

Semi-Definite Programming for Power Output Control in a Wind Energy Conversion System

Zhiqiang Jin, *Student Member, IEEE*, Fangxing Li, *Senior Member, IEEE*,
Xiao Ma, *Student Member, IEEE*, and Seddik M. Djouadi, *Member, IEEE*

Abstract—One of the key issues in wind energy is the control design of the wind energy conversion system to achieve an expected performance under both the power system and mechanical constraints which are subject to stochastic wind speeds. Quadratic control methods are widely used in the literature for such purposes, e.g., linear quadratic Gaussian and model predictive control. In this paper, the chance constraints are considered to address the stochastic behavior of the wind speed fluctuation on control inputs and system outputs instead of deterministic constraints in the literature. Also considered are the two different models: the first one assumes that the wind speed measurement error is Gaussian, where the chance constraints can be reduced to the deterministic constraints with Gaussian statistics; whereas the second one assumes that the error is norm bounded, which is likely more realistic to the practicing engineers, and the problem is formulated as a min–max optimization problem which has not been considered in the literature. Then, both of the models are formulated as semi-definite programming optimization problems that can be solved efficiently with existing software tools. Finally, the simulation results are provided to demonstrate the validity of the proposed method.

Index Terms—Chance constraints, Gaussian distribution, min–max optimization, norm bounded, semi-definite programming (SDP), wind energy conversion systems (WECSs).

I. INTRODUCTION

RECENTLY, wind energy has been growing fast in many countries because it is a critical solution in reducing the pollutant emissions generated by conventional thermal power plants. Another reason is that the prices of fossil fuel are rising and their supplies are increasingly unstable. Different wind energy conversion system (WECS) configurations have been deployed during the last 20 years. Induction and synchronous generators have both been used in WECSs. Based on the power electronic converters' development, the variable-speed and variable-pitch WECSs provide better performance compared to the fixed-speed or fixed-pitch WECSs, since the generator torque and the pitch angle of the turbine blades can be controlled independently and simultaneously, which makes them broadly utilized.

Manuscript received June 11, 2013; revised October 26, 2013; accepted November 28, 2013. This work made use of Engineering Research Center Shared Facilities supported by the NSF/DOE Engineering Research Center (ERC) Program under NSF Award Number EEC-1041877 and the CURENT Industry Partnership Program, as well as NSF Grant CPS-1239366.

The authors are with the Department of Electrical Engineering and Computer Science, The University of Tennessee, Knoxville, TN 37996 USA (e-mail: fi6@utk.edu).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TSTE.2013.2293551

Even so, there are still many challenges to the wind power industry at present. These challenges include wind speed randomness which may cause fluctuations in output power, as well as the undesired dynamic loading of the drive train during the periods of high turbulence wind. Therefore, a sophisticated control strategy plays an important role in the WECSs. A good WECS can generate more efficient electrical energy, provide a better power quality, and lower the cost by alleviating the aerodynamic and mechanical loads.

A variable-speed variable-pitch WECS has two operating regions, which can be divided into partial load region and full load region by the wind speed. The partial load region is defined as the region in which the wind speed is higher than the cut-in speed (the speed at which the WECS starts to produce power) and lower than the rated speed (the speed at which the WECS produces rated power output). In this region, the WECS is expected to extract the maximum power and the pitch angle is kept constant. The full load regime is defined as the wind speed that is higher than the rated speed but lower than the cut-out speed (the speed at which the WECS stops producing power). In this region, the WECS is expected to extract the rated power regardless of the wind speed through pitch angle adjustment.

Several control methods and techniques have been used in the partial load region [2]–[6]. In [2]–[4], the proportional-integral (PI) controllers are employed to achieve power control. Since the operation conditions may vary greatly, a specific set of control gains may work well for one scenario but not in other scenarios. Thus, a tuned set of control gains under various scenarios is necessary, as is demonstrated in [4], where the small signal stability problem is analyzed with the tuned PI controllers to obtain suitable control gains under different operating conditions. Moreover, a gain-scheduling control method to control the variable-speed WECS in the context of linear parameter-varying systems has been proposed in [5]. A linear quadratic Gaussian (LQG) approach is used in [6].

Even more research works have been carried out in the full load region [7]–[12]. LQG controllers have been designed to deal with the stochastic characteristic of wind speed for power control in order to maintain a constant power output of the WECS with respect to mechanical constraints [7]–[10]. Also, papers focusing on both partial and full load regions have been published [12], [13].

Model predictive control (MPC), a relatively new control technique, has been used for WECS control in [1] and [13]. In both papers, the authors propose an MPC-based, multi-variable WECS control strategy, in which the WECS is modeled as a linear system and the wind speed is modeled as a stochastic

process. However, this MPC-based model needs the prediction of disturbances in a finite horizon based on the past estimates during the computation of the control input. Thus, both [1] and [13] use a state-space model for the disturbance while the parameters of the model are assumed known, which is somewhat limited in practice even for the Gaussian disturbances. Further, the distribution of disturbance may not be well understood or even available to the practicing engineers.

In this paper, to tackle the above challenges, variable-speed variable-pitch WECS power control is addressed in the full load region through a constrained stochastic linear quadratic control method. The disturbance will be modeled in two ways: as a Gaussian distribution and as a norm-bounded model without an assumed or known distribution. Correspondingly, two problem formulations are proposed and then solutions are provided. A brief description is as follows.

- 1) In Problem 1, the disturbance is assumed Gaussian, so a stochastic problem is considered by minimizing the expectation of the cost function dealing with both the input energy and desired outputs. Further, different from the previous work [1], [13], in which a state-space model is employed to predict the disturbance, this paper solves the problem through Gaussian statistics.
- 2) In Problem 2, no assumption is made regarding the distribution of the disturbance; instead, a bound on the amplitude of the disturbance is assumed. This is arguably more realistic in practice. Thus, the measured output will be stochastic due to the random disturbances, such that the constraints on the output can only be enforced in a probabilistic sense, e.g., chance constraints. Then, the problem is written as a min–max problem to compute the optimal control that minimizes the largest cost that can be generated in the disturbance space.

Both problems are converted to semi-definite programming (SDP) optimization, which has not been previously employed for WECS control as the min–max problem.

The rest of this paper is organized as follows. Section II presents the model of variable-speed variable-pitch WECS. Section III formulates the problem in a general setting for the SDP approach. Section IV describes the proposed control strategy under two different problem formulations of the disturbance, with a Gaussian distribution and with finite bounds but no assumed distribution. Section V discusses the simulation results and Section VI presents the conclusion and future work.

II. MODELING VARIABLE-SPEED VARIABLE-PITCH WECS

A typical WECS is shown in Fig. 1. The wind power captured by the wind turbine is converted into mechanical torque, then goes through the drive train, and is finally transformed to electrical power and delivered to the grid by a doubly fed induction generator.

A. Wind Speed Model

Wind speed is modeled as two components to mimic a real-time wind speed as follows:

$$v(t) = v_m(t) + v_t(t) \quad (1)$$

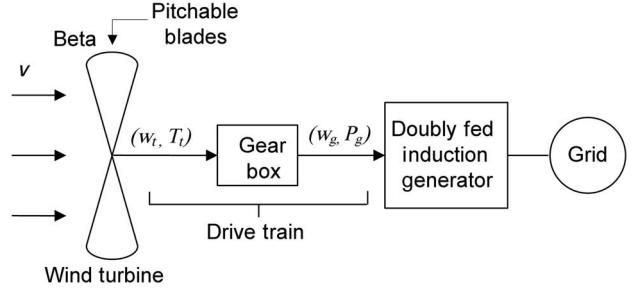


Fig. 1. A typical wind energy conversion system.

where v_m is the long-term, low-frequency variable component and v_t is the turbulence representing the high-frequency variable component [1], [13].

