at $0.250\lambda + 0.357\lambda - 0.500\lambda = 0.107\lambda$ on the WTL scale, and here

$$z_L = 0.82 - j0.39.$$ 

Therefore $Z_L = z_L Z_0 = (0.82 - j0.39) \times 50 \Omega = (41.0 - j19.5) \Omega$.

**Problem 2.44** At an operating frequency of 5 GHz, a 50-Ω lossless coaxial line with insulating material having a relative permittivity $\varepsilon_r = 2.25$ is terminated in an antenna with an impedance $Z_L = 150 \Omega$. Use the Smith chart to find $Z_m$. The line length is 30 cm.

**Solution:** To use the Smith chart the line length must be converted into wavelengths. Since $\beta = 2\pi/\lambda$ and $\nu_p = \omega/\beta$,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi \nu_p}{\omega} = \frac{c}{\sqrt{\varepsilon_r f}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25} \times (5 \times 10^9 \text{ Hz})} = 0.04 \text{ m}.$$

Hence, $l = \frac{0.30 \text{ m}}{0.04 \text{ m}} = 7.5\lambda$. Since this is an integral number of half wavelengths,

$$Z_m = Z_L = 150 \Omega.$$ 

**Section 2-10: Impedance Matching**

**Problem 2.45** A 50-Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25) \Omega$. At 0.3λ from the load, a resistor with resistance $R = 30 \Omega$ is connected as shown in Fig. 2-43 (P2.45(a)). Use the Smith chart to find $Z_m$.

![Figure P2.45: (a) Circuit for Problem 2.45.](image)
Solution: Refer to Fig. P2.45(b). Since the 30-Ω resistor is in parallel with the input impedance at that point, it is advantageous to convert all quantities to admittances.

\[ z_L = \frac{Z_L}{Z_0} = \frac{(50 + j25) \Omega}{50 \Omega} = 1 + j0.5 \]

and is located at point Z-LOAD. The corresponding normalized load admittance is at point Y-LOAD, which is at 0.394λ on the WTG scale. The input admittance of the load only at the shunt conductor is at 0.394λ + 0.300λ − 0.500λ = 0.194λ and is denoted by point A. It has a value of

\[ Y_{inA} = 1.37 + j0.45. \]
CHAPTER 2

The shunt conductance has a normalized conductance

\[ g = \frac{50 \, \Omega}{30 \, \Omega} = 1.67. \]

The normalized admittance of the shunt conductance in parallel with the input admittance of the load is the sum of their admittances:

\[ y_{inR} = g + y_{inA} = 1.67 + 1.37 + j0.45 = 3.04 + j0.45 \]

and is located at point B. On the WTG scale, point B is at 0.242λ. The input admittance of the entire circuit is at 0.242λ + 0.300λ − 0.500λ = 0.042λ and is denoted by point Y-IN. The corresponding normalized input impedance is at Z-IN and has a value of

\[ z_{in} = 1.9 - j1.4. \]

Thus,

\[ Z_{in} = z_{in}Z_0 = (1.9 - j1.4) \times 50 \, \Omega = (95 - j70) \, \Omega. \]

**Problem 2.46**  A 50-Ω lossless line is to be matched to an antenna with

\[ Z_L = (75 - j20) \, \Omega \]

using a shorted stub. Use the Smith chart to determine the stub length and the distance between the antenna and the stub.

**Solution:** Refer to Fig. P2.46(a) and Fig. P2.46(b), which represent two different solutions.

\[ z_L = \frac{Z_L}{Z_0} = \frac{(75 - j20) \, \Omega}{50 \, \Omega} = 1.5 - j0.4 \]

and is located at point Z-LOAD in both figures. Since it is advantageous to work in admittance coordinates, \( y_L \) is plotted as point Y-LOAD in both figures. Y-LOAD is at 0.041λ on the WTG scale.

For the first solution in Fig. P2.46(a), point Y-LOAD-IN-1 represents the point at which \( g = 1 \) on the SWR circle of the load. Y-LOAD-IN-1 is at 0.145λ on the WTG scale, so the stub should be located at 0.145λ − 0.041λ = 0.104λ from the load (or some multiple of a half wavelength further). At Y-LOAD-IN-1, \( b = 0.52 \), so a stub with an input admittance of \( y_{stub} = 0 - j0.52 \) is required. This point is Y-STUB-IN-1 and is at 0.423λ on the WTG scale. The short circuit admittance

