Congestion and Price Prediction Under Load Variation

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***Abstract*—In market-based planning and operation, it is very useful to have the information of generation dispatch, congestion, and price as load increases. Therefore, it is beneficial to system planners if the congestion or locational marginal pricing (LMP) versus load is readily available. This can certainly be obtained by repetitively running an optimization model at different load levels. However, this approach is too brute-force to be practical. In this paper, an efficient algorithm is proposed to identify the new binding constraint and the new marginal unit set when the system load increases from the present load level. It addresses the challenge of step changes in generation dispatch when a generation or transmission limit becomes binding. The algorithm also gives the new sensitivity of the new marginal units. Therefore, the generation dispatch, congested lines, and LMP at a new critical load level (CLL) can be easily calculated. Test results are presented in matrix formulation to clearly demonstrate and verify the proposed algorithm. Since the proposed approach is based on linearized model, it should be particularly suitable for short-term planning or operation, although application to long-term planning is also possible.**

***Index Terms*—Congestion management, critical load level (CLL), energy markets, generation sensitivity, locational marginal pricing (LMP), optimal power flow (OPF), power markets.**

I. Introduction

THE locational marginal pricing (LMP) methodology has been a dominant approach in energy market operation and planning to identify the nodal price and to manage the transmission congestion. For system operators and planners, it is always important to know the future price and possible new binding limits as the system load grows. This information can be used for congestion mitigation and load management in both short-term and long-term. Meanwhile, for generation companies, it is also important to predict the future price and possible congestion, as evidenced by the adoption of optimal power flow (OPF)-based market simulators with transmission fully modeled. This stimulates the research presented in this paper: to explore an efficient algorithm to identify congestion and LMP versus load levels such that an overview of the impact to congestion and price with respect to different system load levels can be easily provided.

Challenges arise because there is a step change of LMP when load grows to a certain level [1]. This is caused by the occurrence of a new binding limit, either a transmission line reaching its limit or a generator reaching its limit. Then, there will be a change of the marginal unit set and the sensitivity of marginal generation with respect to load. Fig. 1 shows a typical LMP versus load curve with a given growth pattern [1] for a sample system slightly modified from the well-known PJM 5-bus system.

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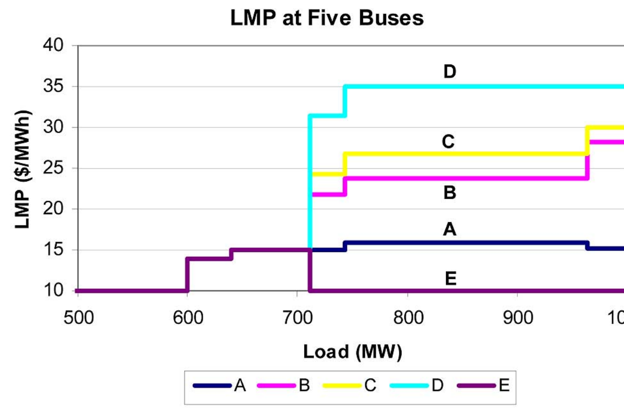

Fig. 1. LMP at all buses with respect to different system loads.

Certainly, the curve can be obtained if we repetitively run an optimization model and then LMP calculation at many different load levels. This approach of repetitive optimization runs can be relatively time-consuming, especially for short-term applications. Even though it may be still fast enough in practice for a one-scenario application such as a real-time dispatch, it will be worse if many different scenarios need to be run. For instance, a short-term market participant or system planner may want to run multiple scenarios with different load growth patterns and/or different transmission and generation maintenances. Then, multiple curves similar to Fig. 1 need to be obtained. This will make the running time of the repetitive optimization-run approach much loner. A more efficient algorithm is highly desired.

The previous work in [1] discusses a modified pricing scheme of LMP. An important step in [1] is to efficiently identify the next critical load level (CLL), defined as a load level at which a step change occurs, and the corresponding new binding limit, either a congested transmission line or a marginal unit reaching generation limit. This step is carried out by applying a perturbation to the energy balanced constraints as well as the present binding transmission constraints. While this approach can find the next CLL, however, several questions related to the system status at the next CLL still remain unresolved such as:

- which unit will be the next new marginal unit?

- what is the new generation sensitivity of each marginal unit? (Note: The generation sensitivity of the previous marginal units will change.)
- what is the new LMP at each bus?

Reference [1] suggests that another optimization run can be performed from scratch to obtain the new dispatch results, and therefore the new marginal unit and LMP at the next CLL. This will be improved in this paper such that congestion and LMP versus load growth can be obtained without running multiple optimizations by extending the approach in [1]. Although the proposed approach can be used to improve the continuous LMP (CLMP) methodology in [1], it is probably more important to emphasize that the proposed work can predict congestion and price with respect to load growth under the presently dominant LMP paradigm. Hence, the algorithm in this paper has immediate and important application in the present market-based operation and planning.

This paper is organized as follows. Section II reviews the basic model of LMP simulation. Section III presents the fundamental formulation of the proposed algorithm to express marginal variables and objective function in nonmarginal variables. Section IV presents the formulation under load variation. Section V presents the calculation of new binding limit, new critical load level, new set of marginal generators, new generation sensitivity of all marginal generators, and new prices, as load reaches the next critical level. Section VI presents an easy-to-follow example to illustrate the algorithm in matrix formulation based on the PJM 5-bus system. Section VII presents the performance speedup results with the PJM 5-bus, the IEEE 30-bus, and the IEEE 118-bus systems. Section VIII presents some discussion, and Section IX summarizes the paper and points out possible future works.

II. Review of LMP Model Based on DCOPF

To address the issues raised previously, the general formulation of Locational Marginal Price (LMP) is briefly reviewed here. LMP is usually decomposed into three components including marginal energy price, marginal congestion price, and marginal loss price [2]–[4]. A common model of LMP simulation is based on DC model and Linear Programming (LP) as shown in [2], which can easily incorporate marginal congestion and marginal losses.

There are several assumptions and simplifications for the discussion in this paper. They are listed as follows.

1) Each bus has one generator and one load for notational convenience.
2) The generation cost/bid has a piece-wise-linear curve, which is aligned with the industrial practice.
3) There is only one piece (or block) in the cost curve of each unit for notational convenience. The costs of any two units shall be different to avoid multiple solutions.
4) The direction problem of constraints is ignored in the derivation and equations in this paper for notational convenience, with the assumption that we consider positive directions only. The actual implementation considers both directions by constructing a pair of constraint equations to address the positive and negative directions.
5) The generic formulation using Generation Shift Factors (GSF) can address not only transmission thermal limits, but security (contingency) and nomogram limits by precalculating different sets of GSFs corresponding to the system topology for each contingency case or nomogram case. This paper shows a regular GSF for line thermal limits only for notational convenience.

The above assumptions and simplifications are mainly for notational convenience and do not change the mathematical kernel of this work. Actual implementation for the tests presented in Sections VI and VII has employed more complicate models such as multiple generators at a bus and bi-directional limits of transmission lines.

With the above assumptions, the generic dispatch model based on Linear Programming can be written as

$$\text{Min} \quad \sum_{i=1}^{N} C_i \times G_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} G_i - \sum_{i=1}^{N} D_i = 0 \tag{2}$$

$$\sum_{i=1}^{N} GSF_{k-i} \times (G_i - D_i) \leq F_k^{\max}, \quad \text{for } k \in \{\text{all lines}\} \tag{3}$$

$$G_i^{\min} \leq G_i \leq G_i^{\max} \text{ for } i \in \{\text{all generators}\} \tag{4}$$

where

N	number of buses;
C_i	generation cost at Bus i (\$/MWh);
G_i	generation dispatch at Bus i (MWh);
D_i	demand at Bus i (MWh);
GSF_{k-i}	generation shift factor to line k from bus i;
$F_k^{\max}$	transmission limit of line k.

