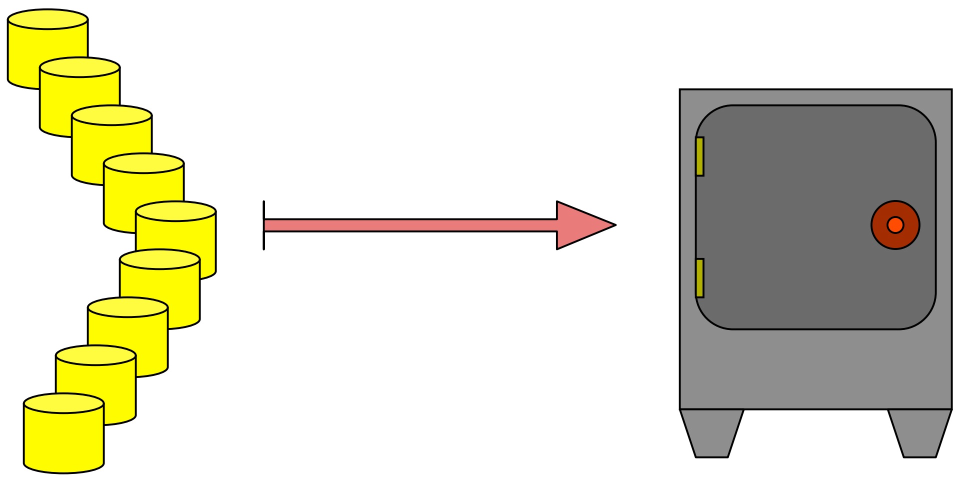
- *k* is big, *m* huge
- Speed of decoding the critical factor.
- MDS is not a concern.



Who Cares?

#5: Archival Storage

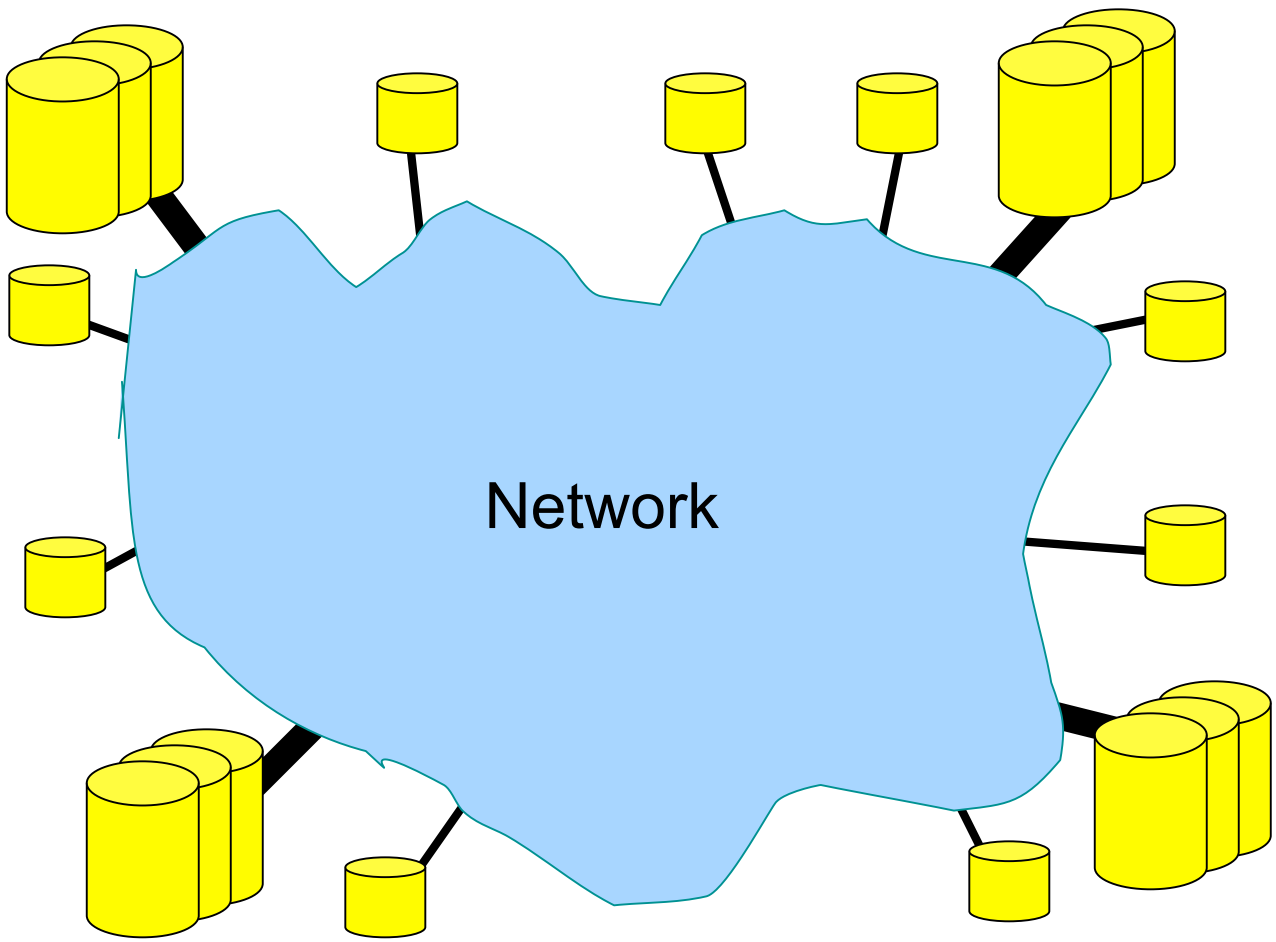
- *k? m?*
- Data availability the only concern.



