Exact Penalty Function Based Constraint Relaxation Method for Optimal Power Flow Considering Wind Generation Uncertainty

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Abstract—This letter presents a constraint relaxation optimal power flow (OPF) model to tackle the issues when traditional OPF is infeasible under large variations such as wind generation output. In this model, the original hard constraints are relaxed into soft constraints and the objective function is adjusted for the cost of constraint violations. To guarantee the equivalence to the original OPF model when there are feasible solutions, an exact penalty function method is introduced to justify the selection of penalty factor of constraint violations. By solving an optimization problem, the lower bound of the proper penalty factor is obtained. The results of a 6-bus test system show that the proposed method achieves the same solution when the original OPF has feasible region, and an optimal solution can be obtained with minimum constraint violation when original OPF has no feasible region. Lastly, three large IEEE systems are tested to verify the effectiveness of proposed method.

Index Terms—Bi-level programming, constraint relaxation, exact penalty function, Karush-Kuhn-Tucker (KKT) conditions, optimal power flow (OPF), wind power.

I. INTRODUCTION

Optimal power flow (OPF) is widely used in economic dispatch (ED) [1], in which the objective is to minimize the traditional generation cost with the consideration of specific constraints such as generator physical limits and transmission facility capacity limits. Usually, the ED problem is mathematically formulated as a quadratic programming (QP) problem. When the uncertainty of wind power output is taken into account, the OPF problem often becomes unsolvable under scenarios where actual wind output varies in a large range. Selected hard constraints can be relaxed to soft constraints to help reach a solution. This approach is often taken in industrial practices. However, it is usually unjustified how the penalty function and its penalty factor should be chosen. Therefore, a constraint relaxation method with exact penalty function is proposed in this work. The OPF problem considering uncertain wind power injection can be formulated as (1a)–(1d) (network losses are neglected for simplicity):

\[
\begin{align*}
\min & \sum_{i=1}^{N_B} \left( c_i + b_i P_i + a_i P_i^2 \right) \\
\text{s.t.} & \sum_{i=1}^{N_B} P'_{g,i} + \sum_{i=1}^{N_W} P'_{w,i} - \sum_{i=1}^{N_L} P_{l,i} = 0
\end{align*}
\]  

(1a) (1b)

where \( P'_{g,i} \) is the conventional (non-wind) generation; \( P'_{w,i} \) is the generation shift factors; \( a_i, b_i, c_i \) can be softened and are a pre-determined vector of binary variables and \( P_{l,i} \) is the load; \( N_B, N_W, N_L \) are the minimum and maximum limit of generation and \( P'_{g,i} \) is the load, respectively; \( (a_i, b_i, c_i) \) are the triplet coefficients of quadratic cost function of generator \( i \); \( T \) is the generation shift factors; \( P_{g,i}^{\text{min}} \) and \( P_{g,i}^{\text{max}} \) are the minimum and maximum limit of generator output, respectively; and \( P_{w,i}^{\text{min}} \) and \( P_{w,i}^{\text{max}} \) are the lower and upper limit of transmission capabilities, respectively.

Intuitively, for any given \( P_{w,i} \) in the range of \( [P_{w,i}^{\text{min}}, P_{w,i}^{\text{max}}] \), we may solve the OPF problem in (1) to get the corresponding optimal solution, which is therefore a function of \( P_{w,i} \).

II. EXACT PENALTY FUNCTION BASED CONSTRAINT RELAXATION METHOD

In the above OPF model, all constraints are considered as “hard” constraints, which cannot be violated. However, with wind output being an uncertain variable with stochastic features, sometimes the OPF model can be unsolvable because the feasible region defined by those hard constraints becomes empty. In practice, some constraints can be violated, which are called “soft” constraints, to reach a feasible solution. When soft constraints are allowed, a slack variable for each of the soft constraints is added in the OPF model, and the optimization objective function includes an additional term to represent the cost of constraint violations through penalty factors. To justify the OPF model with soft constraints, three principles are presented as follows:

1) Soft constraints should be physically meaningful and safe in practice for a short duration.
2) Soft constraint violation should not be excessively big.
3) The OPF with soft constraints should yield the same optimal solution as that of the original OPF, when the original OPF itself has feasible solution(s).

Based on the above principles, we introduce the OPF with constraint relaxation as follows:

\[
\begin{align*}
\min & \sum_{i=1}^{N_B} \left( c_i + b_i P_i + a_i P_i^2 \right) + \rho \| \varepsilon \otimes \delta \|_\infty \\
\text{s.t.} & \sum_{i=1}^{N_B} P'_{g,i} + \sum_{i=1}^{N_W} P'_{w,i} - \sum_{i=1}^{N_L} P_{l,i} = 0
\end{align*}
\]  

(2a) (2b)

where \( P_{w,i}^{\text{min}} \leq T (P_{g,i} + P_{w,i} - P_i) \leq P_{w,i}^{\text{max}} \) (2c)

\[
0 \leq \varepsilon \leq \varepsilon^{\text{max}}, \delta \in [0,1]^T, \varepsilon \otimes \delta = [\varepsilon_1 \delta_1, \ldots, \varepsilon_n \delta_n]^T \]  

(2d)

where \( \delta \) is a pre-determined vector of binary variables and \( \delta_i = 1 \) if constraint \( i \) can be softened and \( \delta_i = 0 \) otherwise; \( \varepsilon \) is a relaxation vector; \( \rho \) is the penalty factor for violation; and \( m \) is the number of transmission lines.

