at $0.250\lambda + 0.357\lambda - 0.500\lambda = 0.107\lambda$ on the WTL scale, and here

$$z_L = 0.82 - j0.39.$$

Therefore $Z_L = z_L Z_0 = (0.82 - j0.39) \times 50\ \Omega = (41.0 - j19.5)\ \Omega$.

Problem 2.44 At an operating frequency of 5 GHz, a 50-Ω lossless coaxial line with insulating material having a relative permittivity $\varepsilon_r = 2.25$ is terminated in an antenna with an impedance $Z_L = 150\ \Omega$. Use the Smith chart to find Z_{in}. The line length is 30 cm.

Solution: To use the Smith chart the line length must be converted into wavelengths. Since $\beta = 2\pi/\lambda$ and $u_p = \omega/\beta$,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi u_p}{\omega} = \frac{c}{\sqrt{\varepsilon_r} f} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25} \times (5 \times 10^9 \text{ Hz})} = 0.04 \text{ m}.$$

Hence, $l = \frac{0.30 \text{ m}}{0.04 \text{ m}}\lambda = 7.5\lambda$. Since this is an integral number of half wavelengths,

$$Z_{in} = Z_L = 150\ \Omega.$$

HW5:P1

Section 2-10: Impedance Matching

Problem 2.45 A 50-Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25)\ \Omega$. At 0.3λ from the load, a resistor with resistance $R = 30\ \Omega$ is connected as shown in Fig. 2-43 (P2.45(a)). Use the Smith chart to find Z_{in}.

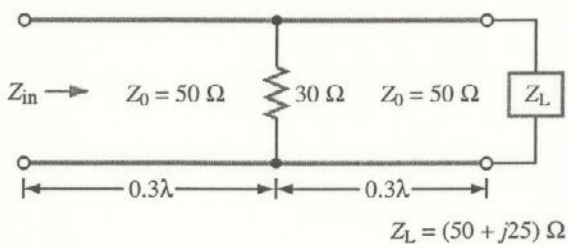

Figure P2.45: (a) Circuit for Problem 2.45.

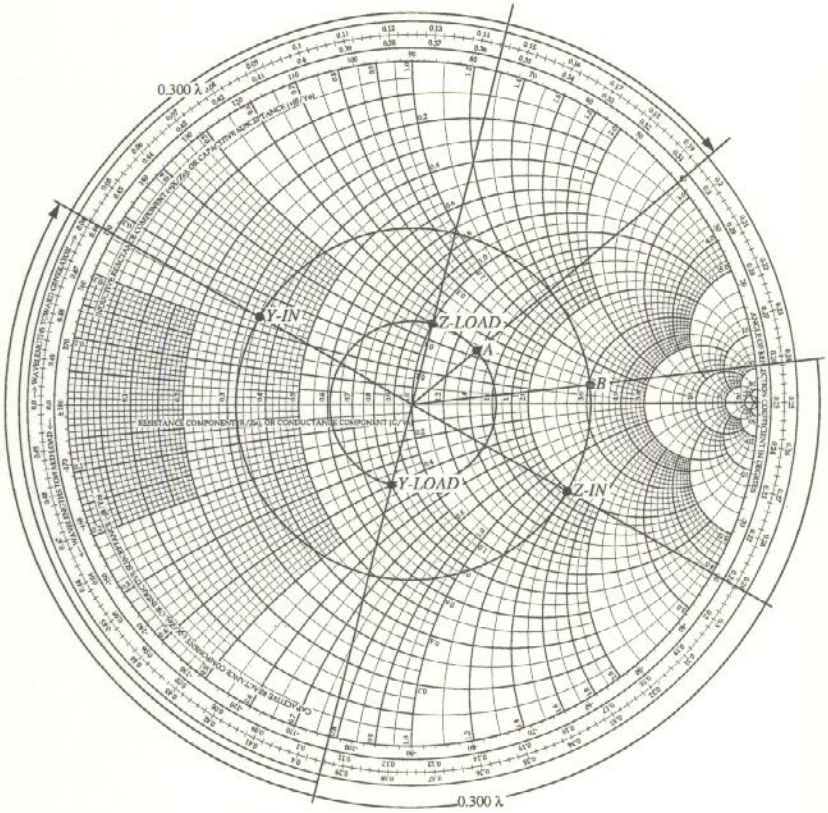

Figure P2.45: (b) Solution of Problem 2.45.

