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Fully reference-independent LMP decomposition using reference-independent loss factors

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ABSTRACT

The decomposition of locational marginal price (LMP) under the popular DCOPF framework generally depends on the choice of the reference bus. A previous work has achieved reference independence for the overall LMP and LMP congestion component, but not all individual LMP components. This paper proposes a method to obtain a truly reference-independent LMP decomposition such that all three components of LMP at each bus will be invariant w.r.t. the choice of the system reference bus. This is achieved with loss factors based on a new AC-based distribution factor model, which depends on the network topology and the present operating condition only, but not the system reference bus. This model gives reference-independent loss prices, which can serve for a better loss hedging financial transmission rights, since the choice of reference bus will not change the loss prices. Further, this paper uses the fictitious nodal demand (FND) model to obtain loss distribution factors (LDFs). FND gives more reasonable power flows since losses should be distributed in each individual line, rather than at load buses when the load-weighted LDFs are applied. Also, the proposed reference-independent distribution factors and loss factors may have great potentials in other areas of power system analysis.

1. Introduction

The locational marginal price (LMP) methodology has been implemented or is under consideration at a number of US RTOs or ISOs such as PJM, New York ISO, ISO-New England, California ISO, ERCOT, and Midwest ISO. LMP at a given Bus *B* can be decomposed into three components: marginal energy price, marginal congestion price, and marginal loss price [1–5]. This can be written as

$$\mathrm{LMP}_B = \mathrm{LMP}^{\mathrm{energy}} + \mathrm{LMP}_B^{\mathrm{cong}} + \mathrm{LMP}_B^{\mathrm{loss}} \quad (1)$$

A number of earlier works [6–18] have reported LMP-related research results. In particular, Refs. [6–13] discussed the modelling of LMPs and its decomposed components. Also, a distributed reference bus model is discussed in [14]. The sensitivity of LMP is discussed in [13,15]. The comparison of AC-based and DC-based results is discussed in [9,13]. A modification of LMP methodology is proposed in [16]. Forecasting of LMP considering load variation and uncertainty is presented in [17,18]. Loss hedging rights are discussed in [19,20].

Ref. [10] presents an ACOPF-based decomposition which is independent of the choice of energy reference bus. Ref. [8] presents an approach to calculate reference-independent LMP and its congestion component based on DCOPF using linear programming (LP). Since LP-based DCOPF is popularly employed in industrial practices for real-time LMP calculation at a number of RTOs/ISOs, this paper uses DC models to extend the previous LMP research in [8].

In the decomposition model in [8], LMP congestion component at Bus *B*, i.e., $\mathrm{LMP}_B^{\mathrm{cong}}$, remains invariant w.r.t. different reference buses, and the combination of the other two components, i.e., $\mathrm{LMP}^{\mathrm{energy}} + \mathrm{LMP}_B^{\mathrm{loss}}$, is also reference-independent. The $\mathrm{LMP}_B^{\mathrm{cong}}$ is needed for financial transmission rights to hedge the transmission congestion cost, while $\mathrm{LMP}_B^{\mathrm{loss}}$ is useful for the proposed loss price hedging in [19,20]. Since the previous works did not provide a separation of $\mathrm{LMP}^{\mathrm{energy}}$ and $\mathrm{LMP}_B^{\mathrm{loss}}$, this can be controversial when loss price hedging is considered. Based on this motivation, this paper will present a decomposition model that makes three individual LMP components fully independent of the choice of reference. This is achieved by using a loss factor model based on a proposed new AC-based distribution factor model, which depends on the network topology and the operating condition only and does not require a system reference bus. Also, the loss factor and the LMP loss component at the man-made reference bus will not be zero, while the literature gives zero values. The non-zero values are more reasonable since in reality there is no reference bus and every bus should have some contribution to losses.

There are a few assumptions of the formulations in this paper that are listed here to avoid confusion: (1) each bus has one generator and one load for simplicity of discussion; (2) each transmission

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constraint (thermal, contingency, nomogram, etc.) may have bidirectional limits in reality, but it is modelled as if a unidirectional limit for simple formulation; (3) a single-block generation cost or bid model is assumed, while in reality a monotonically increasing multi-block model is commonly used; and (4) the demand elasticity is not explicitly modelled since unserved loads can be simply viewed as generation resources.

2. Review of DCOPF formulations for LMP calculation

2.1. Model 1 (lossless)

Earlier studies of LMP calculation with linearized OPF ignore line losses. Thus, the energy price and the congestion price follow a perfect linear model given by:

$$\text{Min} \quad \sum_{i=1}^{N} c_i \times G_i \tag{2}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} G_i = \sum_{i=1}^{N} D_i \tag{3}$$

$$\sum_{i=1}^{N} \text{GSF}_{k-i} \times (G_i - D_i) \le Limit_k, \quad \text{for } k = 1 \sim M \tag{4}$$

$$G_i^{\min} \le G_i \le G_i^{\max}, \quad \text{for } i = 1 \sim N \tag{5}$$

The LMP decomposition of this model is straightforward and ignored here. It is well known that the actual GSF values depend on the choice of reference bus. However, the line flow models in (4) based on the reference-dependent GSF are reference-independent. So, this model produces the same power flow results and hence the same LMP regardless of the choice of reference bus. This model can be acceptable for estimation purposes or as a starting point for market-related research. However, it may not be preferred in operation because of the lack of the loss component. Also, the energy and congestion components will vary w.r.t. different choices of reference.

2.2. Model 2

When losses are considered, the key to consider marginal loss price is marginal loss factor (LF) and the marginal delivery factor (DF), defined as:

$$\text{DF}_i = 1 - \text{LF}_i = 1 - \frac{\partial P_{\text{Loss}}}{\partial P_i} \tag{6}$$

LFs and DFs will be one of the main topics in this paper. For now, they are assumed available. Then, we can formulate different DC-based OPF models. A straightforward approach presented in the past is to multiply DF by nodal injections to account for losses in the energy balance equation while keeping (2)–(5) unchanged.

$$\text{Min} \quad \sum_{i=1}^{N} c_i \times G_i \tag{7}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \text{DF}_i \times G_i - \sum_{i=1}^{N} \text{DF}_i \times D_i + offset = 0 \tag{8}$$

$$\sum_{i=1}^{N} \text{GSF}_{k-i} \times (G_i - D_i) \le Limit_k, \quad \text{for } k = 1 \sim M \tag{9}$$

$$G_i^{\min} \le G_i \le G_i^{\max}, \quad \text{for } i = 1 \sim N \tag{10}$$

The LMP decomposition of this model is straightforward and can be found in [8,13]. It has been observed that the marginal DF may produce doubled losses in Eq. (8) without *offset*. Ref. [13] rigorously proves that in a fully DC-based model the value of *offset* should be the estimated total system losses. Without *offset*, generators may output doubled losses. Also, *offset* may consider errors in initial loss estimation.