Who Cares?

#6: Clusters and Grids

Mix & match
from the others.



Who cares about erasure codes?

- Fran does *(part of the "Berman pyramid")*
- Tony does *(access to datasets and metadata)*
- Joel does *(Those sliced up mice)*
- Phil does *(Where the *!!#$'s my data?)*
- Ken does *(Scheduling on data arrival)*
- Laurent does *(Mars and motorcycles)*

They just may not know it yet.

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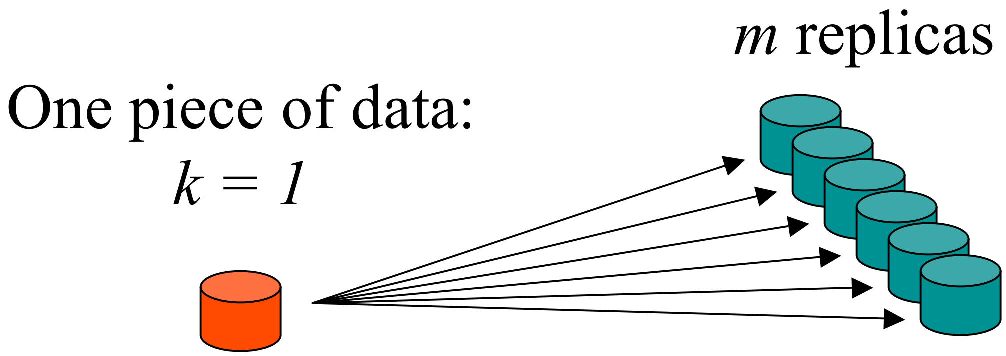
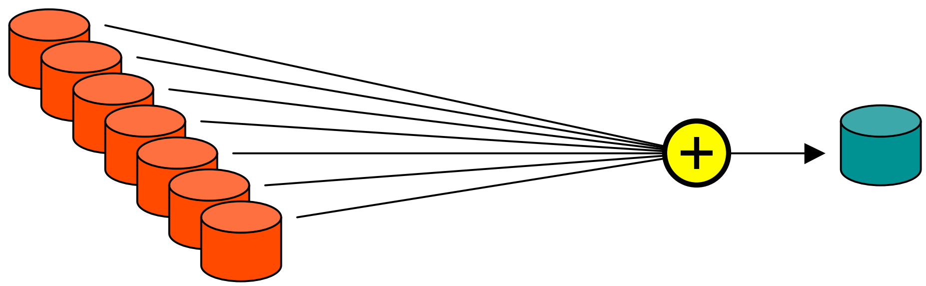
Trivial Example: Replication



Can tolerate any *m* erasures.

- MDS
- Extremely fast encoding/decoding/update.
- Rate: $R = 1/(m+1)$ - Very space inefficient

Less Trivial Example: RAID Parity



- MDS
- Rate: $R = k/(k+1)$ - Very space efficient
- Optimal encoding/decoding/update:
- Downside: $m = 1$ is limited.

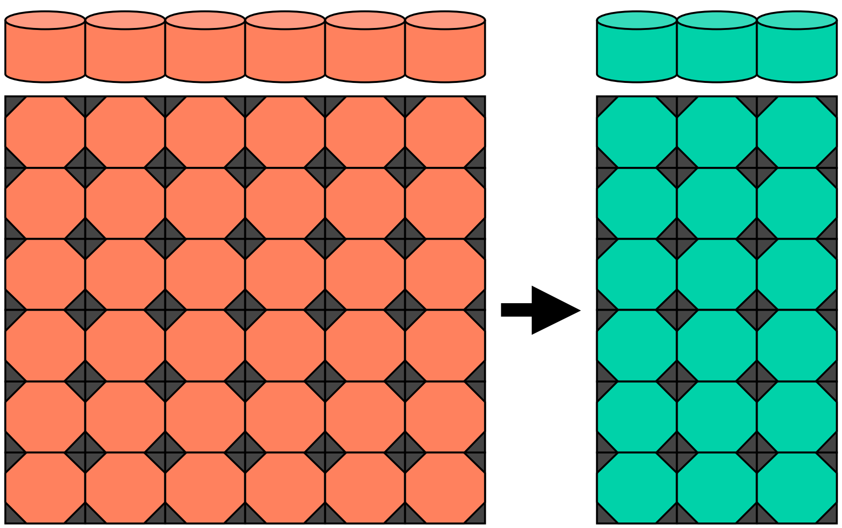
The Classic: Reed-Solomon Codes

- Codes are based on linear algebra over $GF(2^w)$.
- General-purpose MDS codes for all values of k,m.
- Slow.