Solution: Refer to Fig. P2.45(b). Since the 30-Ω resistor is in parallel with the input impedance at that point, it is advantageous to convert all quantities to admittances.

$$z_L = \frac{Z_L}{Z_0} = \frac{(50 + j25)\ \Omega}{50\ \Omega} = 1 + j0.5$$

and is located at point *Z-LOAD*. The corresponding normalized load admittance is at point *Y-LOAD*, which is at 0.394λ on the WTG scale. The input admittance of the load only at the shunt conductor is at $0.394\lambda + 0.300\lambda - 0.500\lambda = 0.194\lambda$ and is denoted by point *A*. It has a value of

$$y_{inA} = 1.37 + j0.45.$$

The shunt conductance has a normalized conductance

$$g = \frac{50\ \Omega}{30\ \Omega} = 1.67.$$

The normalized admittance of the shunt conductance in parallel with the input admittance of the load is the sum of their admittances:

$$y_{inB} = g + y_{inA} = 1.67 + 1.37 + j0.45 = 3.04 + j0.45$$

and is located at point B. On the WTG scale, point B is at 0.242λ. The input admittance of the entire circuit is at $0.242\lambda + 0.300\lambda - 0.500\lambda = 0.042\lambda$ and is denoted by point *Y-IN*. The corresponding normalized input impedance is at *Z-IN* and has a value of

$$z_{in} = 1.9 - j1.4.$$

Thus,

$$Z_{in} = z_{in}Z_0 = (1.9 - j1.4) \times 50\ \Omega = (95 - j70)\ \Omega.$$

HW5:P2

Problem 2.46 A 50-Ω lossless line is to be matched to an antenna with

$$Z_L = (75 - j20)\ \Omega$$

using a shorted stub. Use the Smith chart to determine the stub length and the distance between the antenna and the stub.

Solution: Refer to Fig. P2.46(a) and Fig. P2.46(b), which represent two different solutions.

$$z_L = \frac{Z_L}{Z_0} = \frac{(75 - j20)\ \Omega}{50\ \Omega} = 1.5 - j0.4$$

and is located at point *Z-LOAD* in both figures. Since it is advantageous to work in admittance coordinates, y_L is plotted as point *Y-LOAD* in both figures. *Y-LOAD* is at 0.041λ on the WTG scale.

For the first solution in Fig. P2.46(a), point *Y-LOAD-IN*-1 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-1 is at 0.145λ on the WTG scale, so the stub should be located at $0.145\lambda - 0.041\lambda = 0.104\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-1, $b = 0.52$, so a stub with an input admittance of $y_{stub} = 0 - j0.52$ is required. This point is *Y-STUB-IN*-1 and is at 0.423λ on the WTG scale. The short circuit admittance