2.3. Model 3

This is the optimization model named LP2 in [8] for LMP calculation considering losses. It can be written as:

$$\text{Min} \quad \sum_{i=1}^{N} c_i \times G_i \tag{11}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} G_i - \sum_{i=1}^{N} D_i - P_{\text{Loss}} = 0 \tag{12}$$

$$P_{\text{Loss}} - \sum_{i=1}^{N} \text{LF}_i \times G_i + \sum_{i=1}^{N} \text{LF}_i \times D_i + offset = 0 \tag{13}$$

$$\sum_{i=1}^{N} \text{GSF}_{k-i} \times (G_i - D_i - \text{LDF}_i \times P_{\text{Loss}}) \le Limit_k, \quad \text{for } k = 1 \sim M \tag{14}$$

$$G_i^{\min} \le G_i \le G_i^{\max}, \quad \text{for } i = 1 \sim N \tag{15}$$

As previously mentioned, $\text{LMP}_B^{\text{cong}}$ is reference-independent, and the combination of the other two components, i.e., $\text{LMP}^{\text{energy}} + \text{LMP}_B^{\text{loss}}$, is also reference-independent. But each of $\text{LMP}^{\text{energy}}$ or $\text{LMP}_B^{\text{loss}}$ is still reference-dependent.

2.4. Model 4

Another model of LMP with losses is proposed in [13] using an iterative approach. It can be written as:

$$\text{Min} \quad \sum_{i=1}^{N} c_i \times G_i \tag{16}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \text{DF}_i^{\text{est}} \times G_i - \sum_{i=1}^{N} \text{DF}_i^{\text{est}} \times D_i + P_{\text{Loss}}^{\text{est}} = 0 \tag{17}$$

$$\sum_{i=1}^{N} \text{GSF}_{k-i} \times (G_i - D_i - E_i^{\text{est}}) \le Limit_k, \quad \text{for } k = 1 \sim M \tag{18}$$

$$G_i^{\min} \le G_i \le G_i^{\max}, \quad \text{for } i = 1 \sim N \tag{19}$$

Here an iterative approach is used. Initially, $\text{DF}_i^{\text{est}} = 1$, $\text{E}_i^{\text{est}} = 0$, and $P_{\text{Loss}}^{\text{est}} = 0$; and essentially a lossless DCOPF is performed. Then, DF_i, E_i and P_{Loss} are updated to start a new DCOPF. This is repeated till convergence. It is also discussed that a two-iteration simplification, i.e., lossless model in (2)–(5) for the first iteration and then (16)–(19) for the second iteration, can produce good enough results. The LMP decomposition of this model is the same as Model 2, as shown in [13]. An important feature of this formulation is the fictitious nodal demand (FND) model to mimic line losses. For each line, 50% of the line loss is assigned to each connected bus as an extra demand, represented by E_i. Hence, the losses are distributed in each line. As shown in Section 5, this FND model can be used to give fairer and more reasonable loss distribution factors (LDFs) to improve Model 3.

3. Reference-independent distribution factors and loss factors

3.1. Motivation for reference-independent LF

When losses are considered, LF is commonly employed as shown in Models 2–4. However, LF in the previous works depends on the reference choice. In general, this leads to reference-dependent decomposition.

Model 3 is the present state of the art in terms of achieving reference-independence $\text{LMP}_B^{\text{cong}}$ and $(\text{LMP}^{\text{energy}} + \text{LMP}_B^{\text{loss}})$ using DC model. The LMP decomposition is given by:

$$\text{LMP}^{\text{energy}} = \tau \tag{20}$$

$$\text{LMP}_B^{\text{loss}} = -\tau \times \text{LF}_B \tag{21}$$

$$\text{LMP}_B^{\text{cong}} = \lambda - \tau + \sum_{k=1}^{M} \mu_k \times \text{GSF}_{k-B} = -\sum_{k=1}^{M}\left(\mu_k \times \sum_{i=1}^{N} \text{LDF}_i \times \text{GSF}_{k-i}\right) + \sum_{k=1}^{M}(\mu_k \times \text{GSF}_{k-B}) \tag{22}$$

From (20) to (21), the combination of LMP energy and loss components can be written as

$$\text{LMP}^{\text{energy}} + \text{LMP}_B^{\text{loss}} = \tau - \tau \times \text{LF}_B = \tau \times (1 - \text{LF}_B) \tag{23}$$

There are only two variables in (23). Hence, if we can find a way to make one variable (such as LF_B) reference-independent, the other variable, τ, will be reference-independent as well. Therefore, a fully reference-independent LMP decomposition can be achieved.

3.2. Basic model for loss factors

As shown in [13], LF can be modelled as:

$$P_{\text{Loss}} = \sum_{k=1}^{M} F_k^2 \times R_k \tag{24}$$

$$\text{LF}_i = \frac{\partial P_{\text{Loss}}}{\partial P_i} = \frac{\partial}{\partial P_i}\left(\sum_{k=1}^{M} F_k^2 \times R_k\right) = \sum_{k=1}^{M} R_k \times 2F_k \times \frac{\partial F_k}{\partial P_i} \tag{25}$$

In the above equations, the line flow F_k should be obtained from state estimation results in operation. If we assume a perfect data measurement and state estimation to simplify our discussion, F_k will be the same as the results from economic dispatch by solving ACOPF or a close approximation (a lossless DCOPF as an extreme simplification). Although either ACOPF or lossless DCOPF needs a voltage zero-angle reference, the line flow F_k should be always reference-independent. Hence, seeking reference-independent loss factors (LF_i) is converted to seeking a reference-independent $\partial F_k/\partial P_i$, the distribution factor (a.k.a. sensitivity factor) of line flow w.r.t. bus injection. Please note $\partial F_k/\partial P_i$ involves real power only.

The real-power-only $\partial F_k/\partial P_i$ naturally makes one to consider a linear lossless DC network, in which a line flow is usually considered as the aggregation of the contribution from all power sources (generation as positive and load as negative) based on superposition. This can be written as

$$F_k = \sum_{j=1}^{N} \text{GSF}_{k-j} \times (G_j - D_j) = \sum_{j=1}^{N} \text{GSF}_{k-j} \times P_j \tag{26}$$

From (26), we have

$$\frac{\partial F_k}{\partial P_i} = \text{GSF}_{k-i} \tag{27}$$

Hence, the conventional LF_i in (25) is reference-dependent because the above DC-based distribution factor, namely GSF in this paper, is reference-dependent. For this reason, it is not likely to have DC-based reference-independent distribution factors. Thus, we may have to explore other approaches like AC-based model, which will be intensively studied next.