The general formulation of LMP at Bus i can be written as follows:

$$LMP_i = LMP^{\text{energy}} + LMP_i^{\text{cong}} + LMP_i^{\text{loss}} \tag{5}$$

$$LMP^{\text{energy}} = \lambda \tag{6}$$

$$LMP_i^{\text{cong}} = \sum_{k=1}^{M} GSF_{k-i} \times \mu_k \tag{7}$$

$$LMP_i^{\text{loss}} = \lambda \times (DF_i - 1) \tag{8}$$

where

M	number of lines;
λ	Langrangian multiplier of the equality constraint, i.e., system energy balance equation in (2);
μ_k	Lagrangian multiplier of the kth transmission constraint;
DF_i	delivery factor at bus i.

It should be noted that the optimization model in (1–4) ignores losses, hence we have $DF_i = 1$ and $LMP_i^{loss} = 0$ in (8). The actual solution of LMP calculation, especially LMP loss component, remains a challenging task because delivery factors and actual generation dispatches are mutually dependent. Reference [2] proposes an iterative approach to address DCOPF-based LMP calculation. In the discussion in this work, the loss price is ignored to avoid the complicated issue with delivery factors and to emphasize the main point to be presented.

It should be also noted that other LMP formulations have been presented in [5]–[11]. They are not used here because the DCOPF based LMP decomposition is explicitly aligned with the published formulation at a number of ISOs [4], [12] and a number of commercial market simulators [3], [14], [15] and is acceptable in most cases compared with ACOPF. Also, it is noteworthy to mention other algorithms for reference-bus-independent LMP decomposition are presented in [4], [7] and LMP sensitivity presented in [2], [6]. In addition, [16] presents many practical issues to address in LMP simulation for market studies.

It is very important to be noted that conventional sensitivity analyses in [2], [6], and [7] give the sensitivity when there is a small perturbation and no change of marginal units under load variation is assumed. They do not address the issue when load continuously grows beyond the next CLL where a new binding limit occurs. In contrast, this paper will present a systematic approach, without a new optimization run, based on a simple matrix formulation to identify new CLL, new marginal units, new congested lines, and new nodal prices when load growth leads to a new binding constraint and a step change of LMP and congestion. This is the main mathematical significance of this paper. Apparently, the proposed works are also very different from the previous works [16], [17], both of which solve LMP at different hours (hence different load levels) using chronological optimization runs [16] or artificial intelligence [17]. This paper starts from the present optimum and finds the solution at the next CLL directly by utilizing the unique features of the optimal dispatch model. Hence, it avoids repetitive optimization runs and should be more computationally efficient.

III. Fundamental Formulation of the Proposed Algorithm

In this section, first, slack variables are applied to all inequality transmission constraints to convert them to equality constraints. Then, a matrix formulation is presented to rewrite all constraints such that marginal variables are expressed with nonmarginal variables. It should be noted that in this paper a single vector is typically denoted in bold font, while a vector composed of multiple vectors or a matrix is denoted in bold font with brackets.

Assume at the present load level, we have N_{MG} marginal units. Hence, we should have $N_{MG}-1$ congested lines, because the total number of marginal units is one more than the total number of congested lines [1], [18]. This can be written as

$$N_{MG} = M_{CL} + 1 = M - M_{UL} + 1 \tag{9}$$

where

N_{MG} number of marginal units;

M_{CL} number of congested lines;

M_{UL} number of uncongested lines.

Hence, we have energy balance equality constraint as

$$\sum_{j\in\{\mathsf{MG}\}} MG_j + \sum_{j\in\{\mathsf{NG}\}} NG_j = \sum_{i\in\{\mathsf{N}\}} D_i \tag{10}$$

where $\{\mathsf{MG}\}$, $\{\mathsf{NG}\}$, and $\{\mathsf{N}\}$ represent the marginal unit set, the nonmarginal unit set, and the all bus set, respectively.

The transmission inequality constraints can be written as equality constraints by introducing a nonnegative slack variable. If we use UL_k to represent the slack variables of uncongested lines and CL_k for congested lines, we have

$$\sum_{j\in\{\mathsf{MG}\}} GSF_{k-j} \times MG_j + \sum_{j\in\{\mathsf{NG}\}} GSF_{k-j} \times NG_j - \sum_{i\in\{\mathsf{N}\}} GSF_{k-i} \times D_i + CL_k = F_k^{\max}, \quad \forall\, k \in \{\mathsf{CL}\} \tag{11}$$

$$\sum_{j\in\{\mathsf{MG}\}} GSF_{k-j} \times MG_j + \sum_{j\in\{\mathsf{NG}\}} GSF_{k-j} \times NG_j - \sum_{i\in\{\mathsf{N}\}} GSF_{k-i} \times D_i + UL_k = F_k^{\max}, \quad \forall\, k \in \{\mathsf{UL}\} \tag{12}$$

where

$\{\mathsf{CL}\}$ set of congested lines;

$\{\mathsf{UL}\}$ set of uncongested lines.

Note: $CL_k = 0$ for each congested (binding) transmission constraint.

Equations (10)–(12) can be rewritten in matrix formulation as follows:

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{A_{11}} & \mathbf{A_{12}} \\ \mathbf{A_{21}} & \mathbf{A_{22}} \end{bmatrix} \times \begin{bmatrix} \mathbf{MG} \\ \mathbf{UL} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{p_1} \\ \mathbf{p_2} \end{bmatrix} + \begin{bmatrix} \mathbf{1} \\ \mathbf{q_1} \\ \mathbf{q_2} \end{bmatrix} \times \mathbf{D} + \begin{bmatrix} \mathbf{-1} \\ \mathbf{r_1} \\ \mathbf{r_2} \end{bmatrix} \times \mathbf{NG} + \begin{bmatrix} \mathbf{0} \\ \mathbf{t_1} \\ \mathbf{t_2} \end{bmatrix} \times \mathbf{CL} \tag{13}$$

where

MG $= N_{MG} \times 1$ vector representing marginal generator output;

NG $= N_{NG} \times 1$ vector representing nonmarginal generator output;

$\mathbf{UL}$ $= M_{UL} \times 1$ vector representing the slack variables of uncongested branches (lines);

$\mathbf{D}$ $= [D_1 D_2 \ldots D_N]^T = N \times 1$ vector representing all loads with the assumption that each bus has a load for notational simplicity;

$\mathbf{CL}$ $= M_{CL} \times 1$ vector representing the slack variables of congested lines; and it is a zero vector for the base case;

$\mathbf{1}$ = row vector of 1's (dimension is case dependent);

$\mathbf{0}$ = row vector of 0's (dimension is case dependent);

$[\mathbf{A_{11}}] = [\mathbf{GSF_{CL-MG}}]$ $= M_{CL} \times N_{MG}$ matrix representing the GSF of $M_{CL}(= N_{MG} - 1)$ congested lines w.r.t. marginal unit buses;

$[\mathbf{A_{12}}] = [\mathbf{0}]$ $= M_{CL} \times M_{UL}$ zero matrix;

$[\mathbf{A_{21}}] = [\mathbf{GSF_{UL-MG}}]$ $= M_{UL} \times N_{MG}$ matrix representing the GSF of uncongested lines w.r.t. marginal unit buses;

$[\mathbf{A_{22}}] = [\mathbf{I}]$ $= M_{UL} \times M_{UL}$ identity matrix;

$\mathbf{p}_1 = \mathbf{F_{CL}^{max}}$ $= M_{CL} \times 1$ vector representing the line flow limit of congested lines;

$\mathbf{p_2} = \mathbf{F_{UL}^{max}}$ $= M_{UL} \times 1$ vector representing the line flow limit of uncongested lines;

$[\mathbf{q_1}] = [\mathbf{GSF_{CL-N}}]$ $= M_{CL} \times N$ matrix representing the GSF of congested lines w.r.t. all buses (load buses);

$[\mathbf{q_2}] = [\mathbf{GSF_{UL-N}}]$ $= M_{UL} \times N$ matrix representing the GSF of noncongested lines w.r.t. all buses (load buses);

$[\mathbf{r_1}] = [-\mathbf{GSF_{CL-NG}}]$ $= M_{CL} \times N_{NG}$ matrix, representing the negative of GSF of congested lines w.r.t. nonmarginal unit buses;

$[\mathbf{r_2}] = [-\mathbf{GSF_{UL-NG}}]$ $= M_{UL} \times N_{NG}$ matrix of the GSF of congested lines w.r.t. nonmarginal unit buses;

$[\mathbf{t_1}] = [-\mathbf{I}]$ $= M_{CL} \times M_{CL}$ negative unity matrix;

$[\mathbf{t_2}] = [\mathbf{0}]$ $= M_{UL} \times M_{CL}$ zero matrix.