B. Pitch Actuator Model

A first-order dynamic system is used to model the pitch actuator system as in [1] and [13]

$$\dot{\beta} = -\frac{1}{\tau}\beta + \frac{1}{\tau}\beta_d \quad (2)$$

where τ is the time constant of the pitch system and β is the blade pitch angle.

The constraints of β and its derivative are given by

$$\beta_{\min} \leq \beta \leq \beta_{\max} \quad (3)$$

$$\dot{\beta}_{\min} \leq \dot{\beta} \leq \dot{\beta}_{\max}. \quad (4)$$

C. Aerodynamic System

The output of the aerodynamic system can be expressed as

$$P_t = C_p(\lambda, \beta) \frac{\rho \pi R^2}{2} v^3 \quad (5)$$

where P_t is the mechanical output power of the wind turbine, C_p is the performance coefficient of the wind turbine, ρ is the air density, R is the radius of wind turbine rotor, v is the wind speed, λ is the tip speed ratio of the rotor blade tip speed to the wind speed, $\lambda = \omega_t R/v$, and ω_t is the speed of the low-speed shaft.

The turbine torque Γ_t at the low-speed shaft is given by

$$\Gamma_t = \frac{P_t}{\omega_t} = \frac{C_p(\lambda, \beta) \rho \pi R^3}{\lambda} v^2. \quad (6)$$

A generic equation is used to model $C_p(\lambda, \beta)$

$$C_p(\lambda, \beta) = 0.5176 \left(\frac{116}{\lambda_i} - 0.4\beta - 5 \right) e^{-21/\lambda_i} + 0.0068\lambda \quad (7)$$

with

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1}. \quad (8)$$

D. Drive Train Model

A two-mass drive train model with flexible shaft is used with equations as follows [1], [13]:

$$\frac{d\omega_t}{dt} = -\frac{i}{J_t} \Gamma_{tw} + \frac{1}{J_t} \Gamma_g \quad (9)$$

$$\frac{d\omega_g}{dt} = \frac{i}{J_g} \Gamma_{tw} - \frac{1}{J_g} \Gamma_g \quad (10)$$

$$\begin{aligned} \frac{d\Gamma_{tw}}{dt} &= k_s i \omega_t - k_s \omega_g \\ &\quad - \left(\frac{i^2 B_s}{J_t} + \frac{B_s}{J_g} \right) \Gamma_{tw} + \frac{i B_s}{J_t} \Gamma_g + \frac{B_s}{J_g} \Gamma_g \end{aligned} \quad (11)$$

$$\Gamma_{tw} = k_s \theta_{tw} + B_s (i \omega_t - \omega_g) \quad (12)$$

where i is the gear ratio, Γ_{tw} and Γ_g are the flexible shaft torque and the generator torque, respectively, ω_g is the generator angular velocity, k_s and B_s are the shaft stiffness and damping coefficients, respectively, and θ_{tw} is the shaft twist angle.

E. Generator Model

Generally speaking, the electrical dynamics of the generator are much faster than the mechanical dynamics of the turbine system [15], [16]. In order to achieve the goal of designing a better wind turbine controller, a simple model for the generator and converter system is employed

$$\dot{\Gamma}_g = -\frac{1}{\tau_g} \Gamma_g + \frac{1}{\tau_g} \Gamma_g^* \quad (13)$$

where τ_g is the time constant and Γ_g^* is the generator torque set point.

F. WECS Linearization

The WECS is described by (1)–(13). The nonlinearity of the whole system is the part that describes the performance coefficients. In order to analyze the system using the small signal method, the first step taken is to linearize the system at a specified operating point (OP). This OP can be obtained at a long-term, low-frequency wind speed v_m . Thus, we have

$$\begin{aligned} \Gamma_t &= f(\omega_t, \nu, \beta) = f(\omega_{t,op}, \nu_{op}, \beta_{op}) \\ &\quad + \left[\frac{\partial f}{\partial \omega_t} \Big|_{op} \Delta \omega_t + \frac{\partial f}{\partial \nu} \Big|_{op} \Delta \nu + \frac{\partial f}{\partial \beta} \Big|_{op} \Delta \beta \right] + \dots \end{aligned} \quad (14)$$

where Δ is used for representing the variable's deviation from its OP. With linearization of a small deviation at OP, we have

$$\begin{aligned} \Delta \Gamma_t &= \frac{\partial f}{\partial \omega_t} \Big|_{op} \Delta \omega_t + \frac{\partial f}{\partial \nu} \Big|_{op} \Delta \nu + \frac{\partial f}{\partial \beta} \Big|_{op} \Delta \beta \\ &= L_\omega \Delta \omega_t + L_\nu \Delta \nu + L_\beta \Delta \beta \end{aligned} \quad (15)$$

where

$$L_\omega = \frac{\partial f}{\partial \omega_t} \Big|_{op} = \frac{\rho \pi R^2 \nu^2}{2 \omega_t} \left(-C_p(\lambda, \beta) \frac{\nu}{\omega_t} + R \frac{\partial C_p(\lambda, \beta)}{\partial \lambda} \right) \quad (16)$$

$$L_\nu = \frac{\partial f}{\partial \nu} \Big|_{op} = \frac{\rho \pi R^2 \nu}{2} \left(3C_p(\lambda, \beta) \frac{\nu}{\omega_t} - R \frac{\partial C_p(\lambda, \beta)}{\partial \lambda} \right) \quad (17)$$

$$L_\beta = \frac{\partial f}{\partial \beta} \Big|_{op} = \frac{\rho \pi R^2 \nu^3}{2 \omega_t} \frac{\partial C_p(\lambda, \beta)}{\partial \beta}. \quad (18)$$

The state vector, the control input, and the measure output can be defined as

$$\begin{aligned} \mathbf{x} &= [\Delta \omega_t \quad \Delta \omega_g \quad \Delta \Gamma_{tw} \quad \Delta \Gamma_g \quad \Delta \beta]^T \\ u &= [\Delta \Gamma_g^* \quad \Delta \beta_d]^T \\ y &= [\Delta \omega_g \quad \Delta P_g]^T. \end{aligned}$$

Thus, the linearized WECS can be written as

$$\dot{\mathbf{x}}(t) = \tilde{\mathbf{A}} \mathbf{x}(t) + \tilde{\mathbf{B}}_u u(t) + \tilde{\mathbf{B}}_v \Delta \nu(t) \quad (19)$$

$$y(t) = \tilde{\mathbf{C}} \mathbf{x}(t) \quad (20)$$

where

$$\begin{aligned} \tilde{\mathbf{A}} &= \begin{bmatrix} \frac{L_\omega}{J_t} & 0 & -\frac{1}{J_t} & 0 & \frac{L_\beta}{J_t} \\ 0 & 0 & \frac{1}{J_g} & -\frac{1}{J_g} & 0 \\ ik_s + \frac{i B_s}{J_t} L_\omega & -k_s & -\left(\frac{i^2 B_s}{J_t} + \frac{B_s}{J_g} \right) & 0 & \frac{i B_s}{J_t} L_\beta \\ 0 & 0 & 0 & -\frac{1}{\tau_g} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \\ \tilde{\mathbf{B}}_u &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_g} & 0 \\ 0 & \frac{1}{\tau} \end{bmatrix}, \quad \tilde{\mathbf{B}}_v = \begin{bmatrix} \frac{L_\omega}{J_t} \\ 0 \\ \frac{i B_s}{J_t} L_v \\ 0 \\ 0 \end{bmatrix}, \quad \text{and} \\ \tilde{\mathbf{C}} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \Gamma_{g,\text{rat}} & 0 & \omega_{g,\text{rat}} & 0 \end{bmatrix}. \end{aligned}$$

More details about the model can be found in [1].