To avoid confusion, we first define the generalized AC-based distribution factor of line flow in MVA with respect to nodal injection in MVA, namely, $\partial S_k/\partial S_i$ or ρ_{k-i} for notational convenience. Also, we need to define the real-power distribution factor, $\rho_{k-i,RE}$, as:

$$\rho_{k-i,RE} = \frac{\partial F_k}{\partial P_i} = \frac{(\partial S_k)^{RE}}{(\partial S_i)^{RE}} \tag{28}$$

It should be noted that the above definition takes out the impact of reactive components because the imaginary components in ∂S_k and ∂S_i will partly contribute to the real part of ρ_{k-i}. Hence, (28) will be truly the real power sensitivity. It should be noted that in general we have $\rho_{k-i,RE} \neq (\partial S_k/\partial S_i)^{RE}$. Apparently, the challenge here is to find a reference-independent $\rho_{k-i,RE}$, i.e., AC-based distribution factor $(\partial S_k)^{RE}/(\partial S_i)^{RE}$. The remaining part of this section will show a previous model of AC-based sensitivity, point out an unjustified assumption in its derivation, and then give a mathematically rigorous model with numerical verification.

3.3. Previous model of reference-independent ρ_{k-i}

Ref. [21] shows a well-known derivation of ρ_{k-i}, which is reference-independent. This is shown as follows:

$$\begin{aligned}\rho_{k-i} &= \frac{\partial S_k}{\partial S_i} = \frac{\partial(V_{k1}I_k^*)}{\partial(V_iI_i^*)} = \frac{\partial(V_{k1}\cdot((V_{k1}-V_{k2})^*/z_k{}^*))}{\partial(V_iI_i^*)} \\ &\approx \frac{\partial((V_{k1}-V_{k2})^*/z_k{}^*)}{\partial(I_i^*)} = \frac{1}{z_k^*}\cdot\left(\frac{\partial V_{k1}^*}{\partial I_i^*} - \frac{\partial V_{k2}^*}{\partial I_i^*}\right) \\ &= \frac{Z_{k1,i}^* - Z_{k2,i}^*}{z_k^*}\end{aligned} \tag{29}$$

Regardless of the difference between ρ_{k-i} and $\rho_{k-i,RE}$ that Eq. (29) does not address, the above derivation implies injection as current sources, because it assumes that all bus voltages are held invariant and close enough in both magnitude and angle (i.e., $V_{k1} = V_i$ and they are invariant). The voltage magnitudes may be close to constant, but the angle difference can be considerable. In addition, from the viewpoint of circuit analysis, if there is a current injection change at Bus i, it should change the voltage at every bus as well. Otherwise, if voltages do not change, the current and power injection will not change at all since we have $V = Z_{\text{bus}} \times I$. So, generally speaking, voltages are not independent variables if we consider the network and do not consider some voltage control actions. Hence, the assumption of constant voltage magnitude and angle is not justifiable and should be relaxed.

3.4. Reference-independent $\rho_{k-i,RE}$

The bus voltage at any Bus B can be written as:

$$V_B = \sum_{j=1}^{N}(Z_{B,j}\cdot I_j) \tag{30}$$

And the bus injection power at Bus i is given by

$$S_i = V_i \cdot I_i^* = I_i^* \cdot \sum_{j=1}^{N}(Z_{i,j} \cdot I_j) \tag{31}$$

The existing line flow through Line k at the sending end, i.e., Bus $k1$, is given by

$$\begin{aligned} S_k &= V_{k1} \cdot I_k^* = \frac{V_{k1} \cdot (V_{k1}^* - V_{k2}^*)}{z_k^*} = \frac{\sum_{j=1}^{N}(Z_{k1,j} \cdot I_j)}{z_k^*} \\ &\cdot \left(\sum_{j=1}^{N}(Z_{k1,j}^* \cdot I_j^*) - \sum_{j=1}^{N}(Z_{k2,j}^* \cdot I_j^*)\right) \\ &= \frac{\left[\sum_{j=1}^{N}(Z_{k1,j} \cdot I_j)\right] \cdot \left[\sum_{j=1}^{N}((Z_{k1,j}^* - Z_{k2,j}^*) \cdot I_j^*)\right]}{z_k^*} \end{aligned} \tag{32}$$

As we can see from (31) and (32), both bus injection and line flow are related to the system topology and the initial injections. The former is constant and the latter is independent variables. Hence, bus injection S_i and line flow S_k can be expressed using independent variables I, and there is no intermediate variable like voltages involved in (31) and (32). This will make the derivation below very clear without any possible confusion.

Now we need to consider a small change of I_i, say, ∂x. Although ∂x can be any complex number in theory, here we can consider that ∂x is applied to the magnitude I_i only to keep the same power factor, roughly speaking. To facilitate our derivation, we can take angular shift to make the angle of I_i the reference angle, i.e., 0 degrees for I_i. This is because angular shift (or changing voltage/current reference angle) does not affect bus injection power and line flows, since phase angles are indeed relative measures while power is not.

Without losing generality, we continue to use the symbol I_i for simplicity after the angular shift. Since $I_i^{IM} = 0$, we have

$$(I_i + \partial x)^* - I_i^* = \partial x = (I_i + \partial x) - I_i \tag{33}$$

The change of bus injection power at Bus i is given by

$$\begin{aligned} \partial S_i &= \partial x \cdot \sum_{j=1}^{N}(Z_{i,j} \cdot I_j) + Z_{i,i} \cdot I_i^* \cdot \partial x \\ &= \left(\sum_{j=1}^{N}(Z_{i,j} \cdot I_j) + Z_{i,i} \cdot I_i^*\right) \cdot \partial x \end{aligned} \tag{34}$$

The change of the kth line flow at the $k1$ end is given by

$$\begin{aligned} \partial S_k &= \frac{Z_{k1,i} \cdot \partial x}{z_k^*} \cdot \left(\sum_{j=1}^{N}(Z_{k1,j}^* \cdot I_j^*) - \sum_{j=1}^{N}(Z_{k2,j}^* \cdot I_j^*)\right) \\ &+ \frac{\sum_{j=1}^{N}(Z_{k1,j} \cdot I_j)}{z_k^*} \cdot (Z_{k1,i}^* - Z_{k2,i}^*) \cdot \partial x \end{aligned} \tag{35}$$