Equation (13) can be rewritten as

$$[\mathbf{A}] \times \begin{bmatrix} \mathbf{MG} \\ \mathbf{UL} \end{bmatrix} = \mathbf{p} + [\mathbf{q}] \times \mathbf{D} + [\mathbf{r}] \times \mathbf{NG} + [\mathbf{t}] \times \mathbf{CL}. \quad (14)$$

It should be mentioned that $\mathbf{CL}$, the vector of slack variables for present congested lines, is a vector of 0 at the present load level. However, $\mathbf{CL}$ needs to be kept as a set of variables, rather than constants of 0, in the formulation. The reason is that $\mathbf{CL}$ may change as load grows to beyond the next CLL. In other words, a congested line may become uncongested as load varies. Hence, $\mathbf{CL}$ should be kept here as a vector of variables. This is a critical step to the following analysis.

The above equation (14) can be further simplified by first finding the inverse of the matrix $[\mathbf{A}]$, i.e., $\left[\begin{array}{c|c} \mathbf{1} & \mathbf{0} \\ \mathbf{A_{11}} & \mathbf{A_{12}} \\ \hline \mathbf{A_{21}} & \mathbf{A_{22}} \end{array}\right]$.

This is given by

$$\left[\begin{array}{c|c} \mathbf{1} & \mathbf{0} \\ \mathbf{A_{11}} & \mathbf{A_{12}} \\ \hline \mathbf{A_{21}} & \mathbf{A_{22}} \end{array}\right]^{-1} = \begin{bmatrix} \mathbf{A'_{11}} & \mathbf{0} \\ \mathbf{A_{21}} & \mathbf{I} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A'^{-1}_{11}} & \mathbf{0} \\ -\mathbf{A_{21}} \times \mathbf{A'^{-1}_{11}} & \mathbf{I} \end{bmatrix} \quad (15)$$

where $[\mathbf{A'_{11}}] = \begin{bmatrix} \mathbf{1} \\ \mathbf{A_{11}} \end{bmatrix}$, which is an $N_{MG} \times N_{MG}$ square matrix.

Therefore, only the inverse of $[\mathbf{A'_{11}}]$ needs to be computed. Since there are only a few marginal units, the size of $[\mathbf{A'_{11}}]$ is usually very small. This will not give much computational burden to the algorithm. With the inversion of the $[\mathbf{A}]$ matrix, we could solve (15). Hence, we have

$$\begin{bmatrix} \mathbf{MG} \\ \mathbf{UL} \end{bmatrix} = \mathbf{P} + [\mathbf{Q}] \times \mathbf{D} + [\mathbf{R}] \times \mathbf{NG} + [\mathbf{T}] \times \mathbf{CL} \quad (16)$$

where

$\mathbf{P}$ $= [\mathbf{A}]^{-1}\,\mathbf{p}$; $[\mathbf{Q}] = [\mathbf{A}]^{-1}\,[\mathbf{q}]$; $[\mathbf{R}] = [\mathbf{A}]^{-1}\,[\mathbf{r}]$;

$[\mathbf{T}]$ $= [\mathbf{A}]^{-1}\,[\mathbf{t}]$.

As the above equation shows, (16) consists of three parts:

- 1 equation representing the energy balance equation;
- $M_{CL}(= N_{MG} - 1)$ equations representing the congested lines with slack variables $\mathbf{CL}$ being zeros;
- M_{UL} equations representing the uncongested lines with nonzero slack variables $\mathbf{UL}$.

As we can see, the formulation is written in a matrix form similar to the dictionary format of simplex method to solve linear programming problems. The reason to do this is to rewrite the nonzero variables (or basic variables) like $\mathbf{MG}$ and $\mathbf{UL}$ to the left-hand side, then any small change of load can be expressed as a corresponding change of $\mathbf{MG}$ or $\mathbf{UL}$. Hence, objective function can be written without $\mathbf{MG}$ or $\mathbf{UL}$, as shown below.

Using (16), we can rewrite the original objective function

$$z = \sum_{j \in \mathrm{MG}} C_j \times MG_j + \sum_{j \in \mathrm{NG}} C_j \times NG_j \tag{17}$$

as

$$\begin{aligned} z &= \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times \mathbf{MG} + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}} \times \mathbf{NG} \\ &= \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times (\mathbf{P}_{\mathbf{MG}} + [\mathbf{Q}_{\mathbf{MG}}] \times \mathbf{D} \\ &\quad + [\mathbf{R}_{\mathbf{MG}}] \times \mathbf{NG} + [\mathbf{T}_{\mathbf{MG}}] \times \mathbf{CL}) + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}} \times \mathbf{NG} \\ &= \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times \mathbf{P}_{\mathbf{MG}} + \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{Q}_{\mathbf{MG}}]\right) \times \mathbf{D} \\ &\quad + \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{R}_{\mathbf{MG}}] + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}}\right) \times \mathbf{NG} \\ &\quad + \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{T}_{\mathbf{MG}}] \times \mathbf{CL} \end{aligned} \tag{18}$$

where

$\mathbf{C}_{\mathbf{MG}}$	= column vector of marginal generator costs;
$\mathbf{C}_{\mathbf{NG}}$	= column vector of nonmarginal generator costs;
$[\mathbf{R}_{\mathbf{MG}}]$	= the first N_{MG} rows of the $[\mathbf{R}]$ matrix;
$[\mathbf{T}_{\mathbf{MG}}]$	= the first N_{MG} rows of the $[\mathbf{T}]$ matrix.

IV. Load Variation

If there is a change of system load, with the assumption of linear participating factors [1], we can rewrite the load as

$$\begin{aligned} D_i &= D_i^{(0)} + \Delta D_i = D_i^{(0)} + f_i \times \Delta D_\Sigma \\ D_\Sigma &= \sum_i D_i = \sum_i \left(D_i^{(0)} + f_i \times \Delta D_\Sigma\right) = D_\Sigma^{(0)} + \Delta D_\Sigma \\ \mathbf{\Delta D} &= \mathbf{f} \times \Delta D_\Sigma \end{aligned}$$

where

f_i	$= \frac{\partial D_i}{\partial D_\Sigma}$ (load growth participating factor), and $\sum_{i \in \mathrm{N}} f_i = 1$;
$\mathbf{f}$	$= [\, f_1 \quad f_2 \quad \ldots \quad f_N\,]^{\mathbf{T}}$, an $N \times 1$ column vector;
$\mathbf{\Delta D}$	column vector; ΔD_Σ is a scalar.

With the above load variation model, the change of each bus load follows a linear participating factor with respect to the system load change. This model is reasonable because each bus load can be modeled to have its own variation pattern such that different load characteristics like industrial loads, commercial loads and residential loads can be modeled accordingly. It is also flexible because the variation at each bus load is independent on the initial load. This model is particularly useful for short-term planning.