III. PROBLEM FORMULATION

In this section, the above linearized WECS model is converted into the discrete-time format and then the problem is formulated in a general setting such that the SDP approach can be applied.

A. Discrete-Time Model

The discrete version of the linearized WECS can be written as follows:

$$x(k+1) = A_d x(k) + B_u u(k) + B_d d(k) \quad (21)$$

$$y(k) = C_d x(k) \quad (22)$$

where $A_d = e^{\tilde{A}T}$, $d(k)$ is the discrete version of $\Delta\nu(t)$ representing the disturbance in the discrete-time model, and $[B_u, B_d] = [\int_0^T e^{\tilde{A}T} dt][\tilde{B}_u, \tilde{B}_d]$.

To simplify the notation in this paper, we set

$$\begin{aligned}\mathbf{x} &:= \left[x(1)^T, \dots, x(N)^T \right]^T \\ \mathbf{u} &:= \left[u(0)^T, \dots, u(N-1)^T \right]^T \\ \mathbf{d} &:= \left[d(0)^T, \dots, d(N-1)^T \right]^T.\end{aligned}$$

B. Cost Function

Since this paper focuses on the WECS power control in the full load region, the objective function is to control the outputs (ω_g and P_g) of the drive train part at rated values. Therefore, the cost function is used to minimize the deviations of the generator angular velocity (ω_g) and electric power (P_g) from the rated values. Also included is the change of control vector ($\partial\Gamma_g^*$ and $\partial\beta_d$, where ∂ represents the difference between the values at the current time and the previous time) at each time step. Thus, the cost function can be written as follows:

$$\begin{aligned}\min V_N(x_0, \mathbf{u}, \mathbf{d}) = \frac{1}{2} \left[\sum_{k=1}^N \left(q_1 (\omega_{g,\text{rat}} - \omega_g)_k^2 \right. \right. \\ \left. \left. + q_2 (P_{g,\text{rat}} - P_g)_k^2 \right) \right. \\ \left. + \sum_{k=0}^{N-1} \left(r_1 (\partial\Gamma_g^*)_k^2 + r_2 (\partial\beta_d)_k^2 \right) \right]\end{aligned}\quad (23)$$

subject to

$$\partial\beta_{\min} \leq \partial\beta_d(k) \leq \partial\beta_{\max} \quad (24)$$

$$\beta_{\min} \leq \beta_d(k) \leq \beta_{\max} \quad (25)$$

$$P_{g,\min} \leq P_g(k) \leq P_{g,\max} \quad (26)$$

$$\omega_{g,\min} \leq \omega_g(k) \leq \omega_{g,\max}. \quad (27)$$

For $k = 1, \dots, N$ in (26) and (27), and $k = 0, \dots, N-1$ in (24) and (25), where x_0 is the initial condition at each time step and N is the finite prediction horizon.

Since

$$\begin{aligned}\Delta P_g &= P_g - P_{g,\text{rat}} = \Delta\omega_g\Gamma_{g,\text{rat}} + \Delta\Gamma_g\omega_{g,\text{rat}} \\ \Delta\omega_g &= \omega_g - \omega_{g,\text{rat}}\end{aligned}$$

and

$$y(k) = [\Delta\omega_g \quad \Delta P_g]^T = C_d x(k).$$

Then, after some mathematical manipulations, we can rewrite (23) as follows:

$$\begin{aligned}V_N(x(0), \mathbf{u}, \mathbf{d}) = \left[\sum_{k=1}^N x(k)^T \mathbf{Q} x(k) \right. \\ \left. + \sum_{k=0}^{N-1} (u(k) - u(k-1))^T \mathbf{R} (u(k) - u(k-1)) \right]. \quad (28)\end{aligned}$$

With

$$\begin{aligned}\mathbf{Q} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{q_1+q_2}{\omega_{g,\text{rat}}^2} & 0 & \frac{q_2}{\omega_{g,\text{rat}}\Gamma_{g,\text{rat}}} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{q_2}{\omega_{g,\text{rat}}\Gamma_{g,\text{rat}}} & 0 & \frac{q_2}{\Gamma_{g,\text{rat}}^2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \mathbf{R} &= \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}\end{aligned}$$

where $\mathbf{Q} \geq 0$ (i.e., semi-definite positive matrix) and $\mathbf{R} > 0$ (i.e., positive definite matrix).

C. Formulation of Two Problems

In this work, two different sets of control inputs regarding the disturbance of the wind speed are considered.

The first one assumes that the noise $d(k)$ is Gaussian with the known statistics. Note that with Gaussian noises, the cost function in (28) is random depending on the noise. Unlike the works in the literature, where the noise terms in the cost function are evaluated by the known model through Kalman Filter, here we keep them random, but consider the expectation of the cost function. Moreover, due to these unknown noises, the measurements $y(k) = [\Delta\omega_g \quad \Delta P_g]^T$ are also stochastic which means they are not exactly known. In this sense, constraints in (26) and (27) can only be enforced in a probabilistic form. For example, (26) and (27) can be written as

$$\mathbb{P}(P_{g,\min} \leq P_g(k) \leq P_{g,\max}) \geq \alpha_1 \quad (29)$$

$$\mathbb{P}(\omega_{g,\min} \leq \omega_g(k) \leq \omega_{g,\max}) \geq \alpha_2. \quad (30)$$

Thus, the first problem with Gaussian noise is given as follows.

Problem 1: Find

$$\mathbf{u}(x_0) := \arg \min_{\mathbf{u}} \mathbb{E} V_N$$

subject to (24), (25), (29), and (30), and discretized version of the system in (19), and (20).

In the above formulation, \mathbb{E} is the expectation operator.

The second problem formulation considers a noise variable that does not have a known statistics, but has a bounded set, e.g., $\mathbb{D}_\gamma = \{\mathbf{d} \mid \|\mathbf{d}\|_2 \leq \gamma\}$. In this case, the problem is formulated as a min–max problem to find a control input that minimizes the largest value of the cost function throughout the whole bounded set. The problem formulation is given as follows.

Problem 2: Find

$$\mathbf{u}(x_0) := \arg \min_{\mathbf{u}} \max_{\mathbf{d} \in \mathbb{D}_\gamma} V_N$$

subject to (24), (25), (29), and (30), and discretized version of the system in (19), and (20).

Note that constraints (29) and (30) are not convex; thus, they will be simplified in Section IV.

IV. PROPOSED CONTROL STRATEGY

In this section, the control strategy employed to solve the two problems formulated in Section III will be discussed. Unlike the MPC method, which is quadratic programming, the two problems will be converted to SDP optimization problems. Specifically, in this section, the concept of SDP will be briefly introduced. Then, constraints (29) and (30) will be simplified to linear constraints, which will formulate the two problems as SDP models for solutions.