To obtain $\rho_{k-i,RE}$ we need to combine (34) and (35). Since the perturbation ∂x is a real-number scalar (magnitude only), which can be eliminated, so we have the sensitivity $\rho_{k-i,RE}$ at the sending end Bus $k1$ given by:

$$\rho_{k-i,RE(k1)} = \frac{(\partial S_k)^{RE}}{(\partial S_i)^{RE}} = \frac{\left[(Z_{k1,i}/z_k^*) \cdot \left(\sum_{j=1}^{N}(Z_{k1,j}^* \cdot I_j^*) - \sum_{j=1}^{N}(Z_{k2,j}^* \cdot I_j^*)\right) + \left(\sum_{j=1}^{N}(Z_{k1,j} \cdot I_j)/z_k^*\right) \cdot (Z_{k1,i}^* - Z_{k2,i}^*)\right]^{RE}}{\left[\sum_{j=1}^{N}(Z_{i,j} \cdot I_j) + Z_{i,i} \cdot I_i^*\right]^{RE}} \tag{36}$$

Similar to (31) and (32), (36) involves only the network topology and the initial condition of current injections. There is no intermediate variable, and there is no reference bus involved. Hence, the power distribution factor $\rho_{k-i,RE}$ does not require a reference bus.

Further, if we consider $V_B = \sum_{j=1}^{N}(Z_{B,j} \cdot I_j)$, we can simplify (36) as follows:

$$\rho_{k-i,RE(k1)} = \frac{[(Z_{k1,i}/z_k^*) \cdot (V_{k1}^* - V_{k2}^*) + ((Z_{k1,j}^* - Z_{k2,j}^*)/z_k^*) \cdot V_{k1}]^{RE}}{[V_i + Z_{i,i} \cdot I_i^*]^{RE}} \tag{37}$$

This simplified equation shows the involvement of intermediate state variables of bus voltages, which are essentially determined by the initial condition and the network topology. So, it is still reference-independent. It is not difficult to observe that if we assume voltages are all close to unity, and $V_i(= \sum_{j=1}^{N}(Z_{i,j} \cdot I_j))$ is much greater than $Z_{i,i} \cdot I_i^*$, then (37) is simplified back to (29).

The receiving end sensitivity is similar to (37) except that Bus $k2$ is treated as $k1$. This is given by

$$\rho_{k-i,RE(k2)} = \frac{\left[(Z_{k2,i}/z_k^*) \cdot (V_{k1}^* - V_{k2}^*) + ((Z_{k1,j}^* - Z_{k2,j}^*)/z_k^*) \cdot V_{k2}\right]^{RE}}{[V_i + Z_{i,i} \cdot I_i^*]^{RE}} \tag{38}$$

The sensitivity based on the flow at the center of Line k is the average of (37) and (38). Thus, we have

$$\begin{aligned} \rho_{k-i,RE} &= \frac{\rho_{k-i,RE(k1)} + \rho_{k-i,RE(k2)}}{2} \\ &= \frac{[((Z_{k1,i} + Z_{k2,i}) \cdot (V_{k1}^* - V_{k2}^*) + (Z_{k1,j}^* - Z_{k2,j}^*) \cdot (V_{k1} + V_{k2}))/2z_k^*]^{RE}}{[V_i + Z_{i,i} \cdot I_i^*]^{RE}} \end{aligned} \tag{39}$$

The above analytical derivation of (39) is rigorous and does not need the assumption of invariant voltages implied by (29). Hence, this is the first major contribution of this paper. This can be summarized as below:

The complex power distribution (sensitivity) factor, ρ_{k-i}, and the corresponding real power distribution factor, $\rho_{k-i,RE}$, are determined by the network topology and the present operating condition only, and do not require a system reference.

It should be noted that the derivation of analytical Eq. (39) assumes the bus injections at the present operating point are current sources. This is also implied in Ref. [21] for Eq. (29). This should be a reasonable assumption because of two reasons: some loads behave as current sources indeed as evidenced by many power system dynamic studies; and more important, the voltage or power control cannot have instantaneously effect when a very tiny current perturbation is instantaneously applied to a selected bus.

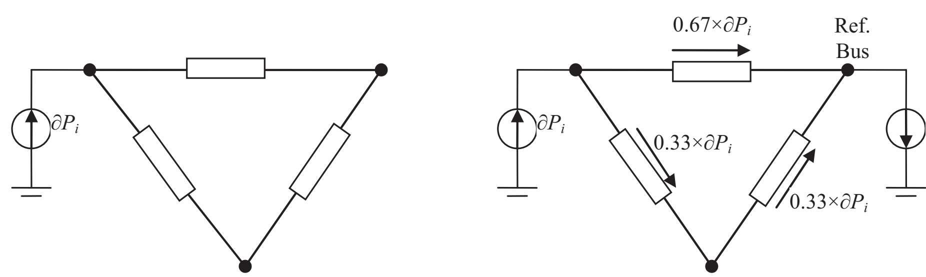
3.5. Numerical verification of reference-independent $\rho_{k-i,RE}$

Numerical tests are presented in this subsection to verify Eq. (39). The test system is a modified PJM 5-bus system with details shown in Section 5 (see Fig. 3 and related data). The test procedure is described as follows:

- First, the current injections for a base case are obtained. This is done with an ACOPF run to find different voltage magnitudes and angles, which should represent the results from economic dispatch. The voltage phasors from Buses A to E are $1.1\angle 0°$, $1.0797\angle -3.3066°$, $1.0855\angle -3.0619°$, $1.0866\angle -2.6929°$, and $1.092\angle 0.7443°$, respectively. Then, solving $I = Z^{-1} \times V$ gives the bus current injections. The injections will be considered as the

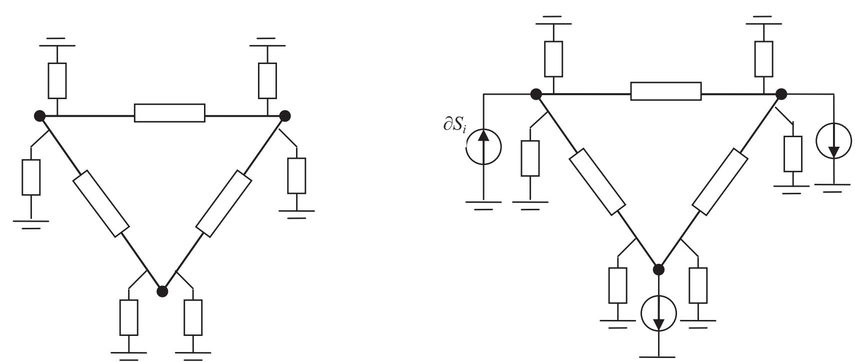
Table 1
Verification of $\rho_{k-i,RE}$ of each line w.r.t. Buses 1 and 4.