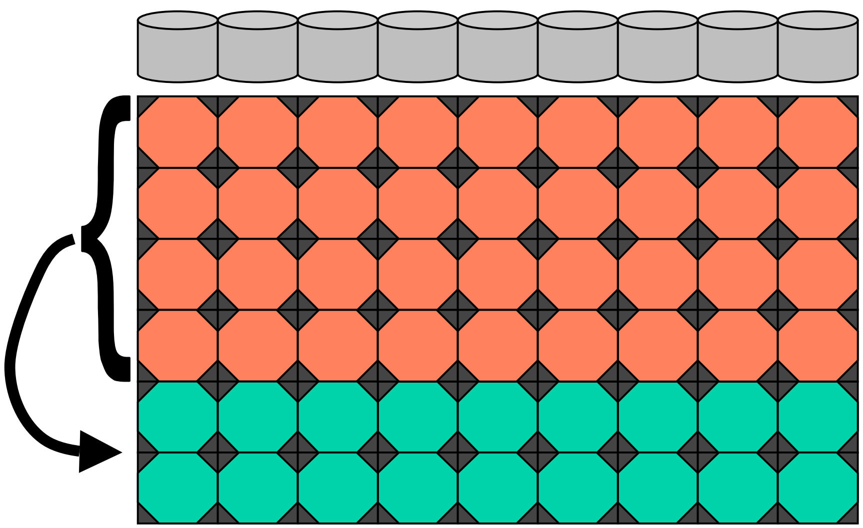
$$k+m\left\{\underbrace{\overbrace{\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\B_{11}&B_{12}&B_{13}&B_{14}&B_{15}\\B_{21}&B_{22}&B_{23}&B_{24}&B_{25}\\B_{31}&B_{32}&B_{33}&B_{34}&B_{35}\end{bmatrix}}^{k}}_{B}\right. * \underbrace{\begin{bmatrix}D_1\\D_2\\D_3\\D_4\\D_5\end{bmatrix}}_{D} = \begin{matrix}\left.\begin{bmatrix}D_1\\D_2\\D_3\\D_4\\D_5\end{bmatrix}\right\}D\\\left.\begin{bmatrix}C_1\\C_2\\C_3\end{bmatrix}\right\}C\end{matrix}$$

The RAID Folks: Parity-Array Codes

- Coding words calculated from parity of data words.
- MDS (or near-MDS).
- Optimal or near-optimal performance.
- Small *m* only (*m*=2, *m*=3, some *m*=4)
- Good names: Even-Odd, X-Code, STAR, HoVer, WEAVER.



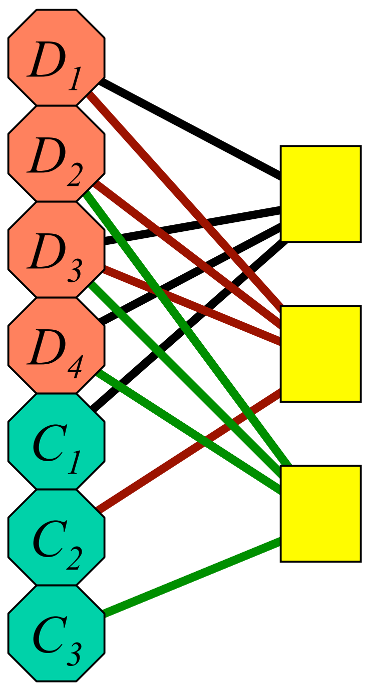
Horizontal



Vertical

The Radicals: LDPC Codes

- Iterative, graph-based encoding and decoding
- Exceptionally fast (factor of k)
- Distinctly non-MDS, but asymptotically MDS



$$D_1 + D_3 + D_4 + C_1 = 0$$

$$D_1 + D_2 + D_3 + C_2 = 0$$

$$D_2 + D_3 + D_4 + C_3 = 0$$

Problems with each:

- Reed-Solomon coding is limited.
  - Slow.
- Parity-Array coding is limited.
  - $m=2$, $m=3$ only well understood cases.
- LDPC codes are also limited.
  - Asymptotic, probabilistic constructions.
  - Non-MDS in the finite case.
  - Too much theory; too little practice.

So……

- Besides replication and RAID, the rest is gray area, clouded by the fact that:
  - Research is fractured.
  - 60+ years of additional research is related, but doesn’t address the problem directly.
  - Patent issues abound.
  - General, optimal solutions are as yet unknown.

The Bottom Line

- The area is a mess:
  - Few people know their options.
  - Misinformation is rampant.
  - The majority of folks use vastly suboptimal techniques (especially replication).

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My Mission:

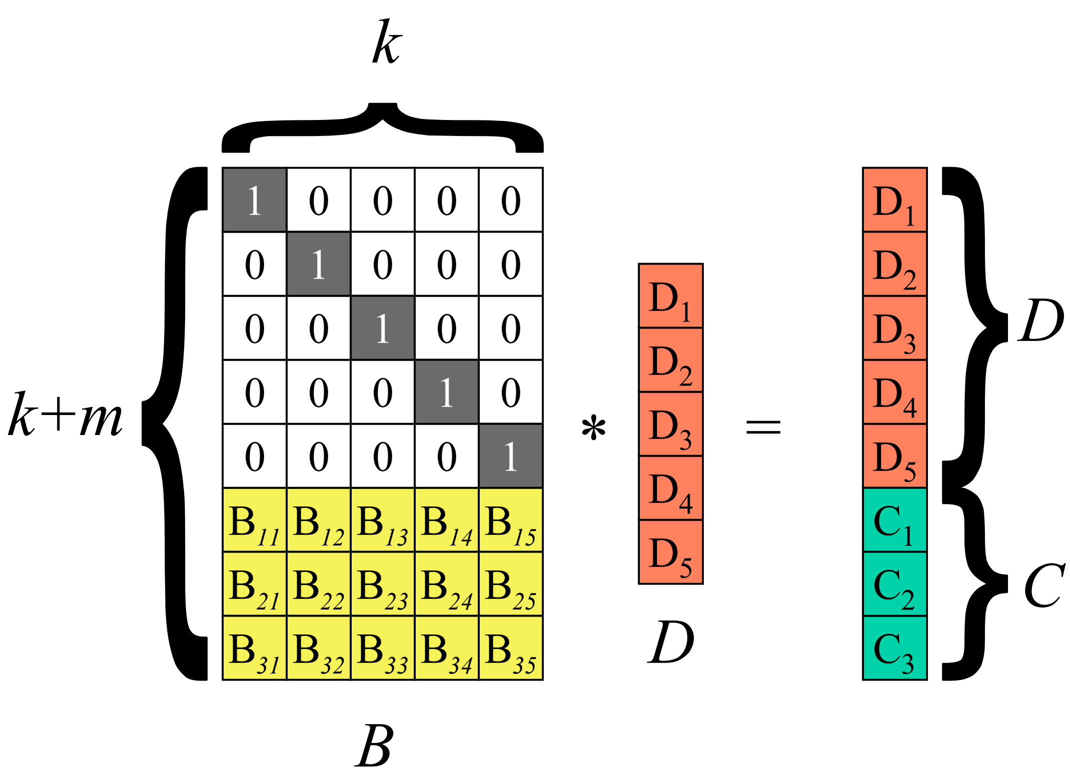
- To unclutter the area using a 4-point, rhyming plan:
  - Elucidate: *Distill from previous work.*
  - Innovate: *Develop new/better codes.*
  - Educate: *Because this stuff is not easy.*
  - Disseminate: *Get code into people's hand.*