Considering the system load will be varied by ΔD_Σ, we have

$$\begin{bmatrix} \mathbf{\Delta MG} \\ \mathbf{\Delta UL} \end{bmatrix} = [\mathbf{Q}] \times \mathbf{\Delta D} + [\mathbf{R}] \times \mathbf{\Delta NG} + [\mathbf{T}] \times \mathbf{\Delta CL}. \tag{19}$$

Also considering the linear participating factors of load variation pattern, i.e., $\mathbf{\Delta D} = \mathbf{f} \times \Delta D_\Sigma$, we can rewrite the above equations to

$$\begin{aligned} \begin{bmatrix} \mathbf{\Delta MG} \\ \mathbf{\Delta UL} \end{bmatrix} &= [\mathbf{Q}] \times \mathbf{f} \times \Delta D_\Sigma + [\mathbf{R}] \times \mathbf{\Delta NG} + [\mathbf{T}] \times \mathbf{\Delta CL} \\ &= \mathbf{Q}' \times \Delta D_\Sigma + [\mathbf{R}] \times \mathbf{\Delta NG} + [\mathbf{T}] \times \mathbf{\Delta CL} \end{aligned} \tag{20}$$

where

$\mathbf{Q}'$ $= [\mathbf{Q}] \times \mathbf{f}$, a $(N_{MG} + M_{UL}) \times 1$ column vector.

We can further decouple the above equation into

$$\begin{aligned} \mathbf{\Delta MG} &= \mathbf{Q}'_{\mathbf{MG}} \times \Delta D_\Sigma + [\mathbf{R}_{\mathbf{MG}}] \times \mathbf{\Delta NG} \\ &\quad + [\mathbf{T}_{\mathbf{MG}}] \times \mathbf{\Delta CL} \end{aligned} \tag{21}$$

$$\begin{aligned} \mathbf{\Delta UL} &= \mathbf{Q}'_{\mathbf{UL}} \times \Delta D_\Sigma + [\mathbf{R}_{\mathbf{UL}}] \times \mathbf{\Delta NG} \\ &\quad + [\mathbf{T}_{\mathbf{UL}}] \times \mathbf{\Delta CL}. \end{aligned} \tag{22}$$

With the above two equations, we can immediately obtain the sensitivity of $\mathbf{MG}$ and $\mathbf{UL}$ with respect to load

$$\nabla_{D_\Sigma} \mathbf{MG} = \mathbf{Q}'_{\mathbf{MG}} \tag{23}$$

$$\nabla_{D_\Sigma} \mathbf{UL} = \mathbf{Q}'_{\mathbf{UL}}. \tag{24}$$

We can also write the change of objective function z as follows:

$$\begin{aligned} \Delta z &= (\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{Q}_{\mathbf{MG}}]) \times \mathbf{f} \times \Delta D_\Sigma \\ &\quad + (\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{R}_{\mathbf{MG}}] + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}}) \times \mathbf{\Delta NG} \\ &\quad + \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{T}_{\mathbf{MG}}] \times \mathbf{\Delta CL} \\ &= \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times \mathbf{Q}'_{\mathbf{MG}}\right) \times \Delta D_\Sigma \\ &\quad + \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{R}_{\mathbf{MG}}] + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}}\right) \times \mathbf{\Delta NG} \\ &\quad + \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{T}_{\mathbf{MG}}] \times \mathbf{\Delta CL}. \end{aligned} \tag{25}$$

Therefore, the sensitivity of objective function w.r.t. $\mathbf{NG}$ and $\mathbf{CL}$ can be written as follows:

$$\nabla_{\mathbf{NG}} z = \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{R}_{\mathbf{MG}}] + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}} \tag{26}$$

$$\nabla_{\mathbf{CL}} z = \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{T}_{\mathbf{MG}}]\,. \tag{27}$$

V. Identification of New Binding Limit, New Marginal Unit, and LMP

The above formulations can be applied to perform three important tasks: 1) identifying the next new binding limit, either generation limit or transmission limit, and the next CLL; 2) identifying the next unbinding limit such as a new marginal unit; 3) finding the new generation sensitivity of all marginal units and the new LMP. These steps provide important information like generation dispatch sensitivity or transmission congestion

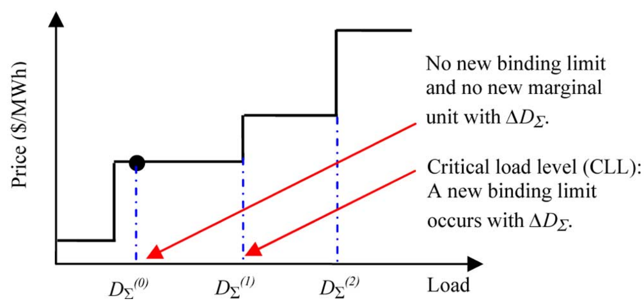

Fig. 2. LMP versus load curve.

prediction, which is aligned with the main goal of this paper, i.e., to find congestion and LMP versus load starting from any initial load level, say $D_\Sigma^{(0)}$, to any load level without running OPF, repetitively.

The Section V-A identifies the next CLL, $D_\Sigma^{(1)}$, as shown in Fig. 2, where a new binding limit will occur. The algorithm utilizes the feature that marginal units and congested lines will remain the same when the load variation ΔD_Σ does not push the load level beyond $D_\Sigma^{(1)}$. Then, the Section V-B will identify the change of binding limits and marginal unit if load grows to the immediate right side of $D_\Sigma^{(1)}$. The algorithm is based on finding the least incremental cost among all possible changes of nonmarginal units or slack variables of congested lines. The algorithm has a very simple final formulation and is very efficient. Essentially, these steps give new dispatches and congested lines at the new CLL, $D_\Sigma^{(1)}$. LMP can then be easily calculated at $D_\Sigma^{(1)}$.

Similarly, starting from $D_\Sigma^{(1)}$, we can easily repeat the above process to find congestion and LMP at the "next-next" CLL, $D_\Sigma^{(2)}$.

A. Identification of New Binding Limit and New Critical Load Level

When load grows and this growth does not lead to any change of marginal units, a nonmarginal unit output should remain its minimum or maximum, and the slack variable of a congested line should remain zero. In other words, $\mathbf{\Delta NG} = \mathbf{\Delta CL} = \mathbf{0}$. Meanwhile, marginal generators and unbinding transmission lines should change and may approach to their respective limits gradually. The one reaching its limit first will be the next binding limit. To analyze this, we have

$$\begin{bmatrix} \mathbf{\Delta MG} \\ \mathbf{\Delta UL} \end{bmatrix} = \begin{bmatrix} \mathbf{Q'_{MG}} \\ \mathbf{Q'_{UL}} \end{bmatrix} \times \Delta D_\Sigma. \tag{28}$$

Since all generation output and the line flows at the present load level are given from the initial OPF, it is not difficult to obtain the present values of the slack variables. In fact, many optimization solvers will give these values as output.

For uncongested transmission lines, the slack variable is $\mathbf{UL}$. Since the sensitivity of $\mathbf{UL}$ with respect to load change ΔD_Σ is given by $\nabla_{D_\Sigma}\mathbf{UL} = \mathbf{Q'_{UL}}$, we can obtain the allowed load growth before a line, say, the kth line, reaches its limit (i.e., UL_k reaching zero). This is given by

$$\Delta D_k^{\text{allowed}} = \frac{UL_k}{Q'_{UL_k}}. \tag{29}$$

For generators, the slack variable is not given explicitly in the previous formulation. However, it can be viewed as

$$\mathbf{MG} + \mathbf{s_{MG}} = \mathbf{MG^{max}} \tag{30}$$

$$s_{MG_i} = MG_i^{\max} - MG_i \tag{31}$$

where s_{MG_i} is the slack variable of the ith marginal generator.

Hence, the allowed load growth corresponding to the ith marginal generator as load grows can be given as

$$\Delta D_i^{\text{allowed}} = \frac{s_{MG_i}}{Q'_{MG_i}} = \frac{MG_i^{\max} - MG_i}{Q'_{MG_i}}. \tag{32}$$

Then, the minimum allowed load growth can be obtained by finding out the minimum value among all $\Delta D^{\text{allowed}}$ given by (29) and (32). Hence, the new critical load is equal to $D_\Sigma^{(0)} + \Delta D_{\min}^{\text{allowed}}$.

