and similarly for the lower branch,

$$Y_{2\text{ in}} = \frac{Y_0^2}{Y_2} = \frac{Z_2}{Z_0^2}.$$

Thus, the total load at the junction is

$$Y_{\text{JCT}} = Y_{1\text{ in}} + Y_{2\text{ in}} = \frac{Z_1 + Z_2}{Z_0^2}.$$

Therefore, since the common transmission line is also quarter-wave,

$$Z_{\text{in}} = Z_0^2/Z_{\text{JCT}} = Z_0^2 Y_{\text{JCT}} = Z_1 + Z_2 = (50 + j50)\ \Omega + (50 - j50)\ \Omega = 100\ \Omega.$$

Section 2-11: Transients on Transmission Lines

HW6:P1

Problem 2.50 Generate a bounce diagram for the voltage $V(z,t)$ for a 1-m long lossless line characterized by $Z_0 = 50\ \Omega$ and $u_p = 2c/3$ (where c is the velocity of light) if the line is fed by a step voltage applied at $t = 0$ by a generator circuit with $V_g = 60$ V and $R_g = 100\ \Omega$. The line is terminated in a load $Z_L = 25\ \Omega$. Use the bounce diagram to plot $V(t)$ at a point midway along the length of the line from $t = 0$ to $t = 25$ ns.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3},$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = \frac{-1}{3}.$$

From Eq. (2.124b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20\ \text{V}.$$

Also,

$$T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5\ \text{ns}.$$

The bounce diagram is shown in Fig. P2.50(a) and the plot of $V(t)$ in Fig. P2.50(b).

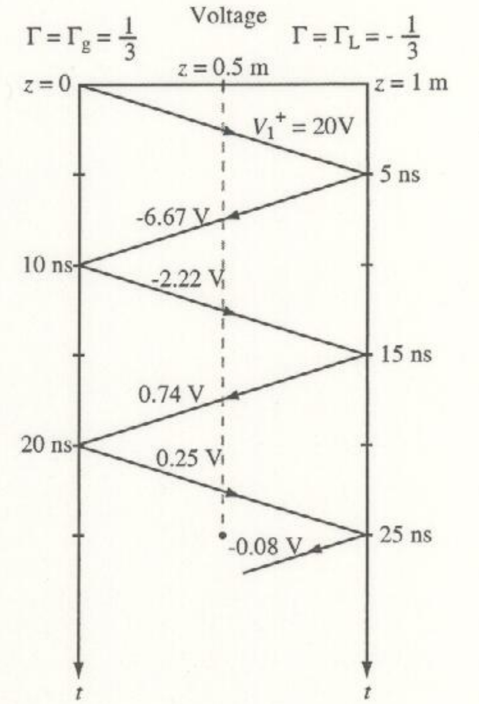

Figure P2.50: (a) Bounce diagram for Problem 2.50.

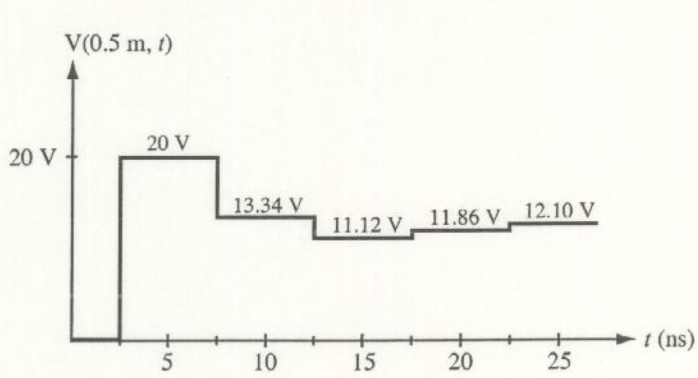

Figure P2.50: (b) Time response of voltage.

HW6:P1

continued

Problem 2.51 Repeat Problem 2.50 for the current I on the line.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100-50}{100+50} = \frac{1}{3},$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25-50}{25+50} = \frac{-1}{3}.$$

From Eq. (2.124a),

$$I_1^+ = \frac{V_g}{R_g + Z_0} = \frac{60}{100+50} = 0.4 \text{ A}.$$

The bounce diagram is shown in Fig. P2.51(a) and $I(t)$ in Fig. P2.51(b).