5 Research Projects

- 1. Improved Cauchy Reed-Solomon coding.
- 2. Parity-Scheduling
- 3. Matrix-based decoding of LDPC's
- 4. Vertical LDPC's
- 5. Reverting to Galois-Field Arithmetic

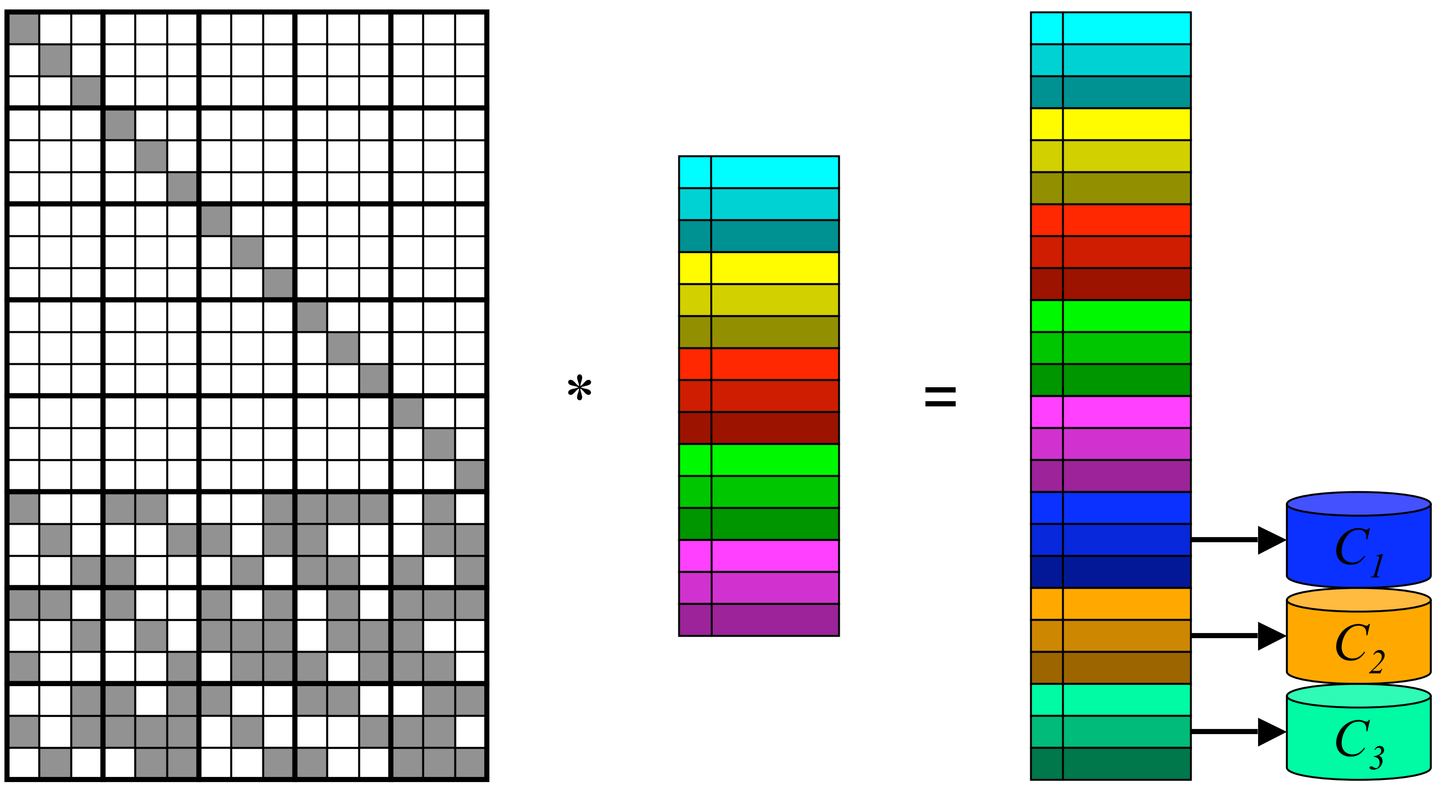
1. Improved Cauchy Reed-Solomon Coding.

- Regular Reed-Solomon coding works on words of size *w*, and expensive arithmetic over $GF(2^w)$.