B. Identification of New Unbinding Constraint

When load increases or decreases, a new binding constraint will occur at the CLL, together with the occurrence of an unbinding constraint, which could be generation or transmission. From the previous subsection, it is known that the new binding constraint can be transmission or generation. These two scenarios will be discussed below.

1) Assume the lth Marginal Unit Becomes Nonmarginal (Binding): Since the lth marginal unit is binding, it cannot grow as the load increase beyond $D_\Sigma^{(1)}$ in Fig. 2. Therefore, the change of load ΔD_Σ, must be offset by either a previously nonmarginal (binding) unit output by ΔNG_j or a previously binding slack variable by ΔCL_k. From (20), we have

$$0 = Q'_{MG_l} \times \Delta D_\Sigma + \mathbf{R_{MG_l}} \times \mathbf{\Delta NG} + \mathbf{T_{MG_l}} \times \mathbf{\Delta CL} \tag{33}$$

where $\mathbf{R_{MG_l}}$ and $\mathbf{T_{MG_l}}$ are the lth row vectors of $[\mathbf{R_{MG}}]$ and $[\mathbf{T_{MG}}]$, respectively.

It is very important to note that when the load is a little bit more than the new CLL, there should be one and only one nonzero variables among all ΔNG_j and ΔCL_k. This is determined by the characteristics of linear programming, because the solution shall move from one vertex to an adjacent vertex (even though the vertices or boundaries themselves of the polytope should also change because of the change of D_Σ). Then, the determination of which ΔNG_j or ΔCL_k should be chosen as the next nonzero variable is based on the change of objective function.

If the jth nonmarginal unit becomes marginal, then we know from (33) that

$$\frac{\partial NG_j}{\partial D_\Sigma} = -\frac{Q'_{MG_l}}{R_{MG_{lj}}}, \quad j \in \{\text{NG}\}. \tag{34}$$

It should be noted that the above sensitivity must give NG_j a possible change that will not violate its limit. For instance, for the case of load increase, if NG_j is already at its maximum, the above sensitivity should be considered only if it is negative. And, if NG_j is at its minimum, the above sensitivity should be considered only if it is positive.

If the kth congested line becomes uncongested, then we know from (33) that

$$\frac{\partial CL_k}{\partial D_\Sigma} = -\frac{Q'_{MG_l}}{T_{MG_{lk}}}, \quad k \in \{\mathsf{CL}\}. \tag{35}$$

Again, the above sensitivity is considered only if it does not push CL_k to negative values. Right at the CLL, CL_k should be zero.

Then, taking (26)–(27), we can easily calculate the expected incremental cost vector $\begin{bmatrix} (\partial z/\partial NG_j) \times (\partial NG_j/\partial D_\Sigma), j \in \{\mathsf{NG}\}\,; \\ (\partial z/\partial CL_k) \times (\partial CL_k/\partial D_\Sigma), k \in \{\mathsf{CL}\} \end{bmatrix}$ that can be expanded as

$$\begin{bmatrix} \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{R}_{\mathbf{MG}}] + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}}\right)_j \times \left(-\frac{Q'_{MG_l}}{R_{MG_{lj}}}\right), j \in \{\mathsf{NG}\} \\ \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{T}_{\mathbf{MG}}]\right)_k \times \left(-\frac{Q'_{MG_l}}{T_{MG_{lk}}}\right), k \in \{\mathsf{CL}\} \end{bmatrix}. \tag{36}$$

Finally, we can choose the smallest positive one in load growth case (or, largest negative one in load drop case), and the corresponding j (or k) will be the new marginal unit (or new uncongested line).

2) Assume the rth Uncongested Line Becomes Congested (Binding): Similar to (33), we have

$$0 = Q'_{UL_r} \times \Delta D_\Sigma + \mathbf{R}_{\mathbf{UL}_r} \times \mathbf{\Delta NG} + \mathbf{T}_{\mathbf{UL}_r} \times \mathbf{\Delta CL}. \tag{37}$$

If the jth nonmarginal unit becomes marginal, then we know from (37) that

$$\frac{\partial NG_j}{\partial D_\Sigma} = -\frac{Q'_{UL_r}}{R_{UL_{rj}}}, \quad j \in \{\mathsf{NG}\}. \tag{38}$$

If the kth congested line becomes uncongested, then we know from (37) that

$$\frac{\partial CL_k}{\partial D_\Sigma} = -\frac{Q'_{UL_r}}{T_{UL_{rk}}}, \quad k \in \{\mathsf{CL}\}. \tag{39}$$

Similar to the discussions below (34) and (35), the sensitivity in (38) and (39) should be considered if and only if it presents a move away from the present binding limit.

Next, we can calculate the incremental cost vector

$$\begin{bmatrix} \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{R}_{\mathbf{MG}}] + \mathbf{C}_{NG}^{\mathbf{T}}\right)_j \times \left(-\frac{Q'_{UL_r}}{R_{UL_{rj}}}\right), j \in \{\mathsf{NG}\} \\ \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{T}_{\mathbf{MG}}]\right)_k \times \left(-\frac{Q'_{UL_r}}{T_{UL_{rk}}}\right), k \in \{\mathsf{CL}\} \end{bmatrix}. \tag{40}$$

Similarly, we should choose the smallest positive one in load growth case (or, largest negative one in load drop case), and the corresponding j (or k) will be the new marginal unit (or new uncongested line).

Note on the new sensitivity of marginal units when load is beyond the next CLL, $D_\Sigma^{(1)}$:

It should be mentioned that the sensitivity of all existing marginal units will also change after the introduction of a new marginal unit at $D_\Sigma^{(1)}$. This can be quantitatively calculated as

$$\left[\frac{\partial \mathbf{MG}}{\partial D_\Sigma}\right] = \mathbf{Q}'_{\mathbf{MG}} + [\mathbf{R}_{\mathbf{MG}}] \times \left[\frac{\partial \mathbf{NG}}{\partial D_\Sigma}\right] + [\mathbf{T}_{\mathbf{MG}}] \times \left[\frac{\partial \mathbf{CL}}{\partial D_\Sigma}\right]. \tag{41}$$

If there is no new marginal unit at CLL (such as load decreases to have a congested line become unbinding), then we have $[\partial \mathbf{NG}/\partial D_\Sigma] = [\mathbf{0}]$ while one and only one variable in $[\partial \mathbf{CL}/\partial D_\Sigma]$ is not zero. Similarly, if there is a new marginal unit, then we have $[\partial \mathbf{CL}/\partial D_\Sigma] = [\mathbf{0}]$ while one and only one variable in $[\partial \mathbf{NG}/\partial D_\Sigma]$ is not zero.

A more straightforward approach is to reformulate (13) using the new $\mathbf{MG}$, $\mathbf{NG}$, $\mathbf{UL}$ and $\mathbf{CL}$ vectors. The change to these vectors should be very little, because only one marginal variable in either $\mathbf{MG}$ or $\mathbf{UL}$ will be switched into $\mathbf{NG}$ or $\mathbf{CL}$. Also, only one nonmarginal variable in either $\mathbf{NG}$ or $\mathbf{CL}$ variable will be switched into $\mathbf{MG}$ or $\mathbf{UL}$. Then, we can apply (20) to obtain new generation sensitivity. By doing so, we can repeat the previous process and eventually identify the "next-next" CLL ($D_\Sigma^{(2)}$), the "next-next" binding limit, etc.

C. LMP at the New Critical Load Level

The above process can identify the new congestion, the new CLL, and the new marginal unit as load grows, but has not addressed the price calculation. Similar approach can be taken since generation sensitivity is the key to calculate LMP. However, there is a little different between the previous steps and this step. In the previous step, the load variation is a "global" scope variation where all load buses are assumed to vary together following some pattern. However, LMP is calculated as the change of cost to supply a "local" change of load at a single bus, after the generation dispatch has been addressed for the "global" change of load. Nevertheless, the LMP calculation can be performed with the essentially same approach, in particular, (20). The only difference is that now we use a different participating factor, expressed as $\mathbf{f} = [0\,0\ldots1\ldots0\,0]$, since LMP is location-dependent.