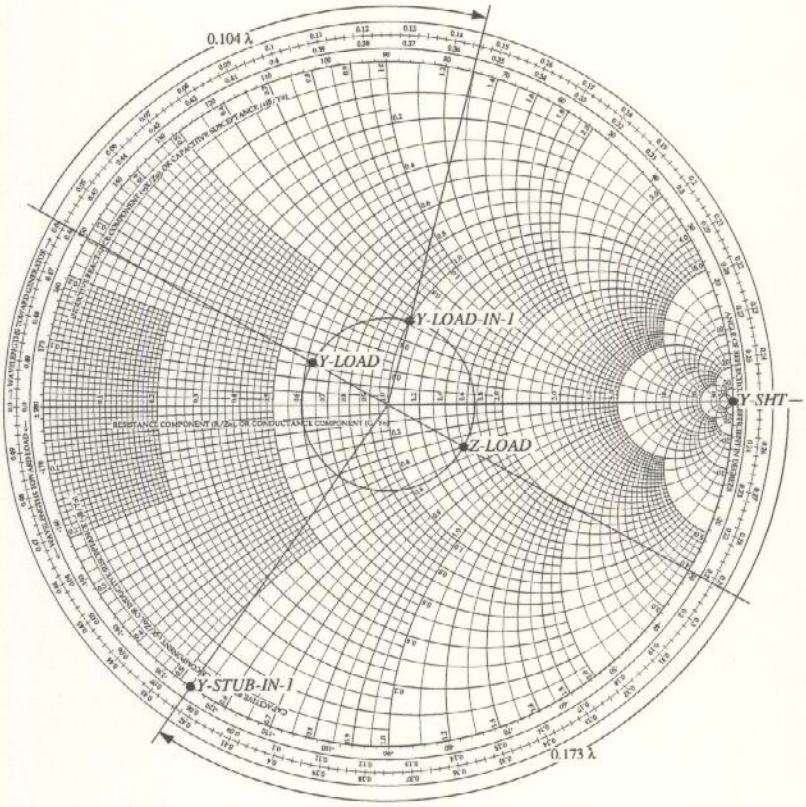

Figure P2.46: (a) First solution to Problem 2.46.

is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.423\lambda - 0.250\lambda = 0.173\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.46(b), point *Y-LOAD-IN*-2 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-2 is at 0.355λ on the WTG scale, so the stub should be located at $0.355\lambda - 0.041\lambda = 0.314\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-2, $b = -0.52$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j0.52$ is required. This point is *Y-STUB-IN*-2 and is at 0.077λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.077\lambda - 0.250\lambda + 0.500\lambda = 0.327\lambda$ long (or some multiple of a half wavelength

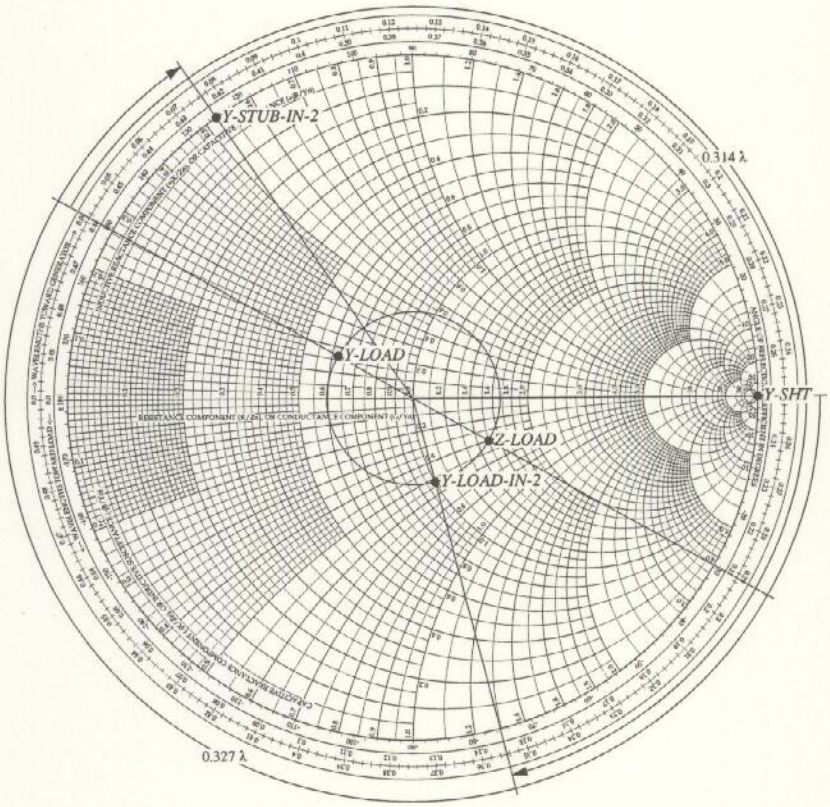

Figure P2.46: (b) Second solution to Problem 2.46.

longer).

HW5:P2

(continued)

Problem 2.47 Repeat Problem 2.46 for a load with $Z_L = (100 + j50)\ \Omega$.

Solution: Refer to Fig. P2.47(a) and Fig. P2.47(b), which represent two different solutions.

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j50\ \Omega}{50\ \Omega} = 2 + j1$$

and is located at point *Z-LOAD* in both figures. Since it is advantageous to work in admittance coordinates, y_L is plotted as point *Y-LOAD* in both figures. *Y-LOAD* is at 0.463λ on the WTG scale.