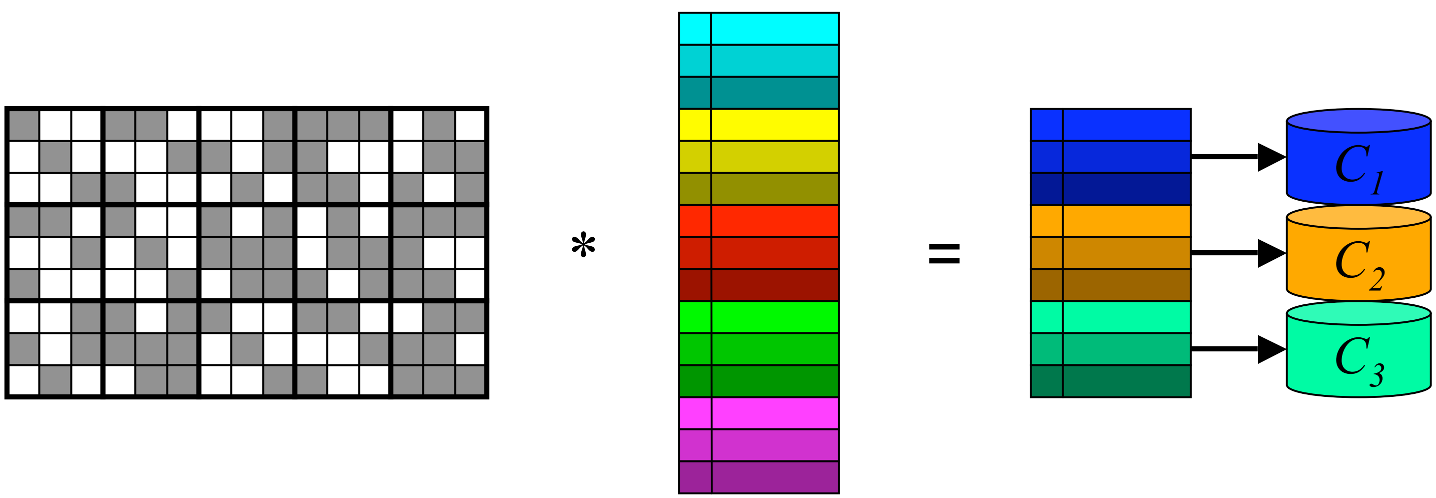
1. Improved Cauchy Reed-Solomon Coding.

- Cauchy RS-Codes expand the distribution matrix over *GF(2)* (bit arithmetic):
- Performance proportional to *number of ones per row*.



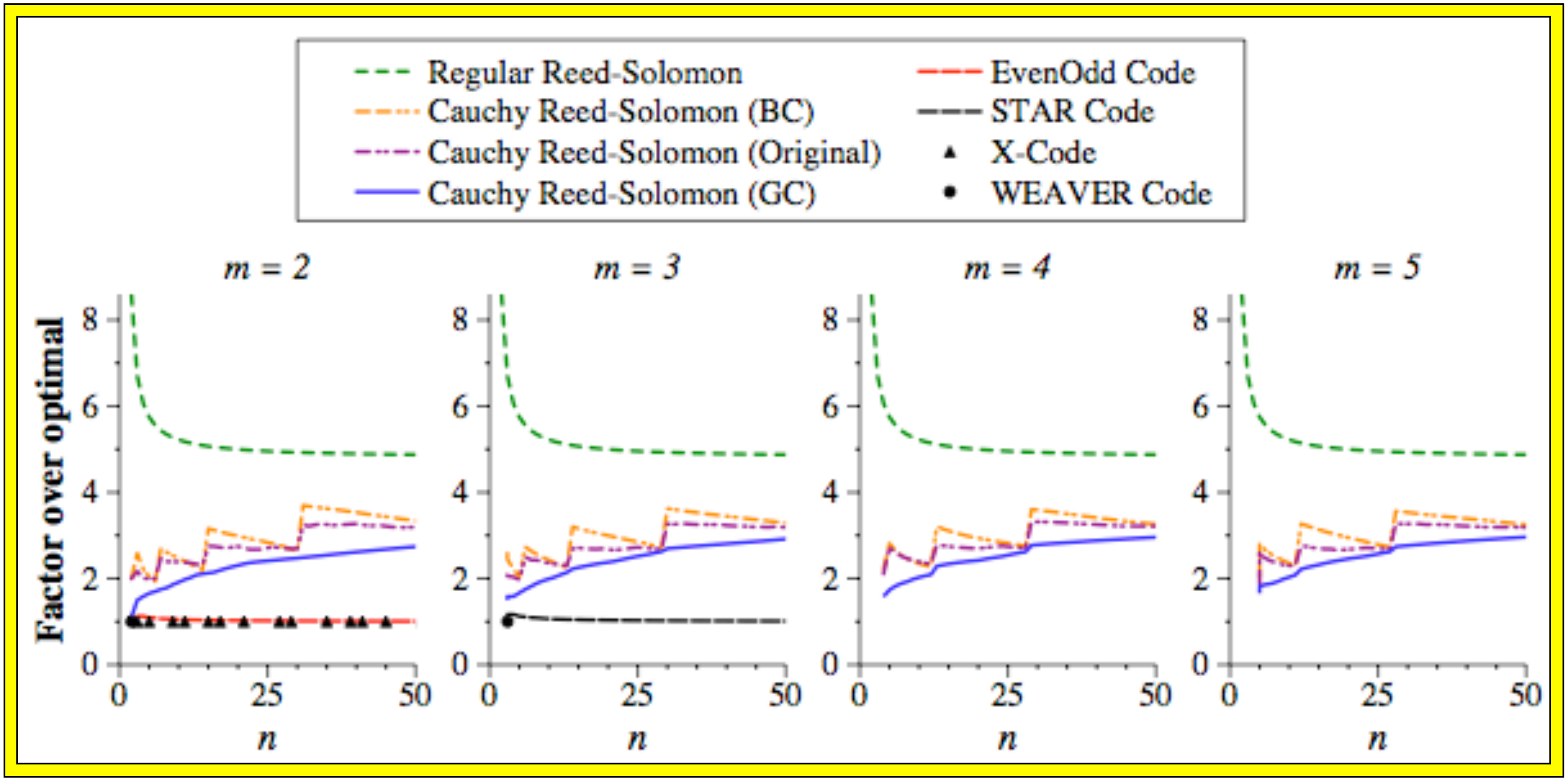
1. Improved Cauchy Reed-Solomon Coding.