A. Brief Introduction of SDP

A semi-definite program has the following form [17]:

$$\begin{aligned} & \text{minimize} \quad c^T x \\ & \text{subject to} \quad F^i(x) = x_1 F_1^i + \cdots + x_n F_n^i \\ & \quad \quad \quad + G^i \leq 0, \quad i = 1, \dots, K \\ & \quad \mathbf{Gx} \leq \mathbf{g}, \quad \mathbf{Ax} = \mathbf{b} \end{aligned}$$

where G^i , $F_1^i, \dots, F_n^i \in \mathbb{R}^{n \times n}$; $x \in \mathbb{R}^n$; \mathbf{G} and \mathbf{A} are the matrices, and \mathbf{g} and \mathbf{b} are the vectors with appropriate dimensions. Note that SDP is subject to constraints such as linear matrix inequalities (LMIs), linear inequalities, and linear equalities.

SDP optimization problems are the convex optimization problems that can be solved efficiently.

B. Approximating Chance Constraints to Linear Constraints

As previously mentioned, two cases regarding the wind speed disturbances are considered in this paper.

- 1) The disturbance to the wind speed $d(k)$ is known to be Gaussian.
- 2) The disturbance distribution is unknown but subject to some norm-bounded set.

Since the disturbance $d(k)$ is random, the state is not exactly known and any constraints on the state could be formulated in a probabilistic sense. Thus, the constraints on the output (P_g and ω_g) can be described by chance constraints which are already given in (29) and (30).

The above constraints are nonconvex and hard to solve directly. In the first case, when the disturbance $d(k)$ is Gaussian, the chance constraints can be reduced to linear inequalities, as shown in [14]. In the second case, if we do not assume any form of the noise, we can approximate the chance constraints by some hard constraints.

For Problem 1, where the disturbance $d(k)$ is Gaussian, e.g., $\mathbf{d} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, (29) is taken as an example to demonstrate how to convert it to a linear inequality. Starting from (29), where

$$\begin{aligned} & \mathbb{P}(P_{g,\min} \leq P_g(k) \leq P_{g,\max}) \\ &= \mathbb{P}(P_{g,\min} \leq P_g(k) \cap P_g(k) \leq P_{g,\max}) \\ &= \mathbb{P}(P_{g,\min} \leq P_g(k)) + \mathbb{P}(P_g(k) \leq P_{g,\max}) \\ &\quad - \mathbb{P}(P_{g,\min} \leq P_g(k) \cup P_g(k) \leq P_{g,\max}) \\ &= \mathbb{P}(P_{g,\min} \leq P_g(k)) + \mathbb{P}(P_g(k) \leq P_{g,\max}) - 1 \geq \alpha_1. \end{aligned}$$

Then, we have $\mathbb{P}(P_{g,\min} \leq P_g(k)) + \mathbb{P}(P_g(k) \leq P_{g,\max}) \geq \alpha_1 + 1$. Thus, to guarantee (29), the above inequality can be separated into two inequalities

$$\mathbb{P}(P_g(k) \leq P_{g,\max}) \geq \frac{1 + \alpha_1}{2} \quad (31)$$

and

$$\mathbb{P}(P_{g,\min} \leq P_g(k)) \geq \frac{1 + \alpha_1}{2}. \quad (32)$$

Note, (31) indicates $\mathbb{P}(P_g(k) > P_{g,\max}) < \frac{1-\alpha_1}{2}$, and $P_g(k) > P_{g,\max}$ can be further written as $G_1 x > g_1$, where $G_1 = [0 \ 1] C_d$ and $g_1 = P_{g,\max} - P_{g,\text{rat}}$. For $k = 1, \dots, N$, we have $\mathbf{G}_1 \mathbf{x} > g_1$ for each k , where $\mathbf{G}_1 = [0, \dots, 0, \underbrace{G_1}_{k\text{th}}, 0, \dots, 0]$. Thus, for each

k , (31) can be represented as

$$\mathbb{P}(\mathbf{G}_1 \mathbf{x} > g_1) < \frac{1 - \alpha_1}{2}. \quad (33)$$

Similarly, (30) and (32) may also be represented in the form of inequalities as (33).

Theorem 1 [14]: Consider a discrete-time linear system with the state equation written as

$$\mathbf{x} = \mathbf{A}_S \mathbf{x}_0 + \mathbf{B}_S \mathbf{u} + \mathbf{C}_S \mathbf{d} \quad (34)$$

which is corresponding to [14]

$$\begin{aligned} \mathbf{A}_S &= \left[(\mathbf{A}_d^1)^T, \dots, (\mathbf{A}_d^k)^T, \dots, (\mathbf{A}_d^N)^T \right]^T \\ \mathbf{B}_S &= \left[\tilde{\mathbf{B}}_0^T, \dots, \tilde{\mathbf{B}}_{N-1}^T \right]^T \\ \tilde{\mathbf{B}}_{k-1} &= \left[(\mathbf{A}_d^{k-1})^T B_u, \dots, B_u, \mathbf{0}_{5 \times 2(N-k)} \right] \\ \mathbf{C}_S &= \left[\tilde{\mathbf{C}}_0^T, \dots, \tilde{\mathbf{C}}_{N-1}^T \right]^T \\ \tilde{\mathbf{C}}_{k-1} &= \left[(\mathbf{A}_d^{k-1})^T B_d, \dots, B_d, \mathbf{0}_{5 \times (N-k)} \right]. \end{aligned}$$

Then, the constraint

$$\mathbf{p}^T \mathbf{u} \leq q \quad (35)$$

where $\mathbf{p} = \mathbf{B}_S^T \mathbf{G}_1$, $q = g_1 - \mathbf{G}_1 \mathbf{A}_S \mathbf{x}_0 - \mathbf{G}_1 \mathbf{C}_S \boldsymbol{\mu} - \|\boldsymbol{\Sigma}^{\frac{1}{2}} \mathbf{C}_S^T \mathbf{G}_1\|_2 \Phi^{-1}(\alpha_1)$ implies the chance constraint (33). Φ is the cumulative distribution function of standard normal variables and Φ^{-1} is its inverse.

Thus, based on the above theorem, the chance constraints (29) and (30) can be reduced to linear inequalities.

For Problem 2, (29) and (30) cannot be transformed to a hard constraint as shown above because the distribution of the disturbance is not known. Thus, a hard constraint can be employed to approximate it [14]. This is elaborated as follows.

First, recall from (33) that we hope $\mathbf{G}_1 \mathbf{x} > g_1$ to be satisfied. Then, by (34), we have

$$\mathbf{G}_1 \mathbf{A}_S \mathbf{x}_0 + \mathbf{G}_1 \mathbf{B}_S \mathbf{u} + \mathbf{G}_1 \mathbf{B}_S \mathbf{d} > g_1$$

which is further implied by

$$\mathbf{G}_1 \mathbf{B}_S \mathbf{u} > g_1 - \mathbf{G}_1 \mathbf{A}_S \mathbf{x}_0 + \|\mathbf{C}_S^T \mathbf{G}_1\|_2 \gamma \quad (36)$$

as $\|\mathbf{d}\|_2 \leq \gamma$.

Note that (36) introduces some conservativeness if compared to the desired constraints, but it is more practical as the distribution of the noise is unknown in many real instances. Similarly, (30) and (32) can also be formulated as linear inequalities as (36).