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The OPF model in (2) attempts to minimize the total production cost and the cost of constraint violation. According to principle 1), some transmission constraints are chosen as soft constraints. Principle 3) is taken into account by (2e).

Selection of penalty factor for such model is most critical and difficult in order to satisfy Principle 3). Applying an extremely big penalty factor in model (2) will make it equivalent to the original OPF model (1). However, when the penalty factor is too large, the cost of constraint violations may dominate the objective function during iterations, which may lead to slow convergence or even retain the no-solution issue for certain scenarios [5]. When the penalty factor is too small, the soft constraints will be easily violated and objective function value will be lowered at the expense of higher level of constraint violations, and dispatch will therefore be skewed.

Therefore, an “exact penalty function” method is proposed in this work to arrive at a reasonable and justified penalty factor that meets the requirement set forth in Principle 3).

According to [2] and [3], in order to guarantee the OPF problems with soft constraints produce the same solution as that of the original OPF problem for any feasible $P_u$, penalty factor $\rho$ must be greater than the 1-norm of the Lagrange multipliers of transmission constraints (1c), denoted by $\lambda$, over all given feasible range of $P_u$, which can be formulated in (3). To simplify the representation, model (1) is transformed into general formulation (4) using matrix expression. Then, (3) can be subsequently converted to a bi-level optimization problem in (5). The lower-level problem can be replaced by its Karush-Kuhn-Tucker (KKT) condition and solved by the branch and bound method [4], to obtain the lower bound of $\rho$:

$$\rho \geq \max_{P_u} ||\lambda||_1.$$  

(3)

General formulation:

$$\min_{P_u} 0.5P_u^T A P_u$$

s.t. $B_{E}P_u = C_{E}P_i + D_{E}P_w$

$B_1P_u \leq C_1P_i + D_1P_w$

given $P_w \subset [P_{w}^{\min} , P_{w}^{\max}]$.

(4)

Maximum Lagrange:

$$\max_{P_u} ||\lambda||_1$$

s.t. $P_u^{\min} \leq P_u \leq P_u^{\max}$

$$\min_{P_u} 0.5P_u^T A P_u$$

s.t. $B_{E}P_u = C_{E}P_i + D_{E}P_w$

$B_1P_u \leq C_1P_i + D_1P_w$

(5)

where $A$ denotes the quadratic cost matrix; $B_{E}, C_{E},$ and $D_{E}$ are the coefficients of equality constraints; and $B_1, C_1,$ and $D_1$ are the coefficients of inequality constraints.

III. NUMERICAL EXAMPLE

The proposed method based on exact penalty function has been verified on a six-bus test system shown in Fig. 1 and implemented with MATPOWER toolbox and CPLEX 12.

Solving model (5) gives the lower bound of $\rho$ as 11.4265 $$/MW, shown in Table I. Assume the constraint maximum relaxation amount of each line capacity is $\epsilon^{\max} = 10$ $$/MW. When $P_{u,1} = 30$ $$/MW, the original OPF is feasible with the objective value $5261.77$, and the constraint relaxation method using $\rho = 10^6$ gives the same solution. However, the iteration increases and numerical problems may occur when $\rho$ is chosen $10^{10}$. When a smaller $\rho$, such as 10, is selected which does not meet the criterion 11.4265, the hard constraint is softened and a different solution is produced. When $P_{w,1} = 25$ $$/MW, the original OPF is infeasible. Using the proposed method, the objective function value is $5351.02$ with two soft constraints (SC), and $5342.84$ with three SCs. Because of the lower penalty factor, lower objective function is achieved with one more SC. It should be pointed out that the same solution will be obtained for any $\rho$ equal or greater than 11.4265. It demonstrates the exact penalty function can give the true lower bound of penalty factor $\rho$ that satisfies principle 3).

Lastly, Table II shows the proposed method can be solved quickly for large IEEE test systems with multiple wind farms.

<p>| $</p><table><thead><tr><th>ho$ ($$/MW)</th><th>0</th><th>6.49</th><th>2.08</th><th>6.35</th><th>47.72</th><th>984.33</th></tr>
</thead>
<tbody><tr><td>Iteration</td><td>3</td><td>6</td><td>7</td><td>12</td><td>8</td><td>6</td></tr></tbody></table>

Note: $\times$ denotes the penalty function is not considered.

TABLE II COMPUTATIONAL TIME OF LARGE SYSTEMS WITH MULTIPLE WIND FARMS

REFERENCES