HW5: P4
Notice that this is answer to Problem 4.
is denoted by point $Y-SHT$, located at $0.250\lambda$. Therefore, the short stub must be $0.423\lambda - 0.250\lambda = 0.173\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.46(b), point $Y-LOAD-IN-2$ represents the point at which $g = 1$ on the SWR circle of the load. $Y-LOAD-IN-2$ is at $0.355\lambda$ on the WTG scale, so the stub should be located at $0.355\lambda - 0.041\lambda = 0.314\lambda$ from the load (or some multiple of a half wavelength further). At $Y-LOAD-IN-2$, $b = -0.52$, so a stub with an input admittance of $Y_{stub} = 0 + j0.52$ is required. This point is $Y-STUB-IN-2$ and is at $0.077\lambda$ on the WTG scale. The short circuit admittance is denoted by point $Y-SHT$, located at $0.250\lambda$. Therefore, the short stub must be $0.077\lambda - 0.250\lambda + 0.500\lambda = 0.327\lambda$ long (or some multiple of a half wavelength.
Problem 2.47  Repeat Problem 2.46 for a load with \( Z_L = (100 + j50) \, \Omega \).

Solution: Refer to Fig. P2.47(a) and Fig. P2.47(b), which represent two different solutions.

\[
Z_L = \frac{Z_L}{Z_0} = \frac{100 + j50 \, \Omega}{50 \, \Omega} = 2 + j1
\]

and is located at point \( Z\text{-LOAD} \) in both figures. Since it is advantageous to work in admittance coordinates, \( y_L \) is plotted as point \( Y\text{-LOAD} \) in both figures. \( Y\text{-LOAD} \) is at 0.463\( \lambda \) on the W TG scale.
For the first solution in Fig. P2.47(a), point $Y$-LOAD-IN-1 represents the point at which $g = 1$ on the SWR circle of the load. $Y$-LOAD-IN-1 is at 0.162$\lambda$ on the WTG scale, so the stub should be located at $0.162\lambda - 0.463\lambda + 0.500\lambda = 0.199\lambda$ from the load (or some multiple of a half wavelength further). At $Y$-LOAD-IN-1, $b = 1$, so a stub with an input admittance of $y_{stub} = 0 - j1$ is required. This point is $Y$-STUB-IN-1 and is at 0.375$\lambda$ on the WTG scale. The short circuit admittance is denoted by point $Y$-SHT, located at 0.250$\lambda$. Therefore, the short stub must be $0.375\lambda - 0.250\lambda = 0.125\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.47(b), point $Y$-LOAD-IN-2 represents the point at which $g = 1$ on the SWR circle of the load. $Y$-LOAD-IN-2 is at 0.338$\lambda$ on the
WTG scale, so the stub should be located at $0.338\lambda - 0.463\lambda + 0.500\lambda = 0.375\lambda$ from the load (or some multiple of a half wavelength further). At $Y$-LOAD-IN-2, $b = -1$, so a stub with an input admittance of $y_{stub} = 0 + j1$ is required. This point is $Y$-STUB-IN-2 and is at $0.125\lambda$ on the WTG scale. The short circuit admittance is denoted by point $Y$-SHT, located at $0.250\lambda$. Therefore, the short stub must be $0.125\lambda - 0.250\lambda + 0.500\lambda = 0.375\lambda$ long (or some multiple of a half wavelength longer).

**Problem 2.48** Use the Smith chart to find $Z_{in}$ of the feed line shown in Fig. 2-44 (P2.48(a)). All lines are lossless with $Z_0 = 50\ \Omega$. 

**HW5: P2**
\[ Z_1 = (50 + j50) \, \Omega \]

\[ Z_2 = (50 - j50) \, \Omega \]

\[ Z_{in} \]

\[ 0.3\lambda \]

\[ 0.7\lambda \]

**Figure P2.48: (a) Circuit of Problem 2.48.**

**Solution:** Refer to Fig. P2.48(b).

\[ z_1 = \frac{Z_1}{Z_0} = \frac{50 + j50}{50} \, \Omega = 1 + j1 \]

and is at point \( Z\text{-LOAD}-1 \).

\[ z_2 = \frac{Z_2}{Z_0} = \frac{50 - j50}{50} \, \Omega = 1 - j1 \]

and is at point \( Z\text{-LOAD}-2 \). Since at the junction the lines are in parallel, it is advantageous to solve the problem using admittances. \( y_1 \) is point \( Y\text{-LOAD}-1 \), which is at 0.412\( \lambda \) on the WTG scale. \( y_2 \) is point \( Y\text{-LOAD}-2 \), which is at 0.088\( \lambda \) on the WTG scale. Traveling 0.300\( \lambda \) from \( Y\text{-LOAD}-1 \) toward the generator one obtains the input admittance for the upper feed line, point \( Y\text{-IN}-1 \), with a value of \( 1.97 + j1.02 \). Since traveling 0.700\( \lambda \) is equivalent to traveling 0.200\( \lambda \) on any transmission line, the input admittance for the lower line feed is found at point \( Y\text{-IN}-2 \), which has a value of 1.97 \( - j1.02 \). The admittance of the two lines together is the sum of their admittances: \( 1.97 + j1.02 + 1.97 \text{ } - j1.02 = 3.94 + j0 \) and is denoted \( Y\text{-JUNCT} \).