When load is beyond $D_\Sigma^{(1)}$, we can first formulate the new $\mathbf{MG}$, $\mathbf{NG}$, $\mathbf{UL}$, and $\mathbf{CL}$ vectors due to the change of marginal units and so on. Then, we can use (20) to calculate the marginal unit sensitivity with respect to a single bus load change using the "local" $\mathbf{f} = [0\,0\ldots1\ldots0\,0]^T$. Therefore, LMP at a particular bus can be easily calculated as

$$LMP_i = \frac{\partial z}{\partial D_i} = \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times \frac{\partial \mathbf{MG}}{\partial D_i} + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}} \times \frac{\partial \mathbf{NG}}{\partial D_i} = \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times \frac{\partial \mathbf{MG}}{\partial D_i} \tag{42}$$

where

∂D_i load change at a single bus i.

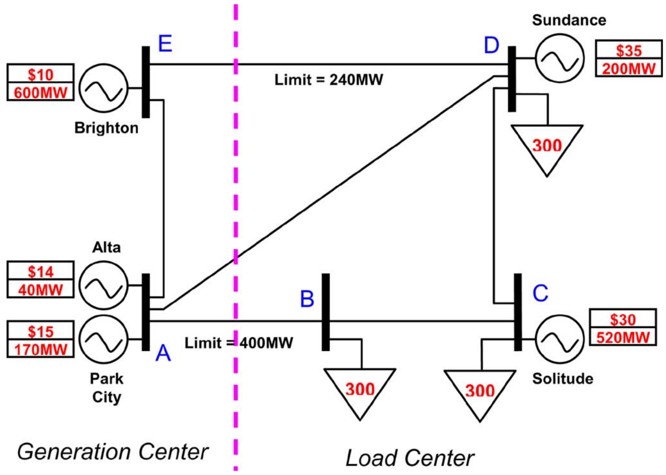

Fig. 3. Base case modified from the PJM five-bus example.

VI. Case Study With the PJM 5-Bus System

In this section, the PJM 5-Bus system with slight modification [1] will be employed to illustrate steps to identify new binding limit, new unbinding limit, new generation sensitivity, and new LMP as load grows. The modifications to the original PJM 5-bus system [11] are as follows.

- The output limit of the Alta unit is reduced from 110 to 40 MW, while the output limit of the Park City unit is increased from 100 to 170 MW;
- The cost of Sundance unit at Bus D is changed from $30/MWh to $35/MWh to differentiate its cost from the Solitude unit;
- Line AB is assumed to have a limit of 400 MW.

These changes are made such that there will be reasonably more binding limits within the investigated range of load levels. Also, we will not have two binding limits that occur at very close load levels. Hence, better illustration will be achieved when the price curves versus load levels are drawn [1].

We assume that the system load change is distributed to each nodal load proportional to its base case load for simplicity.

TABLE I
Line Impedance and Flow Limits

Line	AB	AD	AE	BC	CD	DE
X (%)	2.81	3.04	0.64	1.08	2.97	2.97
Limit (MW)	400	999	999	999	999	240

TABLE II
GSF of Line AB and ED

	A	B	C	D	E
Line AB	0.1939	-0.4759	-0.349	0	0.1595
Line ED	0.3685	0.2176	0.1595	0	0.4805

Therefore, the load change is equally distributed at Buses B, C and D since each has 300 MW load in the base case. Fig. 3 shows the configuration of the system, Table I shows the line reactances and flow limits, and Table II shows Generation Shift Factors of Line AB and ED with respect to all buses.

The basic OPF model for economic dispatch can be written as

$$\min \quad C_1 \cdot G_1 + C_2 \cdot G_2 + C_3 \cdot G_3 + C_4 \cdot G_4 + C_5 \cdot G_5$$
$$\text{s.t.} \quad \begin{cases} G_1 + G_2 + G_3 + G_4 + G_5 = D_\Sigma \\ \sum GSF_{1i} \times G_i + CL_1 = F_1^{\max} + \sum GSF_{1i} \times D_i \\ \sum GSF_{2i} \times G_i + UL_1 = F_2^{\max} + \sum GSF_{2i} \times D_i \\ \sum GSF_{3i} \times G_i + UL_2 = F_3^{\max} + \sum GSF_{3i} \times D_i \\ \sum GSF_{4i} \times G_i + UL_3 = F_4^{\max} + \sum GSF_{4i} \times D_i \\ \sum GSF_{5i} \times G_i + UL_4 = F_5^{\max} + \sum GSF_{5i} \times D_i \\ \sum GSF_{6i} \times G_i + UL_5 = F_6^{\max} + \sum GSF_{6i} \times D_i. \end{cases}$$

After solving the initial case OPF (load $= 900$ MW), there are 2 marginal units and three nonmarginal units as well as five uncongested lines and one congested line. Hence, we have seven nonzero basic variables, two for marginal units and five for uncongested lines. Then, we can rewrite the above equations in matrix formulation as shown in the equation at the bottom of the page, where MG_1 and MG_2 represent unit Sundance and

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.4805 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1595 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.3600 & 0 & 1 & 0 & 0 & 0 \\ 0 & -0.5195 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.1595 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0.1595 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} MG_1 \\ MG_2 \\ UL_1 \\ UL_2 \\ UL_3 \\ UL_4 \\ UL_5 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ -240 \\ 400 \\ 999 \\ -999 \\ 999 \\ -999 \end{bmatrix} + \begin{bmatrix} 1.0000 \\ -0.1257 \\ -0.2750 \\ 0.1493 \\ 0.1257 \\ 0.0584 \\ 0.3917 \end{bmatrix} D_\Sigma + \begin{bmatrix} -1.0000 & -1.0000 & -1.0000 \\ 0.3685 & 0.3685 & 0.1595 \\ -0.1939 & -0.1939 & 0.3490 \\ -0.4376 & -0.4376 & -0.1895 \\ -0.3685 & -0.3685 & -0.1595 \\ -0.1939 & -0.1939 & 0.3490 \\ -0.1939 & -0.1939 & -0.6510 \end{bmatrix} \begin{bmatrix} NG_1 \\ NG_2 \\ NG_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} CL_1$$

Brighton, respectively; NG_1, NG_2 and NG_3 represent unit Alta, Park City and Solitude respectively; CL_1 represents line DE, UL_1 to UL_5 represents remaining Lines, namely, AB, AD, AE, BC, and CD.

The above equation can be rewritten as

$$\begin{bmatrix} MG_1 \\ MG_2 \\ UL_1 \\ UL_2 \\ UL_3 \\ UL_4 \\ UL_5 \end{bmatrix} = \begin{bmatrix} -499.5298 \\ 499.5298 \\ 320.3060 \\ 819.1642 \\ -739.4702 \\ 919.3060 \\ -1078.6940 \end{bmatrix} + \begin{bmatrix} 0.7384 \\ 0.2616 \\ -0.3167 \\ 0.0551 \\ 0.2616 \\ 0.0166 \\ 0.3500 \end{bmatrix} D_\Sigma + \begin{bmatrix} -0.2330 & -0.2330 & -0.6679 \\ -0.7670 & -0.7670 & -0.3321 \\ -0.0716 & -0.0716 & 0.4020 \\ -0.1615 & -0.1615 & -0.0699 \\ -0.7670 & -0.7670 & -0.3321 \\ -0.0716 & -0.0716 & 0.4020 \\ -0.0716 & -0.0716 & -0.5980 \end{bmatrix} \begin{bmatrix} NG_1 \\ NG_2 \\ NG_3 \end{bmatrix} + \begin{bmatrix} -2.0814 \\ 2.0814 \\ -0.3321 \\ -0.7493 \\ 1.0814 \\ -0.3321 \\ -0.3321 \end{bmatrix} CL_1. \quad (43)$$