Line	Analytical: using the proposed model in Eq. (39)		Numerical: 1% perturbation of injecting current		Real part of ρ_{k-i} using the previous work shown in Eq. (29)	
	w.r.t. Bus 1	w.r.t. Bus 4	w.r.t. Bus 1	w.r.t. Bus 4	w.r.t. Bus 1	w.r.t. Bus 4
AB	0.3267	0.0047	0.3267	0.0047	0.2402	0.0416
AD	0.2016	−0.3322	0.2016	−0.3322	0.1490	−0.2934
AE	0.4700	0.2221	0.4700	0.2221	0.3265	−0.0427
BC	0.0009	−0.1688	0.0009	−0.1688	0.0693	−0.1279
CD	−0.1037	−0.2869	−0.1037	−0.2869	−0.0999	−0.2931
DE	−0.1049	0.3839	−0.1049	0.3839	−0.0822	0.2911



(a) No current can flow back to the ground without a reference bus to absorb the injection.
(b) Current can flow to the reference bus through the network.

Fig. 1. Necessity of the reference bus for finding GSF using DC model.



(a) AC network with shunts
(b) Equivalent to have multiple sinks determined by network parameters

Fig. 2. Using AC network to find loss factors.

initial condition here. Note: The voltage angle reference for ACOPF is Bus 1. Changing it to another bus will change the angle values at each individual bus, but not the relative differences between any two phasors. Hence, generation dispatch and line flow remain the same, because angle is a relative measure, but power is not. Hence, generation output and line flows from ACOPF are indeed reference independent.

- Second, a perturbation is performed by applying a 1% current change at a given bus. The system is solved using V' = Z × I' (prime means the perturbed case), since current injections are the independent variables.
- Finally, the ratio of real-power line-flow change versus real-power bus-injection change is calculated using power flows from the base case obtained with ACOPF and the perturbed case. The values of this "numerically" calculated sensitivity, i.e., $(\Delta S_k)^{RE}/(\Delta S_i)^{RE}$, are compared with the "analytically" calculated values, i.e., $(\partial S_k)^{RE}/(\partial S_i)^{RE}$, using (39). The comparison is shown in the second and third columns in Table 1.

Apparently, the analytically calculated $\rho_{k-i,RE}$ using (39) is accurate as verified by the numerically calculated values, i.e.,

$$\frac{(\partial S_k)^{RE}}{(\partial S_i)^{RE}} \approx \frac{(\Delta S_k)^{RE}}{(\Delta S_i)^{RE}}$$

The distribution factors using the real part of ρ_{k-i} from the previous work shown in Eq. (29) is listed in the last column in Table 1. If compared with Eq. (39), Eq. (29) leads to significant difference. This is reasonable because of the nonlinearity of power systems, which means that the sensitivity with the present operating condition ignored, i.e., sensitivity without no source, is certainly different from the condition with loads. This justifies the necessity of using a more complicated analytical model (39) that includes the initial system condition and considers voltages as variables rather than constant. And, more numerical tests, not shown here simply due to space limit, all support Eq. (39).

Combining (39) with (25), the reference-independent loss factor is given as follows:

$$\mathrm{LF}_i = \frac{\partial P_{\mathrm{Loss}}}{\partial P_i} = \sum_{k=1}^{M} R_k \times 2F_k \times \rho_{k-i,RE} \tag{40}$$

where $\rho_{k-i,RE}$ is defined in (39). Since both F_k and $\rho_{k-i,RE}$ are reference-independent, the loss factor LF_i is reference-independent.

3.6. Some illustrative comments

The foundation of the DC power flow model is that the line flow can be simplified to

$$F_k = \frac{\delta_{k1} - \delta_{k2}}{x_k} \tag{41}$$

Hence, only line reactances are needed in the model. Line resistances and shunt capacitances are all ignored. To calculate the GSF of Bus i to Line k, it is assumed that there is a small injection ∂P_i at Bus i. Here ∂P_i can be viewed as a current injection as well since the voltage is held at 1.0 p.u. at every bus. Then the resultant line flow at Line k divided by ∂P_i is the GSF of Bus i to Line k. This is shown in Fig. 1 with the upper-right bus as the reference. As shown in the figure, since we have an injection, we must specify a reference bus to absorb the injection. Otherwise, there is no path to let the ∂P_i injection at Bus i to flow back to ground. This is why a reference/slack bus must be specified. In case we have a distributed reference bus, then the ∂P_i injection will flow back at different buses to ground with the amount decided by reference-bus weighting factors.

As shown in Fig. 1, when the reference bus changes, the GSF of each bus to different lines may change. This is the reason that LF from (25) and (27) is reference-dependent.

In contrast, the AC network model has shunt capacitances that are typically represented by the "pi" model shown in Fig. 2. The distribution factor ρ_{k-i} means the change of MVA flow through Line k when there is a per unit change of an injection at Bus i. As we assume the injections are all current sources, a small change of I_i leads to voltage changes at all buses and then leads to changes of injection power at all buses with injection sources, positive (generation) or negative (load). The changes at other buses are essentially equivalent to multiple absorption sinks corresponding to the perturbation (∂S_i) at Bus i, as illustrated in Fig. 2. The absorption amounts at different buses are different, and they are objectively determined by the system topology and the initial condition (operating point). It is not determined by any user-defined or man-made reference. This is the significant difference between the proposed model and conventional approaches. For instance, the generic Power Transfer Distribution Factor (PTDF) in [22–24] requires a pair of user-defined injection and sink. Also, the traditional (distributed) reference bus is essentially a user-defined sink.

The user-defined or man-made source-sink pair or reference bus(es) will give different incremental line flow with a different choice of sink or reference. However, the equivalent sinks shown in Fig. 2(b) are objectively determined by the network topology and the present operating point, both of which are reference-independent. Therefore, the AC-based sensitivity factor ρ_{k-i} (or $\rho_{k-i,RE}$) and the loss factor LF_i do not need a reference bus defined by users.

Here is another explanation from the mathematical viewpoint. The Z_{bus} matrix in Eq. (39) is the inversion of the Y_{bus} matrix. If all line resistances and shunt branches are ignored, the AC model is simplified to the DC model. That is, $\rho_{k-i} = \rho_{k-i,RE} = \mathrm{GSF}_{k-i}$. Then, we cannot obtain the Z_{bus} matrix because the $N \times N$ Y_{bus} matrix is singular with a rank of $N-1$ due to the ignored shunt branches. Hence, to make the Y_{bus} matrix invertible, we have to specify a reference bus. This means to delete the row and column associated with the reference bus such that the rank of the new $(N-1) \times (N-1)$ Y_{bus} matrix is $N-1$ (i.e., nonsingular). Apparently, with shunt branches and the AC model, the original $N \times N$ Y_{bus} matrix is nonsingular and can be inversed to obtain Z_{bus}.