- Different Cauchy matrices have different numbers of ones.
- Use this observation to derive optimal / heuristically good matrices.



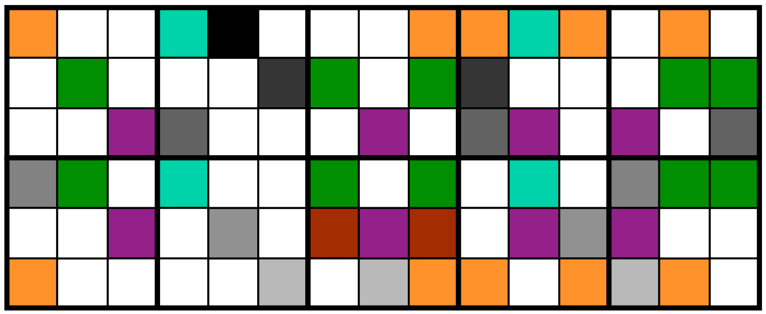
1. Improved Cauchy Reed-Solomon Coding.

- E.g. Encoding performance: (NCA 2006 Paper)



2. Parity Scheduling

- Based on the following observation:



Reduces XORs from 41 to 28 (31.7%). Optimal = 24.

$A = \sum \blacksquare$

$B = \sum \blacksquare$

$C = \sum \blacksquare + B$

$D = \sum \blacksquare$

$E = \sum \blacksquare$

$C_{1,1} = A + E + \blacksquare$

$C_{1,2} = C + \blacksquare + \blacksquare$

$C_{1,3} = D + \blacksquare + \blacksquare + \blacksquare$

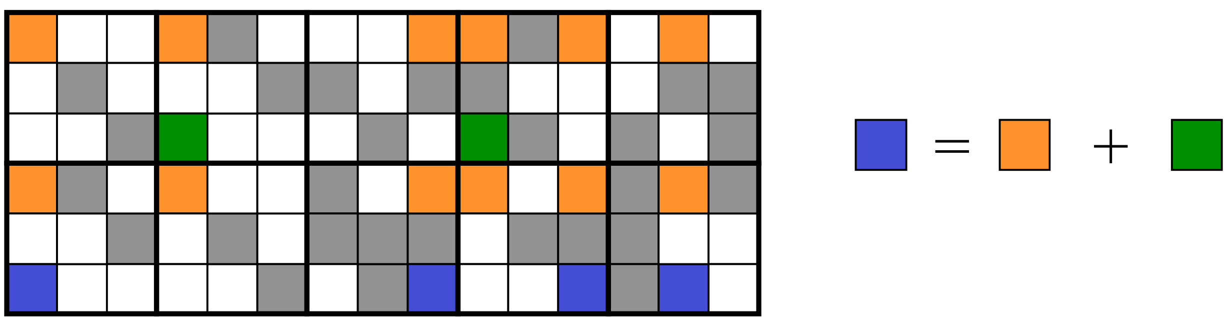
$C_{2,1} = C + E + \blacksquare + \blacksquare$

$C_{2,2} = B + D + \blacksquare + \blacksquare$

$C_{2,3} = A + \blacksquare + \blacksquare + \blacksquare$

2. Parity Scheduling

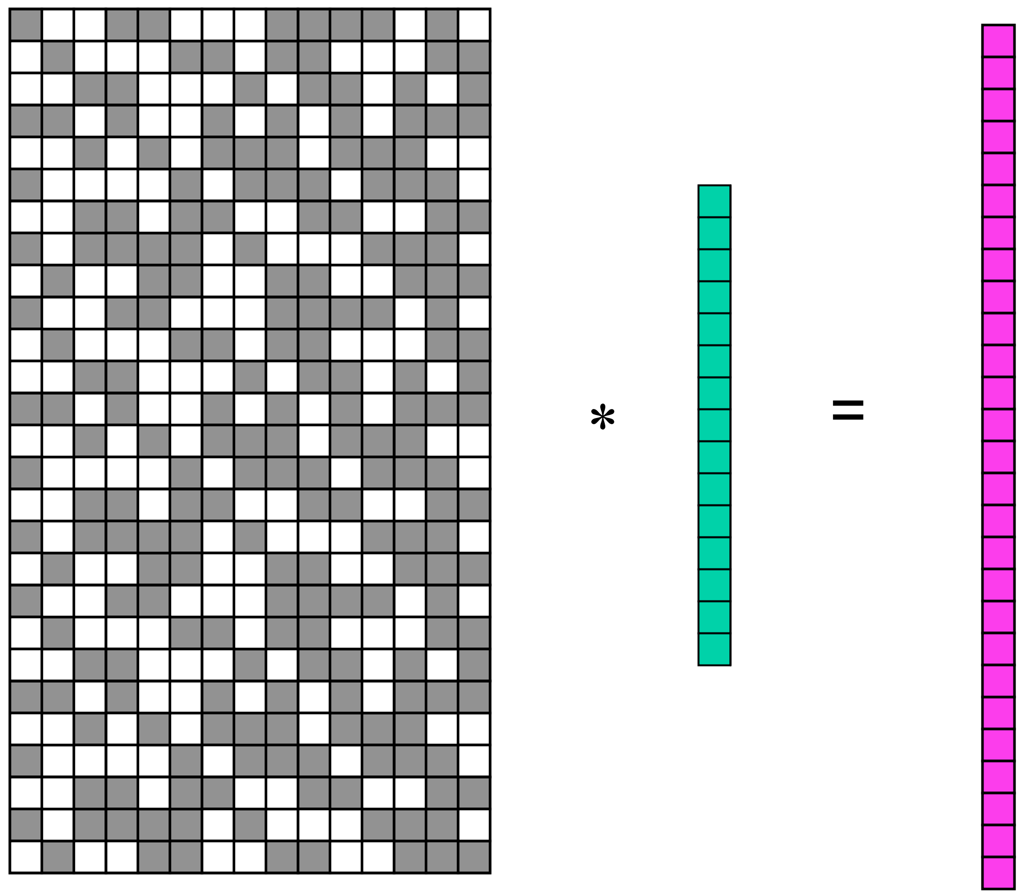
- Relevant for all parity-based coding techniques:
- Start with common subexpression removal.
- Can use the fact that XOR's cancel.



