at $0.250\lambda + 0.357\lambda - 0.500\lambda = 0.107\lambda$ on the WTL scale, and here

$$z_L = 0.82 - j0.39.$$  

Therefore

$$Z_L = z_L Z_0 = (0.82 - j0.39) \times 50 \, \Omega = (41.0 - j19.5) \, \Omega.$$  

**Problem 2.44** At an operating frequency of 5 GHz, a 50-Ω lossless coaxial line with insulating material having a relative permittivity $\varepsilon_r = 2.25$ is terminated in an antenna with an impedance $Z_L = 150 \, \Omega$. Use the Smith chart to find $Z_m$. The line length is 30 cm.

**Solution:** To use the Smith chart the line length must be converted into wavelengths. Since $\beta = 2\pi/\lambda$ and $\nu_p = \omega/\beta$,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi v_p}{\omega} = \frac{c}{\sqrt{\varepsilon_r} \nu_p} = \frac{3 \times 10^8 \, \text{m/s}}{\sqrt{2.25} \times (5 \times 10^9 \, \text{Hz})} = 0.04 \, \text{m}.$$  

Hence, $l = \frac{0.30 \, \text{m} \times 0.04 \, \text{m}}{\text{m}^2} = 7.5\lambda$. Since this is an integral number of half wavelengths,

$$Z_m = Z_L = 150 \, \Omega.$$  

**Section 2-10: Impedance Matching**

**Problem 2.45** A 50-Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25) \, \Omega$. At 0.3λ from the load, a resistor with resistance $R = 30 \, \Omega$ is connected as shown in Fig. 2-43 (P2.45(a)). Use the Smith chart to find $Z_m$.

![Circuit Diagram](image)

Figure P2.45: (a) Circuit for Problem 2.45.
Figure P2.45: (b) Solution of Problem 2.45.

Solution: Refer to Fig. P2.45(b). Since the 30-Ω resistor is in parallel with the input impedance at that point, it is advantageous to convert all quantities to admittances.

\[ z_L = \frac{Z_L}{Z_0} = \frac{(50 + j25) \, \Omega}{50 \, \Omega} = 1 + j0.5 \]

and is located at point Z-LOAD. The corresponding normalized load admittance is at point Y-LOAD, which is at 0.394\( \lambda \) on the WTG scale. The input admittance of the load only at the shunt conductor is at 0.394\( \lambda + 0.300\lambda = 0.500\lambda = 0.194\lambda \) and is denoted by point A. It has a value of

\[ y_{inA} = 1.37 + j0.45. \]
The shunt conductance has a normalized conductance
\[ g = \frac{50 \, \Omega}{30 \, \Omega} = 1.67. \]

The normalized admittance of the shunt conductance in parallel with the input admittance of the load is the sum of their admittances:
\[ y_{inR} = g + y_{inA} = 1.67 + 1.37 + j0.45 = 3.04 + j0.45 \]

and is located at point B. On the WTG scale, point B is at 0.242\( \lambda \). The input admittance of the entire circuit is at 0.242\( \lambda + 0.300\lambda - 0.500\lambda = 0.042\lambda \) and is denoted by point Y-IN. The corresponding normalized input impedance is at Z-IN and has a value of
\[ z_{in} = 1.9 - j1.4. \]

Thus,
\[ Z_{in} = z_{in}Z_0 = (1.9 - j1.4) \times 50 \, \Omega = (95 - j70) \, \Omega. \]

**Problem 2.46** A 50-\( \Omega \) lossless line is to be matched to an antenna with
\[ Z_L = (75 - j20) \, \Omega \]

using a shorted stub. Use the Smith chart to determine the stub length and the distance between the antenna and the stub.

**Solution:** Refer to Fig. P2.46(a) and Fig. P2.46(b), which represent two different solutions.
\[ z_L = \frac{Z_L}{Z_0} = \frac{(75 - j20) \, \Omega}{50 \, \Omega} = 1.5 - j0.4 \]

and is located at point Z-LOAD in both figures. Since it is advantageous to work in admittance coordinates, \( y_L \) is plotted as point Y-LOAD in both figures. Y-LOAD is at 0.041\( \lambda \) on the WTG scale.