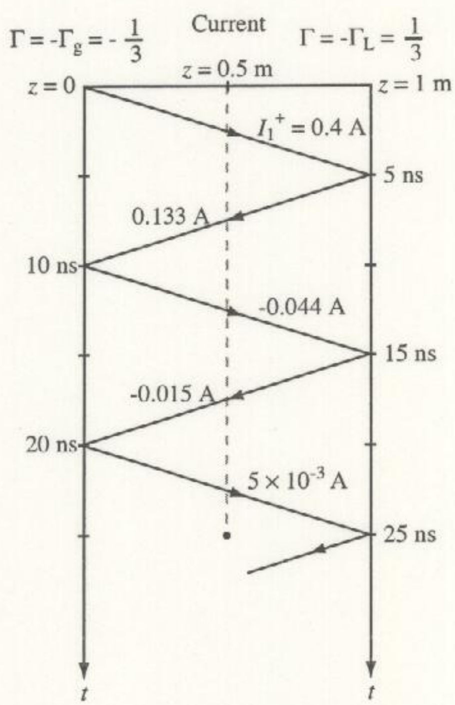

Figure P2.51: (a) Bounce diagram for Problem 2.51.

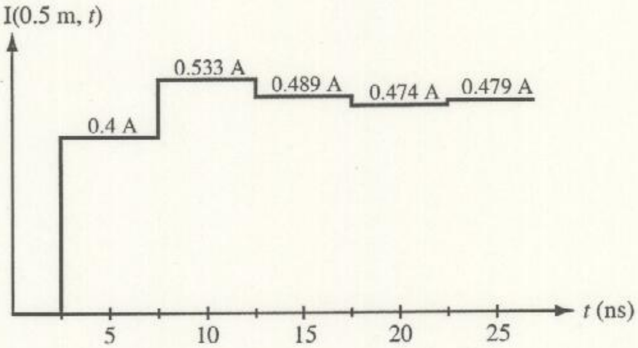

Figure P2.51: (b) Time response of current.

Problem 2.52 In response to a step voltage, the voltage waveform shown in Fig. 2-45 (P2.52) was observed at the sending end of a lossless transmission line with $R_g = 50\ \Omega$, $Z_0 = 50\ \Omega$, and $\varepsilon_r = 2.25$. Determine (a) the generator voltage, (b) the length of the line, and (c) the load impedance.

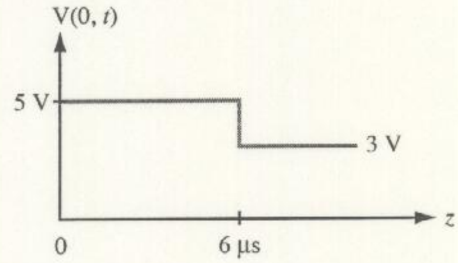

Figure P2.52: Observed voltage at sending end.

Solution:

(a) From the figure, $V_1^+ = 5$ V. Applying Eq. (2.124b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{V_g Z_0}{Z_0 + Z_0} = \frac{V_g}{2},$$

which gives $V_g = 2V_1^+ = 10$ V.

Answer to Problem 2, HW6

$\varepsilon_r = 4$ ➔ $v_p = c/2$

$2T = 7$ µs

$l = v_p T = 525$ m

$Z_L = 0$ ➔ $\Gamma_L = -1$

$v(0, 2T) = V_1^+ - V_1^+ - \Gamma_g V_1^+ = 3$ V

$-\Gamma_g V_1^+ = 3$ V

$V_1^+ = 12$ V

$\therefore\ \Gamma_g = -1/4$

$R_g/Z_0 = (1+\Gamma_g)/(1-\Gamma_g) = 3/5$

$R_g = 50\ \Omega \times 3/5 = 30\ \Omega$

$V_g = V_1^+(R_g+Z_0)/Z_0 = 12\text{ V} \times 8/5 = 19.2$ V