In the previous sections, the chance constraints for both problems are converted into linear constraints, which are convex. In the discussion below, both problems will be formulated as SDP optimization problems.

C. SDP Approach for Problem 1

In this section, the technique in [14] is applied to provide a tractable solution of Problem 1. When the disturbance $d(k)$ is Gaussian, the expectation of the cost function can be computed using the statistics of the Gaussian noise. Then, with the linear inequalities derived above, Problem 1 becomes a convex optimization problem and here Problem 1 is formulated as an SDP problem. However, Problem 2 does not seem tractable directly as it is a min–max problem. In Section IV-D, it is shown that Problem 2 can also be formulated as an SDP problem as in [14]. We embedded wind turbine control problems into this framework, and with these formulations, we are able to obtain feasible control solutions.

An obvious result regarding the cost function is given in the following proposition:

Proposition 1: The cost function (28) can be written as

$$\begin{aligned} V_N(x_0, \mathbf{u}, \mathbf{d}) &= x_0^T \mathbf{A} x_0 + 2\mathbf{b}^T \mathbf{u} + \mathbf{u}^T \mathbf{B} \mathbf{u} \\ &\quad + 2\mathbf{c}^T \mathbf{d} + \mathbf{d}^T \mathbf{C} \mathbf{d} + 2\mathbf{u}^T \mathbf{D} \mathbf{d} \end{aligned} \quad (37)$$

for vectors \mathbf{b} and \mathbf{c} and matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} with appropriate dimensions, and where $\mathbf{B} > 0$ and $\mathbf{C} > 0$.

Proof: The original system can be written in terms of system dynamics, at time k

$$x_k = \tilde{\mathbf{A}}_{k-1} x_0 + \tilde{\mathbf{B}}_{k-1} \mathbf{u} + \tilde{\mathbf{C}}_{k-1} \mathbf{d}$$

where $\tilde{\mathbf{A}}_{k-1} = \mathbf{A}_d^k$.

Then after some manipulations, we can reach the formula of the cost function stated in (37) with

$$\begin{aligned} \mathbf{A} &= \sum_{k=1}^N \tilde{\mathbf{A}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1} \\ \mathbf{B} &= \mathbf{L}^T \text{diag}(R, \dots, R) \mathbf{L} + \sum_{k=1}^N \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{B}}_{k-1} \\ \mathbf{L} &= \begin{bmatrix} 1 & 0 & -1 & & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & \dots & \ddots & \ddots & -1 \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}, \quad \mathbf{C} = \sum_{k=1}^N \tilde{\mathbf{C}}_{k-1}^T Q \tilde{\mathbf{C}}_{k-1} \\ \mathbf{D} &= \sum_{k=1}^N \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{C}}_{k-1}, \quad \mathbf{c} = \left(\sum_{k=1}^N \tilde{\mathbf{C}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1} \right) x_0 \\ \mathbf{b} &= \left(\sum_{k=1}^N \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1} \right) x_0 - \mathbf{R}^T \mathbf{U}, \\ \mathbf{U} &= [0, \dots, 0, u(-1)^T]^T \end{aligned}$$

where \mathbf{L} comes from the difference of control inputs in the cost function (28).

Let $\mathbf{h} = \mathbf{c} - \mathbf{D}^T \mathbf{B}^{-1} \mathbf{b}$ and $\mathbf{F} = \mathbf{B}^{-1/2} \mathbf{D}$, then by eliminating the constant terms and taking $\mathbf{u} = \mathbf{B}^{-1/2} \mathbf{y} - \mathbf{B}^{-1} \mathbf{b}$, the cost function above can be further rewritten as

$$\tilde{V}_N(x_0, \mathbf{y}, \mathbf{d}) = \mathbf{y}^T \mathbf{y} + 2\mathbf{h}^T \mathbf{d} + 2\mathbf{y}^T \mathbf{F} \mathbf{d} + \mathbf{d}^T \mathbf{C} \mathbf{d}. \quad (38)$$

Taking the expectation of the above cost, we have

$$\hat{V}_N(x_0, \mathbf{y}, \mathbf{d}) = \mathbf{y}^T \mathbf{y} + 2\mathbf{h}^T \mu + 2\mathbf{y}^T \mathbf{F} \mu + \text{trace}(\mathbf{C} \Sigma). \quad (39)$$

Again, with the constant terms ignored, the cost to be minimized is $\hat{V}_N(x_0, \mathbf{y}, \mathbf{d}) = \mathbf{y}^T \mathbf{y} + 2\mathbf{y}^T \mathbf{F} \mu$. Then, Problem 1 is equivalent to find $\mathbf{u}(x_0) := \arg \min_{\mathbf{u}} \hat{V}_N$.

Theorem 2: Problem 1 can be solved by the following semi-definite optimization problem:

$$\begin{aligned} &\text{Minimize } z \\ &\text{subject to (24), (25), and (35)} \\ &\begin{bmatrix} \mathbf{I}_N & \mathbf{y} + \mathbf{F} \mu \\ \mathbf{y}^T + \mu^T \mathbf{F}^T & z + (\mathbf{F} \mu)^T \mathbf{F} \mu \end{bmatrix} \geq 0 \end{aligned} \quad (40)$$

in decision variables y and z .

Proof: The proof is given below by following the technique in Theorem 3 in [14].

First, the minimization of $\hat{V}_N(e_0, \mathbf{y}, \omega)$ can be written as

$$\begin{aligned} &\text{Minimize } z \\ &\text{subject to } z - \mathbf{y}^T \mathbf{y} - 2\mathbf{y}^T \mathbf{F} \mu \geq 0. \end{aligned} \quad (41)$$

The constraint (31) can be further written as

$$\begin{aligned} &z - \mathbf{y}^T \mathbf{y} - 2\mathbf{y}^T \mathbf{F} \mu - (\mathbf{F} \mu)^T \mathbf{F} \mu + (\mathbf{F} \mu)^T \mathbf{F} \mu \geq 0 \\ &z + (\mathbf{F} \mu)^T \mathbf{F} \mu - (\mathbf{y} + \mathbf{F} \mu)^T (\mathbf{y} + \mathbf{F} \mu) \geq 0. \end{aligned} \quad (42)$$

Thus, by Schur complement lemma in [17], (42) can be formulated as (40). Moreover, note that (24), (25), and (35) are linear constraints on the control input, which can be added without increasing the complexity. Thus, the statement can be achieved.

D. SDP Approach for Problem 2

In Section IV-C, an exact solution is provided for Problem 1 under the assumption that the disturbance is Gaussian. However, as previously mentioned, the distribution statistics may not be known or follow a regular probability distribution in the real world, but it is not difficult to make a reasonable assumption about the bounds of the disturbance in a given set (e.g., temperatures or wind speed will not go unbounded). Thus, the problem was formulated as Problem 2.

It can be viewed by finding the optimal control that minimizes the worst cost in searching the disturbance bound. The advantage of solving this problem is that it is not necessary to know the distribution of the disturbance, which is used to compute the expected cost. Further, by minimizing the maximum of the cost, it can be guaranteed that the overall cost will be limited in an appropriate range. The solution of Problem 2 can be represented

by the following SDP problem by directly applying the approach in [14] as stated in the next theorem.