- <u>Bottom line</u>: RS coding approaching optimal?

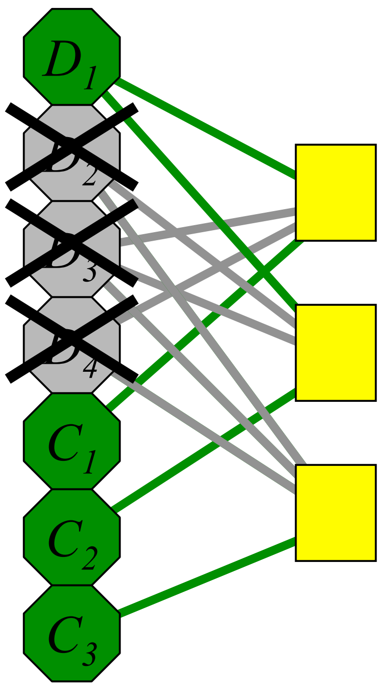
An aside for those who work with linear algebra….

Look familiar?



3. Matrix-Based Decoding for LDPC's

- <u>The crux</u>: Graph-based encoding and decoding are *blisteringly fast*, but codes are *not MDS*, and in fact, *don't decode perfectly*.



$$D_1 + D_3 + D_4 + C_1 = 0$$

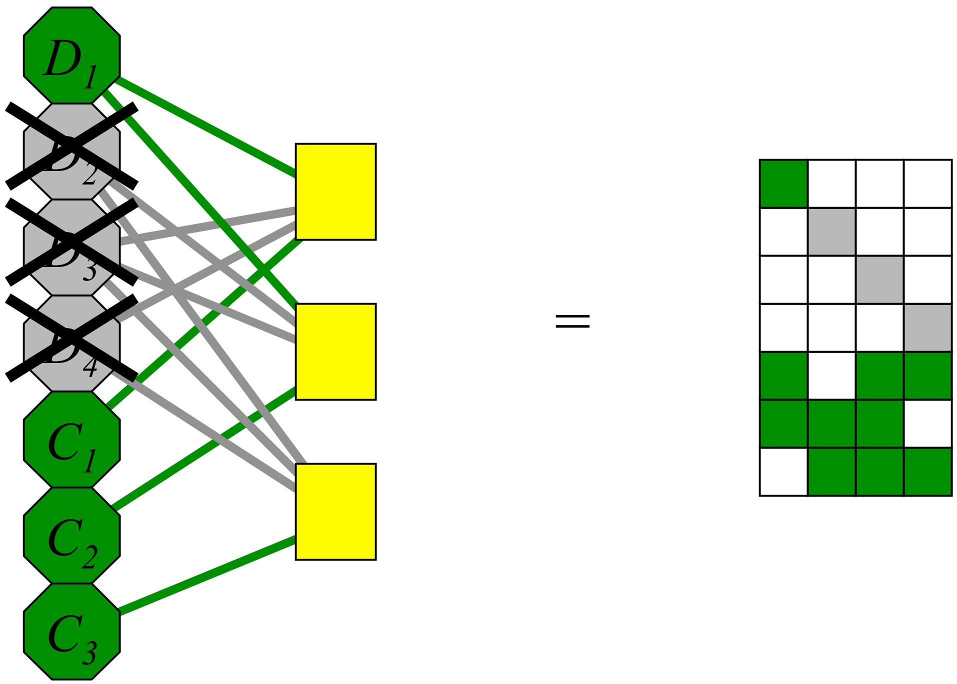
$$D_1 + D_2 + D_3 + C_2 = 0$$

$$D_2 + D_3 + D_4 + C_3 = 0$$

Add all three equations: $C_1 + C_2 + C_3 = D_3$.

3. Matrix-Based Decoding for LDPC's

- Solution: Encode with graph, decode with matrix.



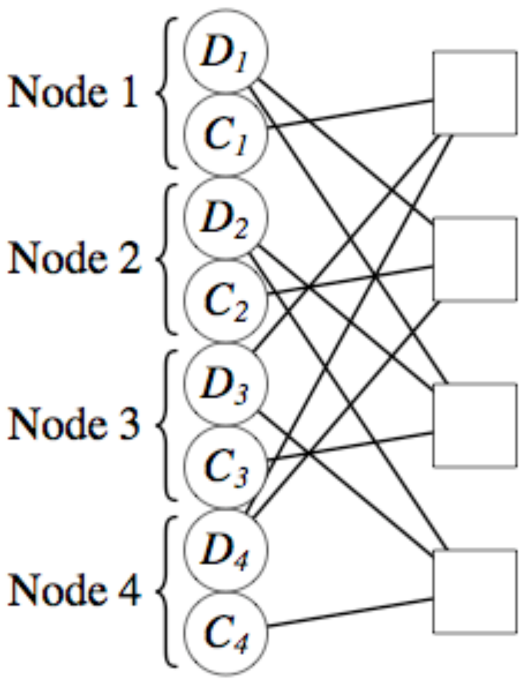
Invertible.