For the first solution in Fig. P2.46(a), point Y-LOAD-IN-1 represents the point at which \( g = 1 \) on the SWR circle of the load. Y-LOAD-IN-1 is at 0.145\( \lambda \) on the WTG scale, so the stub should be located at 0.145\( \lambda - 0.041\lambda = 0.104\lambda \) from the load (or some multiple of a half wavelength further). At Y-LOAD-IN-1, \( b = 0.52 \), so a stub with an input admittance of \( y_{stub} = 0 - j0.52 \) is required. This point is Y-STUB-IN-1 and is at 0.423\( \lambda \) on the WTG scale. The short circuit admittance
is denoted by point $Y-SHT$, located at $0.250\lambda$. Therefore, the short stub must be $0.423\lambda - 0.250\lambda = 0.173\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.46(b), point $Y-LOAD-IN-2$ represents the point at which $g = 1$ on the SWR circle of the load. $Y-LOAD-IN-2$ is at $0.355\lambda$ on the WTG scale, so the stub should be located at $0.355\lambda - 0.041\lambda = 0.314\lambda$ from the load (or some multiple of a half wavelength further). At $Y-LOAD-IN-2$, $b = -0.52$, so a stub with an input admittance of $\gamma_{stub} = 0 + j0.52$ is required. This point is $Y-STUB-IN-2$ and is at $0.077\lambda$ on the WTG scale. The short circuit admittance is denoted by point $Y-SHT$, located at $0.250\lambda$. Therefore, the short stub must be $0.077\lambda - 0.250\lambda + 0.500\lambda = 0.327\lambda$ long (or some multiple of a half wavelength.
Problem 2.47  Repeat Problem 2.46 for a load with $Z_L = (100 + j50) \, \Omega$.

Solution:  Refer to Fig. P2.47(a) and Fig. P2.47(b), which represent two different solutions.

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j50}{50 \, \Omega} = 2 + j1$$

and is located at point $Z\text{-LOAD}$ in both figures. Since it is advantageous to work in admittance coordinates, $y_L$ is plotted as point $Y\text{-LOAD}$ in both figures. $Y\text{-LOAD}$ is at 0.463λ on the WTO scale.
Figure P2.47: (a) First solution to Problem 2.47.

For the first solution in Fig. P2.47(a), point Y-LOAD-IN-1 represents the point at which $g = 1$ on the SWR circle of the load. Y-LOAD-IN-1 is at $0.162\lambda$ on the WTG scale, so the stub should be located at $0.162\lambda - 0.463\lambda + 0.500\lambda = 0.199\lambda$ from the load (or some multiple of a half wavelength further). At Y-LOAD-IN-1, $b = 1$, so a stub with an input admittance of $y_{stub} = 0 - j1$ is required. This point is Y-STUB-IN-1 and is at $0.375\lambda$ on the WTG scale. The short circuit admittance is denoted by point Y-SHT, located at $0.250\lambda$. Therefore, the short stub must be $0.375\lambda - 0.250\lambda = 0.125\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.47(b), point Y-LOAD-IN-2 represents the point at which $g = 1$ on the SWR circle of the load. Y-LOAD-IN-2 is at $0.338\lambda$ on the
WTG scale, so the stub should be located at \(0.338\lambda - 0.463\lambda + 0.500\lambda = 0.375\lambda\) from the load (or some multiple of a half wavelength further). At \(Y\text{-LOAD-IN-2},\)
\(b = -1,\) so a stub with an input admittance of \(y_{\text{stub}} = 0 + j1\) is required. This point is \(Y\text{-STUB-IN-2}\) and is at \(0.125\lambda\) on the WTG scale. The short circuit admittance is denoted by point \(Y\text{-SHT},\) located at \(0.250\lambda\). Therefore, the short stub must be \(0.125\lambda - 0.250\lambda + 0.500\lambda = 0.375\lambda\) long (or some multiple of a half wavelength longer).