Theorem 3: Problem 2 may be solved by

$$\begin{aligned} & \text{Minimize } z \\ & \text{subject to (24), (25), and (36)} \\ & \begin{bmatrix} \mathbf{I}_N & \mathbf{y} & \mathbf{F} \\ \mathbf{y}^T & z - \gamma^2 \lambda & -\mathbf{h}^T \\ \mathbf{F}^T & -\mathbf{h} & \lambda \mathbf{I} - \mathbf{C} + \mathbf{F}^T \mathbf{F} \end{bmatrix} \geq 0 \end{aligned}$$

in decision variables \mathbf{y} , z , and λ .

The optimal control input can be obtained by the transformation $\mathbf{u} = \mathbf{B}^{-1/2} \mathbf{y} - \mathbf{B}^{-1} \mathbf{b}$ after solving the above problem.

V. RESULTS AND DISCUSSION

The performance of the proposed SDP-based control strategy is demonstrated in this section. The rated wind speed used in this paper is 12.5 m/s and the cut-out wind speed is 27.5 m/s. Other parameters used can be found in the Appendix. The simulation is implemented in MATLAB with the LMIs toolbox, which is widely used in the control area.

The results will be divided into three cases. Case 1 gives the response when there is a step change of the wind speed, Case 2 gives the results when the assumed disturbance follows Gaussian distribution, and Case 3 describes the performance when the disturbance is norm bounded.

A. Case 1: Step Change to the Wind Speed

As the first step to test the proposed control strategy, the disturbance is set as a step change to the wind speed. The wind speed is chosen as the midrange of the full load region, which is 20 m/s. The step change is set to $+1$ and -1 m/s, which is considered as the maximum variance in the model of wind forecast error. The prediction horizon N is set to 2. Two different groups of other relevant parameters, as shown below, are chosen for a comparison of the results.

Group 1: $q_1 = 1000$, $q_2 = 0.5$, $r_1 = 0$, and $r_2 = 0.0002$.

Group 2: $q_1 = 1000$, $q_2 = 0.045$, $r_1 = 0$, and $r_2 = 0.002$.

Here, the values of q_1 and q_2 are chosen to normalize the per unit values of active power and angular velocity in the cost function. Also, r_1 is set to zero, whereas r_2 is not, because in this study the change of pitch angle is critical, whereas the change of generator torque is not a focal point.

As shown in Figs. 2 and 3, with different parameters, the results are different. The curves of the first group of parameters have smaller amplitudes of the dynamics, but with several oscillations in the dynamic process. The curves of the second group of parameters have higher amplitudes, but no oscillations in the dynamic process. The reason for these phenomena can be attributed to different parameters giving different weights to different performance characteristics.

The parameters in Group 1 are chosen for the next two case studies because it gives smaller amplitude which is preferable in power system operation.

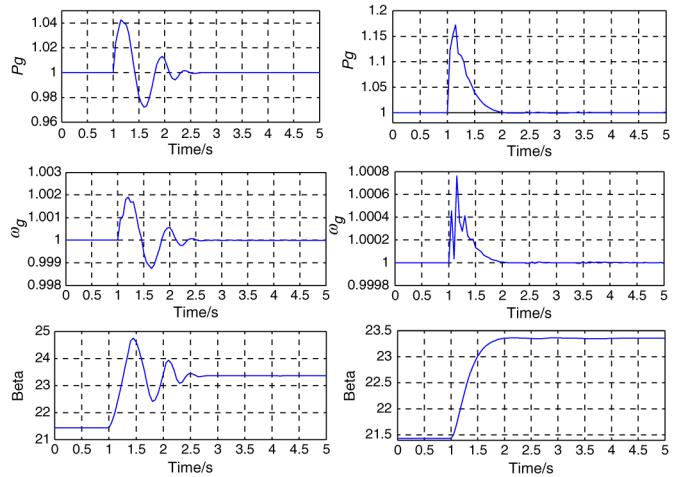


Fig. 2. Results when wind speeds change from 20 to 21 m/s. Group 1 on the left side and Group 2 on the right side.

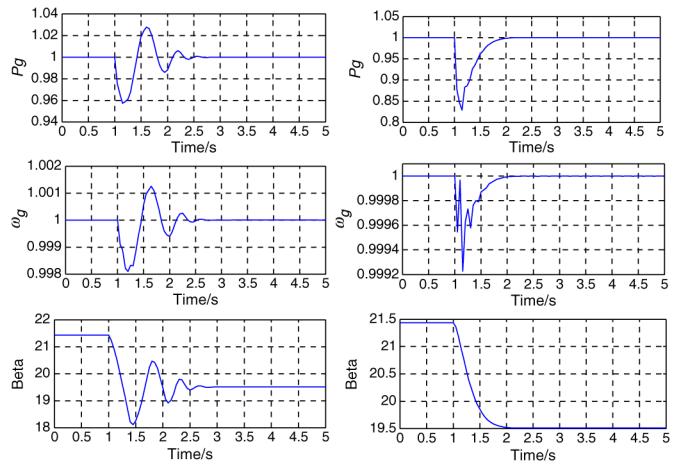


Fig. 3. Results when wind speed changes from 20 to 19 m/s. Group 1 on the left side and Group 2 on the right side.

B. Case 2: The Disturbance Distribution is Gaussian

In this case study, the distribution of the high-frequency variable part of the wind speed is assumed to be Gaussian. All the other parameters are set the same as those in Group 1. Also, the wind speed is sampled every 0.5 s. The chance constraints are set with $\alpha_1 = \alpha_2 = 0.98$. The wind speed, the active power, the angular velocity of the generator, and also the beta curves are shown in Fig. 4. As we can observe from the curves, the wind speed is around low-frequency variable part, which is set to 20 m/s here.

It is perhaps difficult to make a direct comparison between the results presented here and the previous works using MPC in [1] and [13], since random noises subject to the Gaussian distribution are embedded in the wind speed forecast error. Nevertheless, the results presented in Fig. 4 are almost in the same scale as the results from the previous works using MPC in [1] and [13]. For instance, the output wind power (P_g) by the proposed method is within 0.98 and 1.02 per unit of its rated value, as shown in the second diagram in Fig. 4, whereas [1, Fig. 7(b)] shows roughly the same range of P_g variation. Further, P_g in Fig. 4, as well as

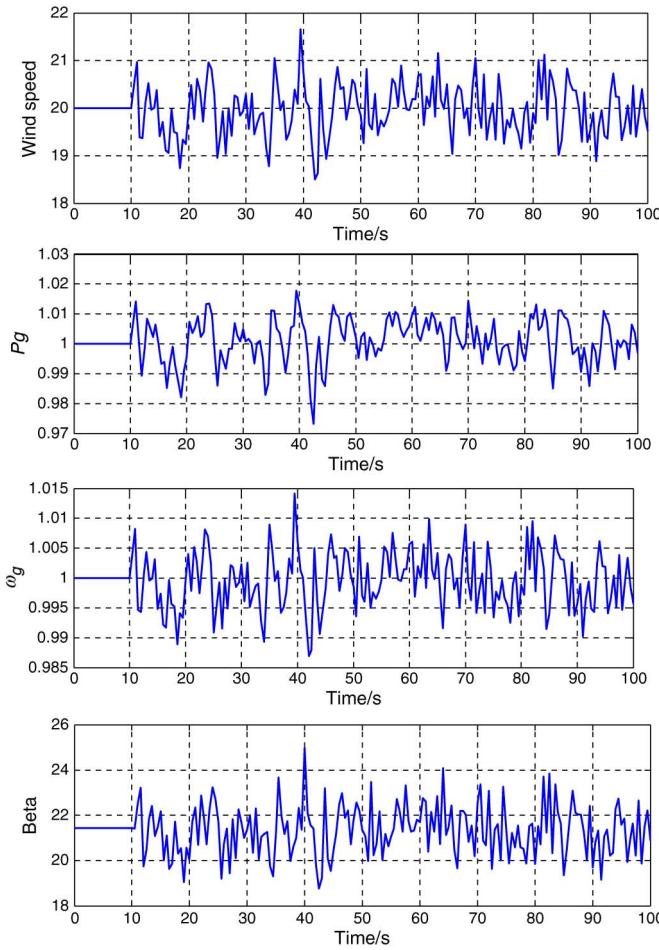


Fig. 4. Results for the Gaussian case.