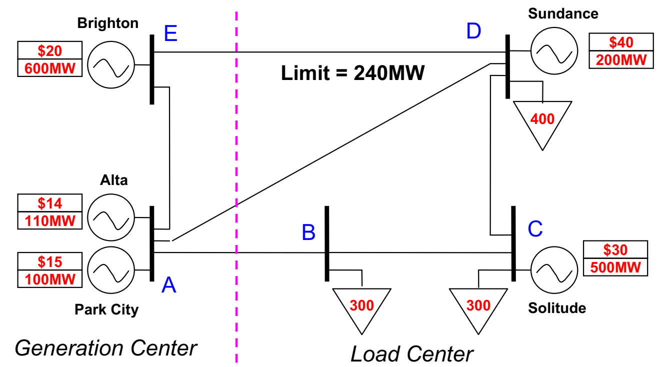


Fig. 3. The base case for simulation test.

4. Proposed LMP model

4.1. Application of FND model for LDF

Beside the proposed reference-independent loss factors expressed in (40) and (39), another proposed improvement of LMP model lies in the representation of loss distribution factors (LDFs). In the previous work [8], LDFs are simply modelled with bus loads as the weighting factors as follows:

$$\mathrm{LDF}_i = \frac{D_i}{D_\Sigma} = \frac{D_i}{\sum_{j=1}^{N} D_j} \tag{42}$$

The proposed improved model applies fictitious nodal demand (FND) to represent LDFs. This can be done by obtaining line flows first via an initial ACOPF or DCOPF. Then, the kth line losses can be calculated as $F_k^2 \times R_k$ and then equally allocated to each of the two connected buses [13]. The accumulated FND at each bus will be used as the weights to calculate LDF_i in Eq. (14). The new LDF model at Bus i can be written as:

$$E_i = \sum_{k=1}^{M_i} \frac{1}{2} \times F_k^2 \times R_k \tag{43}$$

$$\mathrm{LDF}_i = \frac{E_i}{\sum_{j=1}^{N} E_j} \tag{44}$$

where M_i = the number of lines connected to Bus i.

If compared with the LDF model using bus loads as weighting factors in (42), this FND-based model is a more reasonable approach giving better power flow results with very little extra computing and modelling effort, because there is no new variable introduced into the optimization model in (11)–(15).

4.2. Proposed models

Here are two models that will be used in the simulation test in this paper.

- Model 5 (for comparison purpose):
  - Eqs. (11)–(15) for DCOPF-based dispatch;
  - Eq. (40) and (39) for loss factors;
  - Eq. (42) to obtain LDFs using bus loads as weights.

Table 2
Line parameters.

Line	AB	AD	AE	BC	CD	DE
R (%)	0.281	0.304	0.064	0.108	0.297	0.297
X (%)	2.81	3.04	0.64	1.08	2.97	2.97
$B/2$ (% 10^{-3})	3.56	3.29	15.63	9.26	3.37	3.37

- Model 6 (final model):
  - Eqs. (11)–(15) for DCOPF-based dispatch;
  - Eqs. (40) and (39) for loss factors;
  - Eqs. (43) and (44) to obtain LDFs using the FND model.

Both models use (20)–(22) for LMP decomposition. It should be noted that the DC-based GSF is still needed in the dispatch model, i.e., (11)–(15), to model line flows under linear programming because of the nature and advantage of DCOPF in handing transmission constraints and achieving a straightforward LMP decomposition. Otherwise, the final line flows from dispatch model cannot perfectly produce the same power flow results, and the reference-independent LMP decomposition cannot be achieved.

The above models require initial values of system status, such as line flows, generation outputs, and so on, to obtain E_i in (43), LDF_i in (44), *offset* in (13), and LF_i in (40). Since they need to be reference-independent, the initial values of system status need to be reference-independent. Apparently, running an initial ACOPF or lossless DCOPF should fit this need. Then, the DC-based Model 5 or 6 can be applied for LMP calculation with fully reference-independent decomposition.

4.3. Role of the initial OPF

It is important to be noted that running an initial ACOPF is aligned with the popular industrial practice of Ex Post LMP model. In operation, there are three typical steps:

1. An ACOPF or a close approximation, lossless DCOPF as an extreme, is performed for Ex Ante economic dispatch.
2. State estimation is performed to smooth measurement error and to provide input (line flows, generation outputs, etc.) for LMP and other real-time applications.
3. A DCOPF-with-loss model is performed for Ex Post LMP.

If we reasonably assume measurement is perfect to skip the state estimation such that we can focus on LMP models, the above process is basically an ACOPF (or a lossless DCOPF) for dispatch and a DCOPF-with-loss for LMP calculation. Since ACOPF (or lossless DCOPF) gives reference-independent line flows, generation outputs, etc., it is justifiable for the proposed process to use the initial ACOPF results for reference-independent LF_i and then DC-based Model 5 or 6 for reference-independent LMP decomposition.

5. Test results

5.1. Test case

The PJM 5-bus system [1] is used for simulation in this section. The system configuration, generation bids, generation limits, and loads are shown in Fig. 3. Only the Line DE is assumed to have a thermal limit of 240 MW. The line parameters are given in Table 2, where the reactances are from the original case in [1] and the resistances are assumed to be 10% of corresponding line reactances. Each of the two shunt capacitances of a “pi” model transmission line is assumed to have a reactance value of −100 times the line reactance. And the reactive generation limits are simply set to 150 MVar, from leading to lagging. The above data are from [8] except the assumed line resistances, shunt capacitances, and the reactive generation limits, which are not available from [8].

Table 3
Initial dispatch results from ACOPF.

Gen	Alta	Park City	Solitude	Sundance	Brighton
Dispatch (MW)	110	100	325.92	0	468.44

Table 4
Initial MW line flows from ACOPF.

LINE	AB	AD	AE	BC	CD	DE
Sending end	249.94	188.13	−228.07	−51.60	−25.73	−238.54
Receiving end	248.40	187.21	−228.47	−51.65	−25.74	−239.97
Line center	249.17	187.67	−228.27	−51.62	−25.74	−239.25

Table 5
Results from Model 5 using Bus A as the reference bus.