**Problem 2.48** Use the Smith chart to find \(Z_{\text{in}}\) of the feed line shown in Fig. 2-44 (P2.48(a)). All lines are lossless with \(Z_0 = 50\ \Omega.\)
CHAPTER 2

Figure P2.48: (a) Circuit of Problem 2.48.

Solution: Refer to Fig. P2.48(b).

\[ z_1 = \frac{Z_1}{Z_0} = \frac{50 + j50 \ \Omega}{50 \ \Omega} = 1 + j1 \]

and is at point \( Z\text{-LOAD}-1 \).

\[ z_2 = \frac{Z_2}{Z_0} = \frac{50 - j50 \ \Omega}{50 \ \Omega} = 1 - j1 \]

and is at point \( Z\text{-LOAD}-2 \). Since at the junction the lines are in parallel, it is advantageous to solve the problem using admittances. \( y_1 \) is point \( Y\text{-LOAD}-1 \), which is at 0.412\( \lambda \) on the WTG scale. \( y_2 \) is point \( Y\text{-LOAD}-2 \), which is at 0.088\( \lambda \) on the WTG scale. Traveling 0.300\( \lambda \) from \( Y\text{-LOAD}-1 \) toward the generator one obtains the input admittance for the upper feed line, point \( Y\text{-IN}-1 \), with a value of 1.97 + j1.02. Since traveling 0.700\( \lambda \) is equivalent to traveling 0.200\( \lambda \) on any transmission line, the input admittance for the lower line feed is found at point \( Y\text{-IN}-2 \), which has a value of 1.97 – j1.02. The admittance of the two lines together is the sum of their admittances: \( 1.97 + j1.02 + 1.97 - j1.02 = 3.94 + j0 \) and is denoted \( Y\text{-JUNCT} \). 0.300\( \lambda \) from \( Y\text{-JUNCT} \) toward the generator is the input admittance of the entire feed line, point \( Y\text{-IN} \), from which \( Z\text{-IN} \) is found.

\[ Z_{in} = z_{in}z_0 = (1.65 - j1.79) \times 50 \ \Omega = (82.5 - j89.5) \ \Omega. \]
Problem 2.49  Repeat Problem 2.48 for the case where all three transmission lines are $\lambda/4$ in length.

Solution: Since the transmission lines are in parallel, it is advantageous to express loads in terms of admittances. In the upper branch, which is a quarter wave line,

\[ Y_{1 \text{ in}} = \frac{Y_0^2}{Y_1} = \frac{Z_1}{Z_0^2}, \]
and similarly for the lower branch,

\[ Y_{2 \text{in}} = \frac{Y_0^2}{Y_2} = \frac{Z_2}{Z_0^2}. \]

Thus, the total load at the junction is

\[ Y_{\text{CT}} = Y_{1 \text{in}} + Y_{2 \text{in}} = \frac{Z_1 + Z_2}{Z_0^2}. \]

Therefore, since the common transmission line is also quarter-wave,

\[ Z_m = Z_0^2 / Z_{\text{CT}} = Z_0^2 Y_{\text{CT}} = Z_1 + Z_2 = (50 + j50) \, \Omega + (50 - j50) \, \Omega = 100 \, \Omega. \]

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**Section 2-11: Transients on Transmission Lines**

**Problem 2.50** Generate a bounce diagram for the voltage \( V(z, t) \) for a 1-m long lossless line characterized by \( Z_0 = 50 \, \Omega \) and \( u_p = 2c/3 \) (where \( c \) is the velocity of light) if the line is fed by a step voltage applied at \( t = 0 \) by a generator circuit with \( V_g = 60 \, \text{V} \) and \( R_g = 100 \, \Omega \). The line is terminated in a load \( Z_L = 25 \, \Omega \). Use the bounce diagram to plot \( V(i) \) at a point midway along the length of the line from \( t = 0 \) to \( t = 25 \, \text{ns} \).

**Solution:**

\[
\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3},
\]

\[
\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = \frac{-1}{3}.
\]

From Eq. (2.124b),

\[
V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20 \, \text{V}.
\]

Also,

\[
T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5 \, \text{ns}.
\]

The bounce diagram is shown in Fig. P2.50(a) and the plot of \( V(t) \) in Fig. P2.50(b).