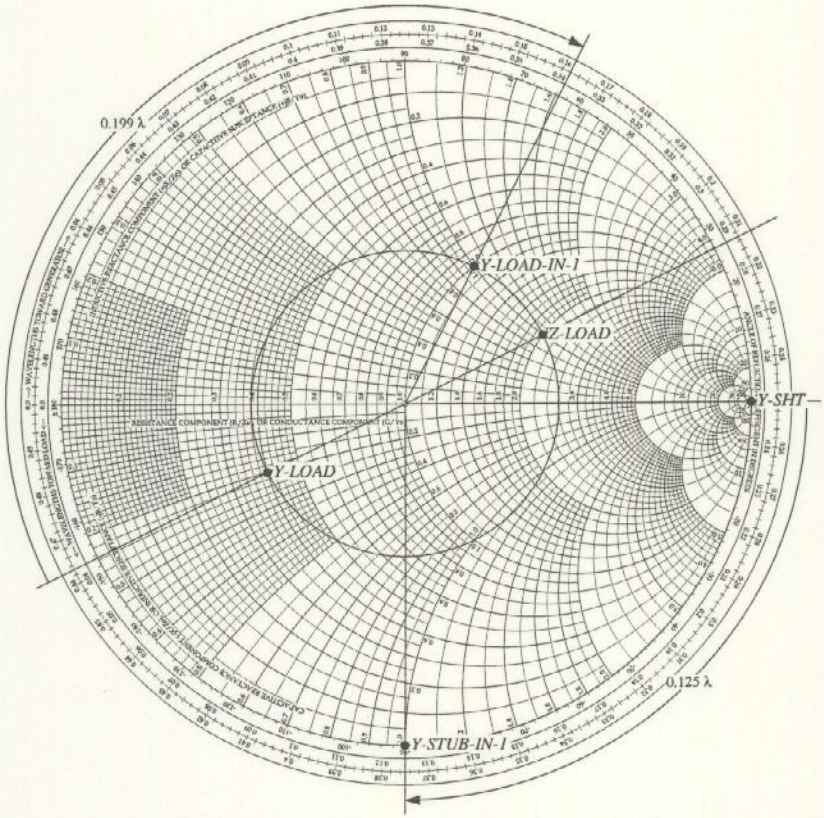

Figure P2.47: (a) First solution to Problem 2.47.

For the first solution in Fig. P2.47(a), point *Y-LOAD-IN*-1 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-1 is at 0.162λ on the WTG scale, so the stub should be located at $0.162\lambda - 0.463\lambda + 0.500\lambda = 0.199\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-1, $b = 1$, so a stub with an input admittance of $y_{\text{stub}} = 0 - j1$ is required. This point is *Y-STUB-IN*-1 and is at 0.375λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.375\lambda - 0.250\lambda = 0.125\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.47(b), point *Y-LOAD-IN*-2 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-2 is at 0.338λ on the