Bus	A	B	C	D	E
Bus gen.	210.0000	0.0000	329.1660	0.0000	465.7886
Bus load	0.0000	300.0000	300.0000	400.0000	0.0000
Bus loss	0.0000	1.4864	1.4864	1.9818	0.0000
Loss factor	0.0071	−0.0176	0.0321	−0.0092	0.0177
GSF	0.0000	−0.1509	−0.2090	−0.3685	0.1120
LDF	0.0000	0.3000	0.3000	0.4000	0.0000
LMP	23.9953	29.7270	30.0000	36.5493	20.0000
LMP energy	32.5590	32.5590	32.5590	32.5590	32.5590
LMP loss	−0.2328	0.5746	−1.0450	0.2996	−0.5756
LMP cong.	−8.3310	−3.4067	−1.5141	3.6906	−11.9834

An ACOPF is applied to obtain the initial economic dispatch solution used for the follow-up LMP calculation. For instance, the results can be used to obtain nodal AC currents and voltages and then the reference-independent $\rho_{k-i,RE}$, as exactly shown in Section 3.5. Then, the reference-independent loss factor, LF_i, can be calculated using Eq. (40). The estimated system losses can be used to set the *offset* in Eq. (13), and the line flows can be used to obtain E_i in (43) and LDF_i in (44). Table 3 shows the generation output. Table 4 shows the MW line flows at the sending end, at the receiving end, and at the line center.

If the results from an initial lossless DCOPF are applied to find reference-independent LF_i, the final LF_i is extremely close to (less than 3% error) the one obtained using results from ACOPF. This is reasonable because linearized DC model in high voltage AC system is usually considered efficient enough.

5.2. Results from Model 5 (bus loads as weights for LDFs)

In this test, LDFs are calculated using bus loads as the weighting factors as specified in Model 5. So, the LDFs are 0, 0.3, 0.3, 0.4, and 0 from Buses A to E, respectively. Here two cases are studied: (1) Bus A as the reference; and (2) distributed reference buses of B, C and D with bus loads as the weights (i.e., 0.3, 0.3, and 0.4).

As shown in Tables 5 and 6, the GSFs are different w.r.t. different references. But the dispatches are the same. Each LMP component is also identical. The LMPs at the marginal unit buses (C and E) are equal to the corresponding marginal unit cost. This meets the principle of LMP modelling.

Fig. 4 also shows the line flows. It can be easily verified that the values of bus loss or bus mismatch (=incoming line flows + generation − outgoing flows − load) accounting for losses are 0, 1.4864, 1.4864, 1.9818, and 0 at Buses A–E, respectively. There is no loss balance at Buses A and E because the LDFs are 0 at these two buses. This is different from reality because losses are distributed in each line, and every bus should balance some losses. The next subsection will show that each bus will absorb some losses

Table 6
Results from Model 5 using distributed reference at B, C, and D.

Bus	A	B	C	D	E
Bus gen.	210.0000	0.0000	329.1660	0.0000	465.7886
Bus load	0.0000	300.0000	300.0000	400.0000	0.0000
Bus loss	0.0000	1.4864	1.4864	1.9818	0.0000
Loss factor	0.0071	−0.0176	0.0321	−0.0092	0.0177
GSF	0.2554	0.1044	0.0464	−0.1131	0.3673
LDF	0.0000	0.3000	0.3000	0.4000	0.0000
LMP	23.9953	29.7270	30.0000	36.5493	20.0000
LMP energy	32.5590	32.5590	32.5590	32.5590	32.5590
LMP loss	−0.2328	0.5746	−1.0450	0.2996	−0.5756
LMP cong.	−8.3310	−3.4067	−1.5141	3.6906	−11.9834

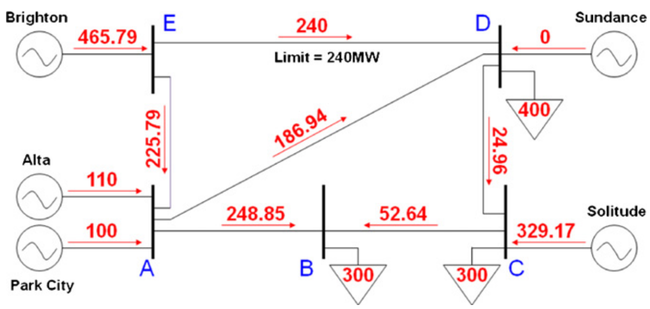


Fig. 4. Line flow results using load-weighted LDF (Model 5).

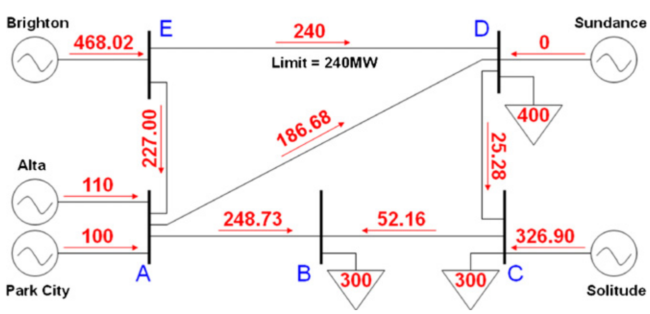


Fig. 5. Line flow results using FND-based LDF (Model 6).

Table 7
Results from Model 6 using Bus A as the reference bus.

Bus	A	B	C	D	E
Bus gen.	210.0000	0.0000	326.9002	0.0000	468.0212
Bus load	0.0000	300.0000	300.0000	400.0000	0.0000
Bus loss	1.5822	0.8910	0.0244	1.4020	1.0218
Loss factor	0.0071	−0.0176	0.0321	−0.0092	0.0177
GSF	0.0000	−0.1509	−0.2090	−0.3685	0.1120
LDF	0.3215	0.1811	0.0049	0.2849	0.2076
LMP	23.9194	29.4972	30.0000	36.3131	20.0000
LMP energy	27.6851	27.6851	27.6851	27.6851	27.6851
LMP loss	−0.1979	0.4886	−0.8885	0.2548	−0.4895
LMP cong.	−3.5678	1.3235	3.2034	8.3731	−7.1957

corresponding to its connecting line losses with the FND model for LDFs.

5.3. Results from Model 6 (FND-based LDFs)

Here the proposed final model (Model 6) is used for another simulation run, in which the FND model is applied for calculating LDFs. Results are shown in Fig. 5 and Tables 7 and 8. Again, the GSFs are different w.r.t. different reference buses; but the dispatch results and the LMP decomposition are identical. Similar to Model 5, the LMPs at marginal unit buses (C and E) are equal to local marginal unit cost.

Table 8
Results from Model 6 using distributed reference at B, C, and D.