0.300\( \lambda \) from \( Y\text{-JUNCT} \) toward the generator is the input admittance of the entire feed line, point \( Y\text{-IN} \), from which \( Z\text{-IN} \) is found.

\[ Z_{in} = z_{in} Z_0 = (1.65 - j1.79) \times 50 \, \Omega = (82.5 - j89.5) \, \Omega. \]
Problem 2.49  Repeat Problem 2.48 for the case where all three transmission lines are \( \lambda/4 \) in length.

Solution: Since the transmission lines are in parallel, it is advantageous to express loads in terms of admittances. In the upper branch, which is a quarter wave line,

\[
Y_{1\text{ in}} = \frac{Y_0^2}{Y_1} = \frac{Z_1}{Z_0^2},
\]
and similarly for the lower branch,

\[ Y_{2 \text{ in}} = \frac{Y_0}{Y_2} = \frac{Z_2}{Z_0}. \]

Thus, the total load at the junction is

\[ Y_{\text{CT}} = Y_{1 \text{ in}} + Y_{2 \text{ in}} = \frac{Z_1 + Z_2}{Z_0}. \]

Therefore, since the common transmission line is also quarter-wave,

\[ Z_m = \frac{Z_0^2}{Z_{\text{CT}}} = Z_0^2 Y_{\text{CT}} = Z_1 + Z_2 = (50 + j50) \Omega + (50 - j50) \Omega = 100 \Omega. \]

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**Section 2-11: Transients on Transmission Lines**

**Problem 2.50** Generate a bounce diagram for the voltage \( V(z,t) \) for a 1-m long lossless line characterized by \( Z_0 = 50 \Omega \) and \( \nu_p = 2c/3 \) (where \( c \) is the velocity of light) if the line is fed by a step voltage applied at \( t = 0 \) by a generator circuit with \( V_g = 60 \text{ V} \) and \( R_g = 100 \Omega \). The line is terminated in a load \( Z_L = 25 \Omega \). Use the bounce diagram to plot \( V(i) \) at a point midway along the length of the line from \( t = 0 \) to \( t = 25 \text{ ns} \).

**Solution:**

\[ \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3}, \]
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = -\frac{1}{3}. \]

From Eq. (2.124b),

\[ V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20 \text{ V}. \]

Also,

\[ T = \frac{l}{\nu_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5 \text{ ns}. \]

The bounce diagram is shown in Fig. P2.50(a) and the plot of \( V(t) \) in Fig. P2.50(b).
P3. Problem 2.74 in 8/E

2.74 A 25 Ω antenna is connected to a 75 Ω lossless transmission line. Reflections back toward the generator can be eliminated by placing a shunt impedance \( Z \) at a distance \( l \) from the load (Fig. P2.74). Determine the values of \( Z \) and \( l \).

\[ Z_0 = 75 \, \Omega \]

\[ Z_L = 25 \, \Omega \]

\[ Z_B = Z_0 / Z \]

\[ y_B + y = 1 \]

\[ Y_B + Y = Y_0 \]

There are an infinite number of solutions \((l, Z)\) that satisfy the above condition. We describe two special ones, where \( Z \) is purely resistive and purely reactive, respectively.

Special solution 1: purely resistive shunt impedance \( Z \)

For \( z_B \) to be purely resistive, we need \( l = \lambda / 4 \), resulting in \( z_B = 1/z_L = 3 \).

\[ y_B + y = 1 \quad \Rightarrow \quad z_B \parallel z = 1 \]

\[ \therefore \quad 3 \parallel \frac{3}{2} = \frac{3 \times \frac{3}{2}}{3 + \frac{3}{2}} = 1 \]

\[ \therefore \quad z = \frac{3}{2} \quad \Rightarrow \quad Z = \frac{3}{2} \times 75 \, \Omega = 112.5 \, \Omega \]

Special solution 2: purely resistive shunt impedance \( Z \)

Using the Smith chart, we find that \( y_B(l) = 1 - 1.75j \) at \( l = 0.333\lambda - 0.250\lambda = 0.083\lambda \).

\[ y_B + y = 1 \quad \Rightarrow \quad y = 1.75j \quad \Rightarrow \quad Z = \frac{Z_0}{1.75j} = -j \frac{75 \, \Omega}{1.75} = -j42.86 \, \Omega \]
\[
y_B(\lambda) = 1 - 1.75 \gamma
\]
\[
l = 0.333 \lambda - 0.250 \lambda = 0.083 \lambda
\]