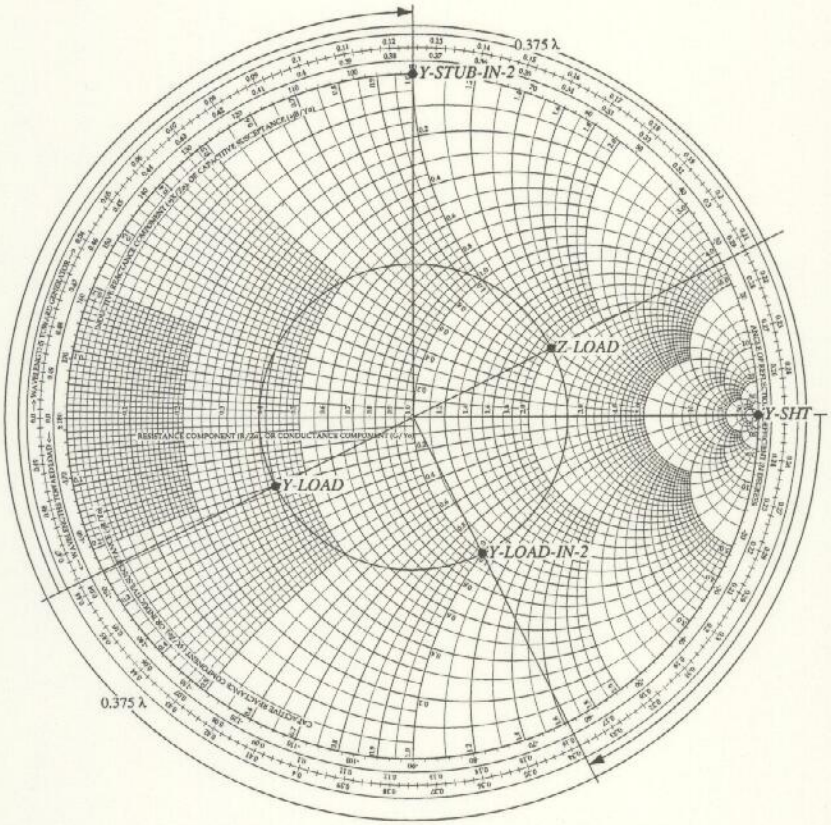

Figure P2.47: (b) Second solution to Problem 2.47.

WTG scale, so the stub should be located at $0.338\lambda - 0.463\lambda + 0.500\lambda = 0.375\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-2, $b = -1$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j1$ is required. This point is *Y-STUB-IN*-2 and is at 0.125λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.125\lambda - 0.250\lambda + 0.500\lambda = 0.375\lambda$ long (or some multiple of a half wavelength longer).

HW5:P3

Problem 2.48 Use the Smith chart to find Z_{in} of the feed line shown in Fig. 2-44 (P2.48(a)). All lines are lossless with $Z_0 = 50\ \Omega$.

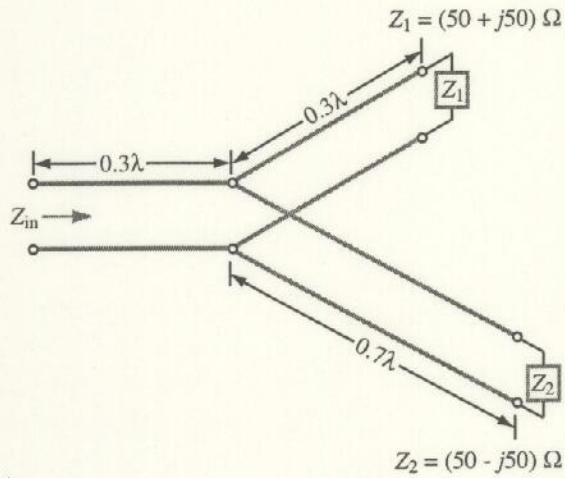

Figure P2.48: (a) Circuit of Problem 2.48.

Solution: Refer to Fig. P2.48(b).

$$z_1 = \frac{Z_1}{Z_0} = \frac{50 + j50\ \Omega}{50\ \Omega} = 1 + j1$$

and is at point *Z-LOAD*-1.

$$z_2 = \frac{Z_2}{Z_0} = \frac{50 - j50\ \Omega}{50\ \Omega} = 1 - j1$$

and is at point *Z-LOAD*-2. Since at the junction the lines are in parallel, it is advantageous to solve the problem using admittances. y_1 is point *Y-LOAD*-1, which is at 0.412λ on the WTG scale. y_2 is point *Y-LOAD*-2, which is at 0.088λ on the WTG scale. Traveling 0.300λ from *Y-LOAD*-1 toward the generator one obtains the input admittance for the upper feed line, point *Y-IN*-1, with a value of $1.97 + j1.02$. Since traveling 0.700λ is equivalent to traveling 0.200λ on any transmission line, the input admittance for the lower line feed is found at point *Y-IN*-2, which has a value of $1.97 - j1.02$. The admittance of the two lines together is the sum of their admittances: $1.97 + j1.02 + 1.97 - j1.02 = 3.94 + j0$ and is denoted *Y-JUNCT*. 0.300λ from *Y-JUNCT* toward the generator is the input admittance of the entire feed line, point *Y-IN*, from which *Z-IN* is found.

$$Z_{in} = z_{in}Z_0 = (1.65 - j1.79) \times 50\ \Omega = (82.5 - j89.5)\ \Omega.$$