At present operating point ($D_\Sigma = 900$), we have the following results from the initial OPF:

$$\begin{bmatrix} NG_1 \\ NG_2 \\ NG_3 \end{bmatrix} = \begin{bmatrix} 40 \\ 170 \\ 0 \end{bmatrix}$$
$$[CL_1] = [0].$$

Therefore, we have

$$\begin{bmatrix} MG_1 \\ MG_2 \\ UL_1 \\ UL_2 \\ UL_3 \\ UL_4 \\ UL_5 \end{bmatrix} = \begin{bmatrix} 116.0757 \\ 573.9243 \\ 20.2495 \\ 834.8262 \\ -665.0757 \\ 919.2495 \\ -778.7505 \end{bmatrix}.$$

If the first two equations in (43) are put into the objective function, as shown in (18), we have

$$\begin{aligned} z &= \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times \mathbf{P}_{\mathbf{MG}} + \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{Q}_{\mathbf{MG}}]\right) \times \mathbf{D} \\ &\quad + \left(\mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{R}_{\mathbf{MG}}] + \mathbf{C}_{\mathbf{NG}}^{\mathbf{T}}\right) \times \mathbf{NG} \\ &\quad + \mathbf{C}_{\mathbf{MG}}^{\mathbf{T}} \times [\mathbf{T}_{\mathbf{MG}}] \times \mathbf{CL} \\ &= -12488.2459 + 28.4595 \times D_\Sigma \\ &\quad + [-1.8256 \quad -0.8256 \quad 3.3015] \times \begin{bmatrix} NG_1 \\ NG_2 \\ NG_3 \end{bmatrix} \\ &\quad + (-52.0344) \times CL_1. \end{aligned}$$

Next, the load increase case will be taken to illustrate the process.

A. Calculate the Next Binding Limit

Assuming a load variation ΔD_Σ, we have the following equation:

$$\begin{bmatrix} \Delta MG_1 \\ \Delta MG_2 \\ \Delta UL_1 \\ \Delta UL_2 \\ \Delta UL_3 \\ \Delta UL_4 \\ \Delta UL_5 \end{bmatrix} = \begin{bmatrix} 0.7384 \\ 0.2616 \\ -0.3167 \\ 0.0551 \\ 0.2616 \\ 0.0166 \\ 0.3500 \end{bmatrix} \Delta D_\Sigma.$$

The allowed load growth corresponding to each uncongested line is given by (29)

$$\Delta D_k^{\text{allowed}} = \frac{-UL_k}{Q'_{UL_k}} \Rightarrow -\begin{bmatrix} \frac{20.2495}{-0.3167} \\ \frac{834.8262}{0.0551} \\ \frac{-665.0757}{0.2616} \\ \frac{919.2495}{0.0166} \\ \frac{-778.7505}{0.3500} \end{bmatrix} = \begin{bmatrix} 63.9391 \\ -15157.0897 \\ 2542.1269 \\ -55265.5401 \\ 2225.2134 \end{bmatrix}.$$

Considering the maximum capacity case in (32), the allowed load growth of each marginal generator is given by

$$\Delta D_i^{\text{allowed}} = \frac{MG_i^{\max} - MG_i}{Q'_{MG_i}} \Rightarrow \begin{bmatrix} \frac{200-116.0757}{0.7384} \\ \frac{600-573.9243}{0.2616} \end{bmatrix} = \begin{bmatrix} 113.6603 \\ 99.6694 \end{bmatrix}.$$

Similarly, for the minimum capacity, the allowed load growth of each marginal generator is given by

$$\Delta D_i^{\text{allowed}} = \frac{MG_i^{\min} - MG_i}{Q'_{MG_i}} \Rightarrow \begin{bmatrix} \frac{0-116.0757}{0.7384} \\ \frac{0-573.9243}{0.2616} \end{bmatrix} = \begin{bmatrix} -157.2035 \\ -2193.7179 \end{bmatrix}.$$

Therefore, the minimum positive value of allowed load growth is 63.94 MW, which corresponds to congestion of line flow AB in the positive direction. So, the next binding limit will be line flow AB at load 963.94 MW.

B. Find the New Marginal Unit At Load $= 963.94$ MW

When the system load grows to 963.94 MW at which a new binding transmission limit (Line AB), the sensitivity of the new nonmarginal generator sensitivity is given by (38)

$$\frac{\partial NG_j}{\partial D_\Sigma} = -\frac{Q'_{UL_r}}{R_{UL_{rj}}} \Rightarrow -\begin{bmatrix} \frac{-0.3167}{-0.0716} \\ \frac{-0.3167}{-0.0716} \\ \frac{-0.3167}{0.4020} \end{bmatrix} = \begin{bmatrix} -4.4260 \\ -4.4260 \\ 0.7879 \end{bmatrix}.$$

Also, we have

$$\begin{aligned}\mathbf{C_{MG}^T} \times [\mathbf{R_{MG}}] + \mathbf{C_{NG}^T} &= [35.0000 \quad 10.0000] \\ &\times \begin{bmatrix} -0.2330 & -0.2330 & -0.6679 \\ -0.7670 & -0.7670 & -0.3321 \end{bmatrix} \\ &+ [14.0000 \quad 15.0000 \quad 30.0000] \\ &= [-1.8256 \quad -0.8256 \quad 3.3015].\end{aligned}$$

The incremental cost vector for $\mathbf{NG}$ is

$$\left(\mathbf{C_{MG}^T} \times [\mathbf{R_{MG}}] + \mathbf{C_{NG}^T}\right)_j \times \left(-\frac{Q'_{MG_l}}{R_{MG_{lj}}}\right) \Rightarrow \begin{bmatrix} -1.8256 \times (-4.4260) \\ -0.8256 \times (-4.4260) \\ 3.3015 \times 0.7879 \end{bmatrix} = \begin{bmatrix} 8.0800 \\ 3.6540 \\ 2.6011 \end{bmatrix}.$$

If we examine the sensitivity of uncongested lines based on (39), we have the equations at the bottom of the page.

If we compare all incremental costs associated with $\mathbf{NG}$ and $\mathbf{UL}$, the smallest positive value is 2.6011, which corresponds to NG_3, the Solitude unit at Bus C. So, the new marginal unit will be Solitude and there is no new congested line in this case.

C. Calculate the New LMP at Load Level 963.94 MW

For new marginal unit set, apply (21) with $\mathbf{f} = [0 \ \ 0 \ldots 1 \ldots 0 \ \ 0]^T$, to calculate $\partial \mathbf{MG}/\partial D_1$ for load variation at each single bus. Again, here the load variation occurs at a specific bus only. Since we have $\left[\frac{\partial \mathbf{MG}}{\partial D_1} \ \frac{\partial \mathbf{MG}}{\partial D_2} \ \frac{\partial \mathbf{MG}}{\partial D_3} \ \frac{\partial \mathbf{MG}}{\partial D_4} \ \frac{\partial \mathbf{MG}}{\partial D_5}\right] = \begin{bmatrix} 0.3519 & -0.3636 & 0.0000 & 1.0000 & 0.0000 \\ 0.8261 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ -0.1780 & 1.3636 & 1.0000 & 0.0000 & 0.0000 \end{bmatrix}$ the LMP at Bus 1 can be calculated as

$$\begin{aligned} LMP_1 &= \mathbf{C}_{MG}^{\mathbf{T}} \times \frac{\partial \mathbf{MG}}{\partial D_1} \\ &= [35 \quad 10 \quad 30] \times \begin{bmatrix} 0.3519 \\ 0.8261 \\ -0.1780 \end{bmatrix} \\ &= 15.2379. \end{aligned}$$

TABLE III
SPEEDUP OF THE PROPOSED ALGORITHM COMPARED WITH THE COMMON PRACTICES OF REPETITIVE DCOPF RUNS

System	Speedup compared with multiple (~10) DCOPF runs
PJM 5-bus	15.2
IEEE 30-bus	30.0
IEEE 118-bus	51.6

Similarly, we can obtain LMP for all buses from 963.94 MW to the next CLL as

$$\mathbf{LMP} = \begin{bmatrix} 15.2379 \\ 28.1818 \\ 30.0000 \\ 35.0000 \\ 10.0000 \end{bmatrix}.$$

VII. PERFORMANCE SPEEDUP

As previously mentioned, this approach is particularly suitable for a short-term or online application. Hence, performance is very important. The advantage of this approach is to start from the present optimal state to directly evaluate the new CLL and the associated congestion and price step changes. This approach avoids repetitive optimization runs by taking advantage of features unique to the optimal dispatch model. Not surprisingly, this approach is computationally much more efficient than the approach of repetitive optimization runs.