Bus	A	B	C	D	E
Bus gen.	210.0000	0.0000	326.9002	0.0000	468.0212
Bus load	0.0000	300.0000	300.0000	400.0000	0.0000
Bus loss	1.5822	0.8910	0.0244	1.4020	1.0218
Loss factor	0.0071	−0.0176	0.0321	−0.0092	0.0177
GSF	0.2554	0.1044	0.0464	−0.1131	0.3673
LDF	0.3215	0.1811	0.0049	0.2849	0.2076
LMP	23.9194	29.4972	30.0000	36.3131	20.0000
LMP energy	27.6851	27.6851	27.6851	27.6851	27.6851
LMP loss	−0.1979	0.4886	−0.8885	0.2548	−0.4895
LMP cong.	−3.5678	1.3235	3.2034	8.3731	−7.1957

Fig. 5 shows that the losses are distributed in each line and eventually balanced at every bus as FNDs (i.e., E_i). The values of bus losses are 1.5822, 0.8910, 0.0244, 1.4020, and 1.0218 at Buses A–E, respectively. It can be easily verified that each bus loss equals to its loss factor multiplied by the system total losses. Hence, this is a fairer and more reasonable model for obtaining LDFs.

5.4. Comparison with Model 3

Models 5 and 6 use the same LDF model as Model 3. The loss factor model is the only difference between Models 5 and 3. The loss factors of Model 3 are reference-dependent; therefore, LF at the reference in Model 3 should be 0. Hence, LMP^{loss} at the reference should be 0 as well. This is shown in the results in [8]. For instance, Tables III and IV in [8] show LF = 0 and LMP^{loss} = 0 at the reference bus. Note that the weighted average values for LF and LMP^{loss} should be used for the distributed reference bus in the case of Table IV in [8].

In contrast, Models 5 and 6 give non-zero loss factors and non-zero LMP^{loss} at all buses. This should be more reasonable than Model 3 because in reality there is no reference (slack) bus and every bus should have some contribution to losses. Hence, LF and LMP^{loss} should not be zero at a given reference which is purely man-made or user-defined.

As previously mentioned, Model 6 further improves Model 5 by using the FND model for a better power flow representation such that losses are distributed into each line, rather than load buses.

5.5. Comparison with ACOPF-based LMP

Although true ACOPF is not commonly used in industrial practices due to the convergence issue, it is a good tool for benchmark purpose because ACOPF gives the exact dispatch results considering all transmission and generation constraints in full AC model. Thus, it gives the accurate LMP at each bus. Therefore, a good approximate, with-loss, DCOPF-based model should produce results close to that from ACOPF.

However, it should be noted that decomposition of the exact ACOPF-based LMP into three LMP components has to take some approximation for linearization because ACOPF only gives the total LMP at each bus, which is the Lagrange multiplier of the corresponding AC power flow constraints [13]. Usually, the generation shift factors and/or the loss factors are involved during the approximate decomposition of LMP. Hence, this leads back to the original question of a fair loss allocation such as being reference-independent. In other words, ACOPF gives the accurate and unique results of the generation dispatch and the total LMP at each bus, but there is no accurate or unique LMP decomposition. An important goal of the LMP research works is to identify more reasonable LMP decompositions such as the proposed decomposition method in this paper.

The ACOPF-based LMPs for the test system are \$23.410, \$28.272, \$30.000, \$34.766 and \$20.000 per MWh, from Buses A to E, respectively. The generation dispatches are 110.00 MW from Alta, 100 MW from Park City, 325.92 MW from Solitude, 0 MW from Sundance, and 468.44 MW from Brighton. If compared with results from Model 5 or 6, the numbers are very close. As a matter of fact, the FND-based with-loss model usually produces the dispatch and LMP results very close to that from ACOPF, as evidenced in [13]. Hence, it is reasonable that the proposed method, which incorporates the FND model, produces the results very close to ACOPF while giving a fully reference independent decomposition.

6. Conclusions

The main contributions of this paper are as follows:

- First, it presents new analytical equations to calculate the AC-based distribution factors and then loss factors that only depend on the system topology and the present operating point. Hence, the proposed new model of distribution factors and loss factors is reference-independent. The rigorous derivation considers the change of bus voltages when there is a perturbation of bus current injection. This leads to more reasonable distribution factors, if compared with (29) from [21] that ignores the nodal voltage changes and the present operation point. The reference-independent LMP loss component can serve for a better loss hedging FTR proposed in [19,20], since it gives LMP loss prices invariable to the energy reference bus.
- Next, this paper plugs the proposed loss factor model into the original LMP Model 3 to achieve a fully reference-independent LMP decomposition. In addition, it also combines the FND model in [13] into Model 3 to obtain new loss distribution factors (LDFs) such that losses are distributed at each line to achieve a fairer and more reasonable model. Therefore, the final model of LMP decomposition using the proposed Model 6 is fully reference-independent, and the system losses are distributed at each individual line giving a better power flow results.

Future works may include the investigation of possible applications of the proposed reference-independent distribution factor and loss factor to other areas in power system analyses.

Acknowledgement

The author would like to thank Eugene Litvinov and Tongxin Zheng for many useful discussions related to LMP.

Appendix A.

List of symbols

λ Lagrangian multiplier of Eq. (12)
τ Lagrangian multiplier of Eq. (13)
μ_k Lagrangian multiplier of the kth trans. constraint in Eq. (14)
ρ_{k-i} AC-based distribution factors from Bus i to Line k
$\rho_{k-i,RE}$ real power distribution factor from Bus i to Line k obtained from AC-based model
δ_i, δ_i voltage angles at Bus i and Bus j
c_i generation cost at Bus i
D_i demand at Bus i
DF_i marginal delivery factor at Bus i
G_i, $G_i^{\max}$, and $G_i^{\min}$ generation output, minimum limit, and maximum limit at Bus i
GSF_{k-i} reference-dependent, DC-based generation shift factor (or just shift factor) of Line k w.r.t. Bus i
E_i fictitious nodal demand at Bus i to represent 50% of the losses of the lines connected to Bus i
F_k line flow through Line k
I_i current injection at Bus i
I_k current through Line k
LDF_i loss distribution factor at Bus i
LF_i marginal loss factor at Bus i
$Limit_k$ limit of the kth transmission constraint
LMP_B ($\mathrm{LMP}^{\mathrm{energy}}$, $\mathrm{LMP}_B^{\mathrm{loss}}$, and $\mathrm{LMP}_B^{\mathrm{cong}}$) LMP at Bus B (energy, loss, and congestion component)
M number of lines
M_i number of lines connected to Bus i
N number of buses
P_i net injection at Bus i
P_{Loss} total system losses
$offset$ the offset in loss balance equation
R_k resistance of Line k
S_i MVA injection at Bus i
S_k MVA flow through Line k
V_i complex voltage at Bus i
x_k reactance of Line k
Y bus admittance matrix
Z bus impedance matrix
z_k impedance of Line k

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