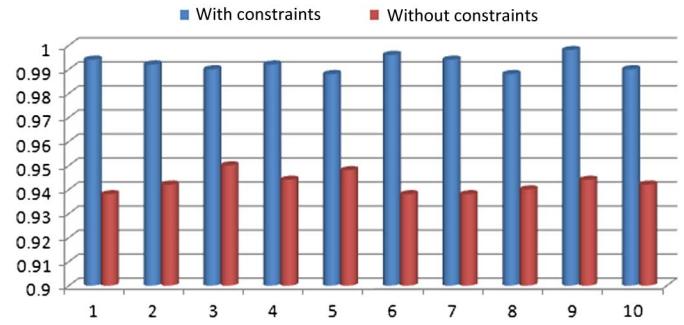
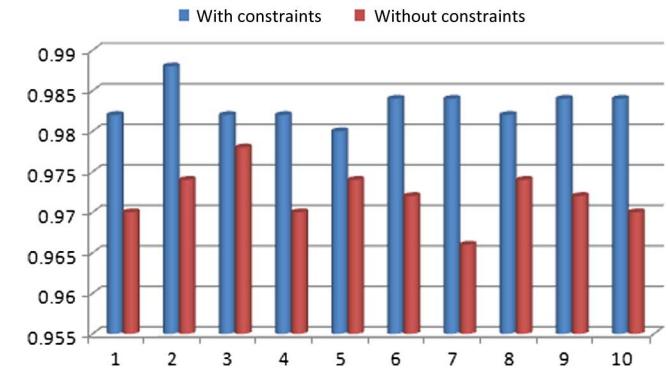
[1, Fig. 7(b)], is much better than the original PI control in [1], which gives a much larger P_g variation range, between about 0.9 and slightly higher than 1.1 per unit.

Note that the proposed method uses the stochastic approach to model the chance constraints and minimizes the expectation of the cost function, whereas the MPC method in [1] and [13] uses the Kalman filter to predict the disturbance to obtain deterministic constraint, which adds complexity to the model. Both the approaches seem to serve their purpose well, as previously discussed.

We also compute the time used to solve this problem for every step, the average time consumed is about 0.05 s, which is much smaller than the sampling rate 0.5 s.

To verify whether the results from the proposed SDP algorithm meet our intention or not, 10 simulation runs with 250 s (i.e., 500 data points) in each run are carried out under two models: with constraints and without constraints. The output variables P_g and ω_g are examined to see if they are within the constraint ranges.

Fig. 5 shows the probability that P_g falls into the desired range with and without chance constraints (10 random tries, with 1000 sample points for each try). With the constraints modeled, the probability that all the P_g values stay within the range is higher than $\alpha_1 = 98\%$. But when those constraints are removed, such probability drops to around 94%. A similar result for ω_g is shown

Fig. 5. Comparison of the probability of P_g within the bounds under two models: with constraints and without constraints, where the x -coordinate represents 10 different tries and the y -coordinate represents the probabilities in the range.Fig. 6. Comparison of the probability of ω_g within the bounds under two models: with constraints and without constraints, where the x -coordinate represents 10 different tries and the y -coordinate represents the probabilities in the range.

in Fig. 6. These plots demonstrate that the proposed method does meet the expectation very well.

C. Case 3: The Disturbance is Norm Bounded

In this case study, the distribution of the disturbance is unknown, but it is norm bounded. As shown in Fig. 7, the high-frequency part of the wind speed is set to random values with its norm at 1 m/s. The active power is shown within 0.97 and 1.03 per unit of its rated value. The angular velocity of the generator is even within 0.99–1.01 per unit. The results are comparable to the previous results in Case 2. Thus, this is a promising approach for the practicing engineers, since it does not require any assumption on the disturbance distribution. This is the unique contribution of the proposed method. The average time for this case is about 0.1 s for each step.

VI. CONCLUSION AND FUTURE WORK

The contribution of this paper can be summarized as follows.

- 1) In this paper, a new control strategy based on the SDP method has been proposed for WECS power control in the full load region. The SDP method solves a stochastic problem by minimizing the expectation of the cost function using the statistics of Gaussian disturbance. Also, the chance constraints are employed to capture the stochastic characteristic of the disturbance which is more practical than the deterministic constraints.

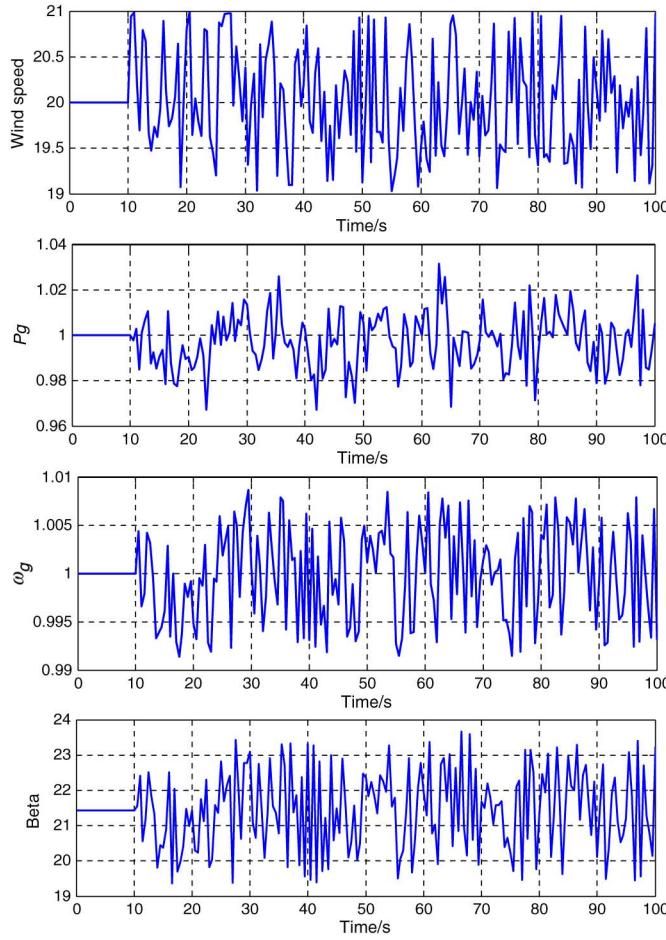


Fig. 7. Results for norm bounded case.