Here it is assumed that the range for the trial-and-error repetition is in 1000 intervals, such as from D_Σ to $D_\Sigma + 1000$ MW with 1 MW as the acceptable accuracy or from D_Σ to $D_\Sigma + 100$ MW with 0.1 MW as the acceptable accuracy. With the most optimistic assumption that there is only one step change during these intervals, we need to execute $\log_2 1000 (\approx 10)$ DCOPF runs in average with a binary search that is the most efficient searching algorithm in this case. With this estimated number of DCOPF runs, Table III shows that the speedup can be up to 51.6 for the IEEE 118-bus system. Here, speedup is defined as the average running time of the repetitive DCOPF-run approach divided by the average running time of the proposed algorithm, which gives the same output as the repetitive DCOPF runs such as CLL, marginal units, congested lines, LMPs, etc. It is more encouraging to observe that the speedup increases with larger systems. This makes the direct approach highly promising to be an online application, compared with the trial-and-error approach of repetitive OPF runs.

$$\frac{\partial CL_k}{\partial D_\Sigma} = -\frac{Q'_{UL_r}}{T_{UL_{rk}}} \Rightarrow -\begin{bmatrix} -0.3167 \\ -0.3321 \end{bmatrix} = [-0.9537]$$

$$\mathbf{C_{MG}^T} \times [\mathbf{T_{MG}}] = [35.0000 \quad 10.0000] \times \begin{bmatrix} -2.0814 \\ 2.0814 \end{bmatrix} = -52.0344$$

$$\left(\mathbf{C_{MG}^T} \times [\mathbf{T_{MG}}]\right)_k \times \left(-\frac{Q'_{UL_r}}{T_{UL_{rk}}}\right) \Rightarrow [-52.0344 \times (-0.9537)] = [49.6277]$$

TABLE IV
MARGINAL UNITS AND CONGESTION VERSUS LOAD GROWTH

Load Range(MW)	Marginal Unit(s)	Congested Line(s)
0~600	Brighton	None
600~640	Alta	None
640~ 711.8083	Park City	None
711.8083~ 742.7965	Park City Brighton	ED
742.7965~ 963.9391	Sundance Brighton	ED
963.9391~ 1137.0152	Solitude Sundance Brighton	AB ED

The test of DCOPF algorithm is implemented with Matlab packages using linear programming function, $linprog()$. Also, sparse matrix is applied for both approaches for the larger systems, the IEEE 30-bus and 118-bus cases. It should be noted that although the kernel of some commercial LP package may have the capability to perform a speeded follow-up LP run if it starts from the results of a previous case with careful data repreparation, the repetitive OPF-run approach shall be still much more time-consuming. The reasons are: 1) there is a need to run OPF multiple times to find the next CLL; and 2) each OPF, even if speeded, should be still slower than the direct algorithm presented in this paper at least due to the overhead such as data repreparation before each OPF. Moreover, if higher resolution is needed for CLL, the number of runs will be even more than the assumed 10 times in the test presented here.

VIII. DISCUSSIONS

The above test illustrates that we can quickly obtain the congestion or binding constraints at the next CLL without repetitively running OPFs at many different load levels. In fact, if we start from zero loads, we can efficiently calculate all binding constraints and prices at different load levels. Table IV shows the marginal units and congested lines corresponding to different CLLs for the PJM 5-bus case, calculated from the proposed approach. The price versus load curve can be easily plotted as well. It is ignored here since it is exactly the same as Fig. 1. This proposed direct approach requires only six runs of the proposed algorithm because there are only six CLLs (i.e., step changes). As a comparison, to obtain Fig. 1 with a similar resolution of CLLs, hundreds of DCOPF runs are needed. This also shows the high efficiency and great potential of the proposed algorithm.

Another note is that the proposed algorithm can be applied to the Continuous LMP (CLMP) methodology in [1] such that it is not necessary to rerun another optimization to obtain LMP at the next CLL, $D_{\Sigma}^{(1)}$, after we calculate ΔD_{Σ} from $D_{\Sigma}^{(0)}$. Nevertheless, as mentioned in Section I, it is more important to emphasize the application of the algorithm in the presently dominant LMP paradigm, because the immediate and important application in congestion and price prediction versus load growth is apparent.

The best application of this work is for short-term operation and planning, when the load change in each bus or area should be close to linear and proportional, and the impact from other factors like unit commitment may not be a significant factor. If applied for long-term planning, the proposed model should be less accurate if compared with the real-time operation. However, there is no existing model that can perform the same work easily and it is sometimes unnecessary to obtain high accuracy for long-term planning. So this work should be still valuable for long-term planning. Nevertheless, it is certain that the impact of unit commitment is an area for future research.

In addition, the generation ramping rate is another factor to consider in the future, especially for short-term applications in which the ramping rate is a possible constraint. Also, other possible future work may lie in different inputs such as nonlinear load variation pattern, generation uncertainty, transmission outage, and so on. If these are coupled with the loss model and ACOPF model, it will be more complicated.

It is true that the running time of the proposed approach, after some modifications to address the above modeling details, will be slower than its present version. However, with all of these complications, the corresponding trial-and-error approach with repetitive optimization runs should be slower as well. Therefore, it is not surprising if the relative speedup will be in the similar scale as shown in Table III.

IX. CONCLUSIONS

It is very useful to market-based operation and planning, especially in short term, if the information of congestion and price versus load can be easily obtained. The proposed algorithm helps the system operators and planners to easily identify possible congestion as the system load grows. It also presents useful information to generation companies to identify the possible congestions and price change as the system load grows, since many of them use OPF model for congestion and price forecasting to achieve better economic benefits. The technical challenges arise if the load variation leads to a change of binding constraint, which will lead to a change of marginal unit set and a step change in LMP. The previous work of the sensitivity of LMP and other variables with respect to load works only for a small load variation without change of binding constraints and cannot work when there is a large variation of system load leading to a new congestion and a step change of LMP.

This paper presents a systematic approach to give a global view of congestion and price versus load, from any given load level to another level, without multiple optimization runs. As shown in the mathematical derivation and the case study, this approach is carried out in the following steps.

- It first expresses marginal variables as a function of other nonmarginal variables.
- Then, it identifies the next binding limit and the next CLL.
- Next, the next unbinding limit such as a new marginal unit can be selected.
- Finally, the new generation output sensitivity at the CLL can be obtained because the objective function is expressed as nonmarginal variables. Therefore, the new LMP can be obtained when the load is greater than the CLL.
- The same procedure can be repeated to run though another CLL.

This approach has great potentials in market-based system operation and planning, especially in short term, for congestion management and price prediction. Future work may lie in the impact of unit commitment, generation ramping rate, different load variation model, uncertainties, inclusion of loss model, and using ACOPF models.

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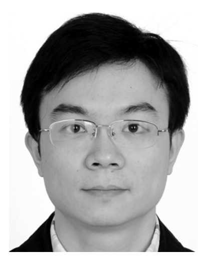

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