HW12:P1

Solution: For $f = 1$ GHz, $\mu_r = 1$, and $\varepsilon_r = 9$,

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r} = \frac{2\pi f}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 10^9}{3 \times 10^8}\sqrt{9} = 20\pi \text{ rad/m},$$

$$\mathbf{E}(y,t) = \hat{\mathbf{x}}6\cos(2\pi \times 10^9 t - 20\pi y + \phi_0) \quad \text{(V/m)}.$$

At $t = 0$ and $y = 2$ cm, $E = 4$ V/m:

$$4 = 6\cos(-20\pi \times 2 \times 10^{-2} + \phi_0) = 6\cos(-0.4\pi + \phi_0).$$

Hence,

$$\phi_0 - 0.4\pi = \cos^{-1}\left(\frac{4}{6}\right) = 0.84 \text{ rad},$$

which gives

$$\phi_0 = 2.1 \text{ rad} = 120.19^\circ$$

and

$$\mathbf{E}(y,t) = \hat{\mathbf{x}}6\cos(2\pi \times 10^9 t - 20\pi y + 120.19^\circ) \quad \text{(V/m)}.$$

Problem 7.3 The electric field phasor of a uniform plane wave is given by $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}\,10e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find (a) the wavelength, (b) the frequency f of the wave, (c) the relative permittivity of the medium, and (d) the magnetic field $\mathbf{H}(z,t)$.

Solution:

(a) From $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}10e^{j0.2z}$ (V/m), we deduce that $k = 0.2$ rad/m. Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi = 31.42 \text{ m}.$$

(b)

$$f = \frac{u_p}{\lambda} = \frac{1.5 \times 10^8}{31.42} = 4.77 \times 10^6 \text{ Hz} = 4.77 \text{ MHz}.$$

(c) From

$$u_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}}, \qquad \varepsilon_r = \frac{1}{\mu_r}\left(\frac{c}{u_p}\right)^2 = \frac{1}{2.4}\left(\frac{3}{1.5}\right)^2 = 1.67.$$

(d)

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \simeq 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{2.4}{1.67}} = 451.94 \quad (\Omega),$$

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \widetilde{\mathbf{E}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \hat{\mathbf{y}}\,10e^{j0.2z} = \hat{\mathbf{x}}\,22.13e^{j0.2z} \quad \text{(mA/m)},$$

$$\mathbf{H}(z,t) = \hat{\mathbf{x}}\,22.13\cos(\omega t + 0.2z) \quad \text{(mA/m)},$$

with $\omega = 2\pi f = 9.54\pi \times 10^6$ rad/s.

HW12:P2

Problem 7.4 The electric field of a plane wave propagating in a nonmagnetic material is given by

$$\mathbf{E} = [\hat{\mathbf{y}}\,3\sin(\pi \times 10^7 t - 0.2\pi x) + \hat{\mathbf{z}}\,4\cos(\pi \times 10^7 t - 0.2\pi x)] \quad \text{(V/m)}.$$

Determine (a) the wavelength, (b) ε_r, and (c) **H**.

Solution:

(a) Since $k = 0.2\pi$,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10 \text{ m}.$$

(b)

$$u_p = \frac{\omega}{k} = \frac{\pi \times 10^7}{0.2\pi} = 5 \times 10^7 \text{ m/s}.$$

But

$$u_p = \frac{c}{\sqrt{\varepsilon_r}}.$$

Hence,

$$\varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3 \times 10^8}{5 \times 10^7}\right)^2 = 36.$$

(c)

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E} = \frac{1}{\eta}\hat{\mathbf{x}} \times [\hat{\mathbf{y}}\,3\sin(\pi \times 10^7 t - 0.2\pi x) + \hat{\mathbf{z}}\,4\cos(\pi \times 10^7 t - 0.2\pi x)]$$
$$= \hat{\mathbf{z}}\frac{3}{\eta}\sin(\pi \times 10^7 t - 0.2\pi x) - \hat{\mathbf{y}}\frac{4}{\eta}\cos(\pi \times 10^7 t - 0.2\pi x) \quad \text{(A/m)},$$