- 2) In the proposed approach, the disturbance to wind speed forecast, which represents the high-frequency variable component of the wind speed, is modeled as a Gaussian distribution and a norm bounded without a known distribution. Both problems are formulated into SDP models in order to be solved.
- 3) When the disturbance is modeled as a Gaussian distribution, SDP gives comparable results to those from the MPC-based method in the literature. Meanwhile, the proposed SDP method requires less information, i.e., the state-space model, which is needed in the MPC model to determine the disturbances in the prediction horizon of the deterministic cost function in MPC.
- 4) When the wind speed error is modeled as norm bounded without a known distribution, it likely represents a more realistic assumption in practice and this has not been previously reported in the wind power control studies. The results are also promising and comparable to the one with a Gaussian distribution.

Future works may include the study of modeling the generator as a doubly fed induction generator and connecting the DFIG-based WECS to a large power system, especially when an energy storage system is connected to the wind plant. Another interesting aspect is the control design for other possible disturbance, e.g., highly colored stochastic process. For such disturbance, the

control algorithm needs to be recomputed to fit the stochastic characteristic of the turbulence.

APPENDIX

Other Parameters of WECS

Wind turbine

$$P_{t\text{-rat}} = 2 \text{ MW}, \quad \omega_{t\text{-rat}} = 3.0408 \text{ rad/s}, \quad R = 33.29 \text{ m}.$$

Pitch actuator

$$\begin{aligned} \tau &= 0.1 \text{ s}, \quad \beta_{\min} = 0^\circ, \quad \beta_{\max} = 45^\circ, \\ \dot{\beta}_{\min} &= -10^\circ/\text{s}, \quad \dot{\beta}_{\max} = 10^\circ/\text{s}. \end{aligned}$$

Drive train

$$\begin{aligned} i &= 74.38, \quad J_t = 1.86E+06 \text{ kg m}^2, \quad J_g = 56.29 \text{ kg m}^2, \\ k_s &= 31.8E+04 \text{ Nm/rad}, \quad B_s = 212.2 \text{ Nm} \cdot \text{s}/\text{rad}. \end{aligned}$$

Generator

$$\tau_g = 20 \text{ ms.}$$

REFERENCES

- [1] M. Soliman, O. P. Malik, and D. T. Westwick, "Multiple model multiple-input multiple-output predictive control for variable speed variable pitch wind energy conversion systems," *IET Renew. Power Gen.*, vol. 5, no. 2, pp. 124–136, Mar. 2011.
- [2] H. Camblong, I. M. de Alegria, M. Rodriguez, and G. Abad, "Experimental evaluation of wind turbines maximum power point tracking controllers," *Energy Convers. Manage.*, vol. 47, pp. 2846–2858, 2006.
- [3] E. A. Bossanyi, "The design of closed loop controllers for wind turbines," *Wind Energy*, vol. 3, pp. 149–163, 2000.
- [4] Y. Mishra, S. Mishra, F. Li, Z. Y. Dong, and R. C. Bansal, "Small-signal stability analysis of a DFIG-based wind power system under different modes of operation," *IEEE Trans. Energy Convers.*, vol. 24, no. 4, pp. 972–982, Dec. 2009.
- [5] F. D. Bianchi, R. J. Mantz, and C. F. Christiansen, "Control of variable-speed wind turbines by LPV gain scheduling," *Wind Energy*, vol. 7, pp. 1–8, 2004.
- [6] I. Munteanu, N. A. Cutululis, A. I. Bratcu, and E. Ceanga, "Optimization of variable speed wind power systems based on a LQG approach," *Control Eng. Pract.*, vol. 13, pp. 903–912, Jul. 2005.
- [7] E. B. Muhando, T. Senju, O. Z. Siagi, and T. Funabashi, "Intelligent optimal control of wind power generating system by a complemented linear quadratic Gaussian approach," in *Proc. IEEE Power Eng. Soc. Conf. Expo. Africa*, Jul. 2007, pp. 1–8.
- [8] E. B. Muhando, T. Senju, N. Urasaki, and T. Funabashi, "Online WTG performance and transient stability enhancement by evolutionary LQG," in *Proc. IEEE Power Eng. Soc. Gen. Meeting*, Jun. 2007, pp. 1–8.
- [9] S. Nouridine, H. Camblong, I. Vechiu, and G. Tapia, "Comparison of wind turbine LQG controllers designed to alleviate fatigue loads," in *Proc. 8th IEEE Int. Conf. Control Autom. (ICCA)*, Jun. 2010, pp. 1502–1507.
- [10] S. Nouridine, H. Camblong, I. Vechiu, and G. Tapia, "Comparison of wind turbine LQG controllers using individual pitch control to alleviate fatigue loads," in *Proc. 18th Mediterranean Conf. Contr. Autom. (Med)*, Jun. 2010, pp. 1591–1596.
- [11] F. D. Bianchi, R. J. Mantz, and C. F. Christiansen, "Power regulation in pitch-controlled variable-speed WECS above rated wind speed," *Renew. Energy*, vol. 29, pp. 1911–1922, 2004.
- [12] Y. She, X. She, and M. E. Baran, "Universal tracking control of wind conversion system for purpose of maximum power acquisition under hierarchical control system," *IEEE Trans. Energy Convers.*, vol. 26, no. 3, pp. 766–775, Sep. 2011.
- [13] M. Soliman, O. P. Malik, and D. T. Westwick, "Multiple model predictive control for wind turbines with doubly fed induction generators," *IEEE Trans. Sustain. Energy*, vol. 2, no. 3, pp. 215–225, Jul. 2011.

- [14] D. Bertsimas and D. Brown, "Constrained stochastic LQC: A tractable approach," *IEEE Trans. Automat. Contr.*, vol. 52, no. 10, pp. 1826–1841, Oct. 2007.
- [15] A. D. Hansen, P. Sorensen, F. Lov, and F. Blaabjerg, "Control of variable speed wind turbines with doubly-fed induction generators," *Wind Eng.*, vol. 28, no. 4, pp. 411–432, 2004.
- [16] E. B. Muhando, T. Senju, A. Yona, H. Kinjo, and T. Funabashi, "Disturbance rejection by dual pitch control and self-tuning regulator for wind turbine generator parametric uncertainty compensation," *IET Contr. Theory Appl.*, vol. 1, no. 5, pp. 1431–1440, Sep. 2007.
- [17] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, U.K.: Cambridge Univ. Press, 2004.



Zhiqiang Jin (S'10) is a Ph.D. candidate in electrical engineering at the University of Tennessee (UT), Knoxville, TN, USA.

His current research interests are stability and control of wind power.



Xiao Ma (S'08) is currently pursuing the Ph.D. degree at the University of Tennessee, Knoxville, TN, USA.

His research interests include control over communication channels and sensor networks, modeling and identification in wireless channels, and localization and navigation in navigation systems.



Seddik M. Djouadi (M'95) received the Ph.D. degree from McGill University, Quebec, Canada, in 1999.

He is currently an Associate Professor in the Department of Electrical Engineering and Computer Science (EECS) at the University of Tennessee, Knoxville (UTK), TN, USA. His research interests include control theory, communication, sensor networks, and smart grid.



Fangxing Li (M'01–SM'05), also known as Fran Li, is currently an Associate Professor at the University of Tennessee, Knoxville, TN, USA. He received the Ph.D. degree from Virginia Tech, in 2001.

His current interests include renewable energy integration, power markets, distributed energy resources, and smart grid.

Dr. Li is an Editor of IEEE TRANSACTIONS ON RENEWABLE ENERGY and a Fellow of IET. He is a registered Professional Engineer in North Carolina.