Issues: incremental decoding, common subex's, etc.

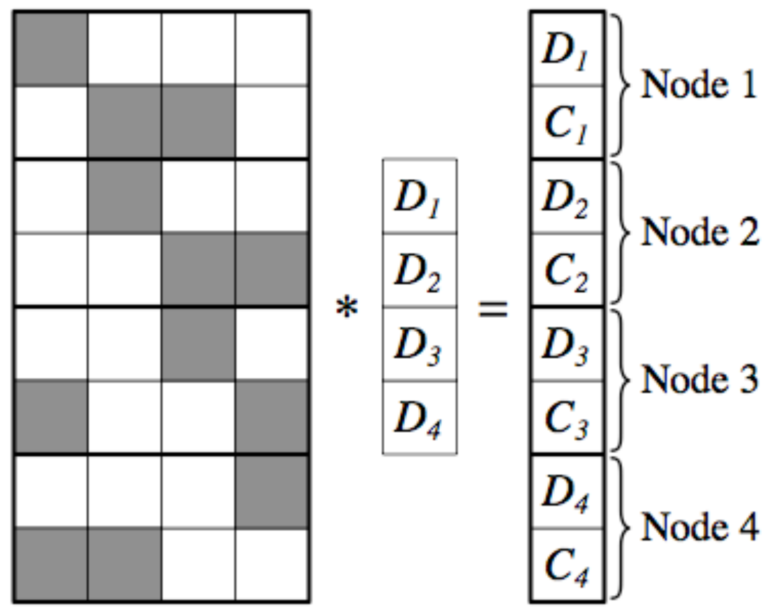
Result: Push the state of the art further.

4. Vertical LDPC’s

- Employ augmented LDPC’s & Distribution matrices to combine benefits of vertical coding/LDPC encoding.



Augmented LDPC



Augmented Binary Distribution Matrix

MDS WEAVER code for $k=2$, $m=2$

5. Reverting to Galois Field Arithmetic

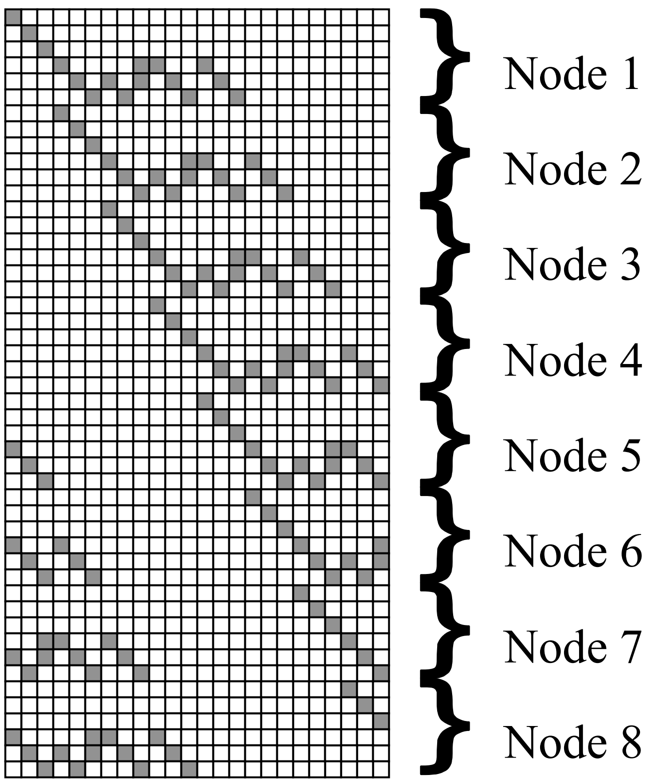
- This is an MDS code for $k=4$, $m=4$ over $GF(2^w)$, $w \geq 3$:

1	0	0	0	0	0	0	0	Node 1
0	1	2	1	1	0	0	0	
0	1	0	0	0	0	0	0	Node 2
0	0	1	2	1	1	0	0	
0	0	1	0	0	0	0	0	Node 3
0	0	0	1	2	1	1	0	
0	0	0	1	0	0	0	0	Node 4
0	0	0	0	1	2	1	1	
0	0	0	0	1	0	0	0	Node 5
1	0	0	0	0	1	2	1	
0	0	0	0	0	1	0	0	Node 6
1	1	0	0	0	0	1	2	
0	0	0	0	0	0	1	0	Node 7
2	1	1	0	0	0	0	1	
0	0	0	0	0	0	0	1	Node 8
1	2	1	1	0	0	0	0	

The kitchen table code

5. Reverting to Galois Field Arithmetic

- If we use the Cauchy Reed-Solomon coding transformation, we get the following Binary Dist. Matrix:



3.33 XORs per coding word.

Best current code is Cauchy RS @ 5.75 XORs per coding word.

At $GF(2^7)$, it's 3.14

And at $GF(2^\infty)$, it's 3.00.

What I Hope You Got From This:

- You pretend to care about erasure codes.
- You understand some of their issues, and that we don't currently live in a perfect world.
- I'm working to push the world more toward perfection.
- Some of this stuff is cool.
- Look for code / papers.

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