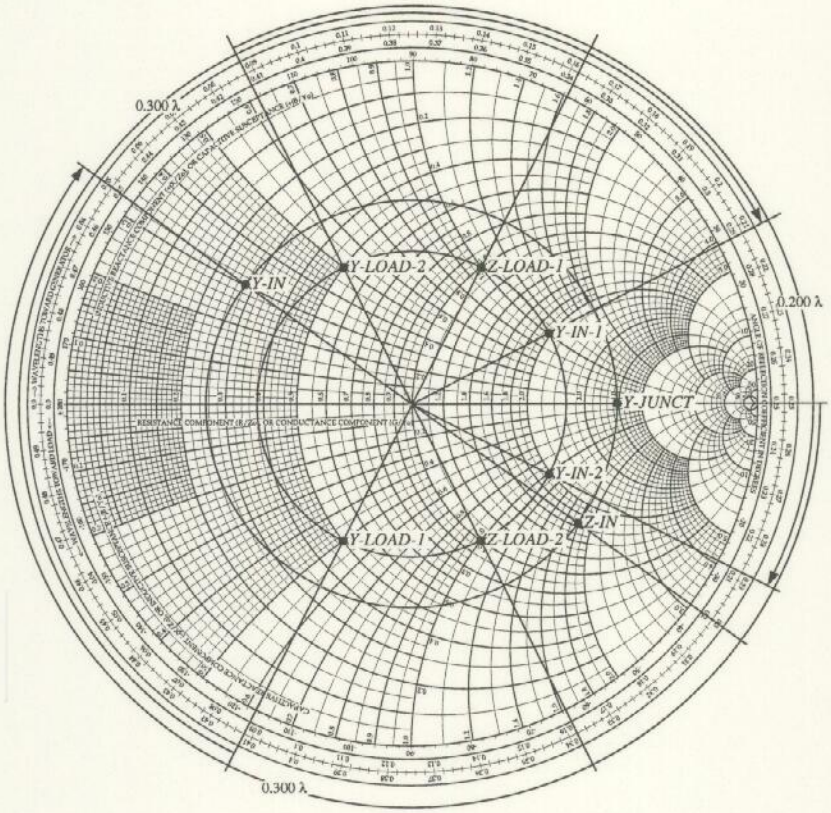

Figure P2.48: (b) Solution of Problem 2.48.

HW5:P3

(continued)

Problem 2.49 Repeat Problem 2.48 for the case where all three transmission lines are $\lambda/4$ in length.

Solution: Since the transmission lines are in parallel, it is advantageous to express loads in terms of admittances. In the upper branch, which is a quarter wave line,

$$Y_{1\,\text{in}} = \frac{Y_0^2}{Y_1} = \frac{Z_1}{Z_0^2},$$

and similarly for the lower branch,

$$Y_{2\,\text{in}} = \frac{Y_0^2}{Y_2} = \frac{Z_2}{Z_0^2}.$$

Thus, the total load at the junction is

$$Y_{\text{JCT}} = Y_{1\,\text{in}} + Y_{2\,\text{in}} = \frac{Z_1 + Z_2}{Z_0^2}.$$

Therefore, since the common transmission line is also quarter-wave,

$$Z_{\text{in}} = Z_0^2/Z_{\text{JCT}} = Z_0^2 Y_{\text{JCT}} = Z_1 + Z_2 = (50 + j50)\ \Omega + (50 - j50)\ \Omega = 100\ \Omega.$$

Section 2-11: Transients on Transmission Lines

Problem 2.50 Generate a bounce diagram for the voltage $V(z,t)$ for a 1-m long lossless line characterized by $Z_0 = 50\ \Omega$ and $u_p = 2c/3$ (where c is the velocity of light) if the line is fed by a step voltage applied at $t = 0$ by a generator circuit with $V_g = 60$ V and $R_g = 100\ \Omega$. The line is terminated in a load $Z_L = 25\ \Omega$. Use the bounce diagram to plot $V(t)$ at a point midway along the length of the line from $t = 0$ to $t = 25$ ns.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3},$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = \frac{-1}{3}.$$

From Eq. (2.124b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20\ \text{V}.$$

Also,

$$T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5\ \text{ns}.$$

The bounce diagram is shown in Fig. P2.50(a) and the plot of $V(t)$ in Fig. P2.50(b).