So far, we have only discussed single-frequency harmonic signals, if we can call them signals at all.

Why do I say this, "If we can call them signals at all?" Because they don't carry any info!

Anything deterministic, which is there forever, doesn't carry any information, coz you know what it will be when detecting it.

So, to transfer information, we modulate the wave.

For example, we can modulate a wave to send a bit.

Now, let's say we send this bit down a transmission line.

Now, you don't have a continuous wave. This envelope travels down the line at speed $v_g$ — the group velocity, which is usually a little different from $v_p$, due to something called "dispersion."
For simplicity, we ignore dispersion, and say \( v_g = v_p \).

If \( Z_L = Z_0 \), this wave, arriving at \( Z_L \), is totally absorbed. This is what we want.

But, if \( Z_L \neq Z_0 \neq Z_g \), things become complicated.

Let's call the time duration of the bit \( \tau \).

First we look at the case \( \tau < \frac{2l}{v_p} \), where \( l \) is the transmission line length.

You will have a lot of echoes.

If you sending a lot of bits, these echoes will interfere with other bits.

Then, we look at the case \( \tau \geq \frac{2l}{v_p} \).
You may not have lots of echoes outside the bit, but the bit is distorted and broadened.

Now, let's consider the situation where we use a matching network to achieve matching.

Without the matching network, if the transmission line is very long, we will have echoes. Recall that, even with the matching network we still have reflection back and forth here. The difference is that now the back & forth reflection is local near the load. You no longer have the bouncing between the sender & receiver.

You eliminate the echoes outside the bit, but the bit envelope is still distorted.

This modulation case is pretty complicated. From now on, we'll use a simple case to show you some quantitative analysis.
Basic assumptions used:

1. Lossless line. $Z_0$ is purely real. Implication of this?

2. Purely resistive load $Z_L = R_L$

3. Therefore, $\Gamma$ is frequency-independent.
   
   (If $R_L = Z_0$, impedance matched for all frequencies)

4. Dispersionless. $v_g = v_p$ for all frequencies.
Consider this case:

\[ R_g \quad V_g \quad Z_0 \quad R_L \]

Notice change in convention.

For \( 0 < t < T = \frac{l}{v_p} \):

The "turn on" event has not reached the load yet. So, it doesn't know anything about \( R_L \).

The transmission line feels like infinitely long.

Therefore, the equivalent circuit is

\[ V_i^+ = \frac{V_g Z_0}{R_g + Z_0} \]

\[ I_i^+ = \frac{V_g}{R_g + Z_0} \]

In other words, there's no reflection yet.

The textbook uses a different convention here.

The source is \( 0 \) at \( z = 0 \) and the load \( 0 \) at \( z = l \).

I follow it to minimize your confusion.
The front edge hits the load at $t = T$.

The reflection voltage

$$V_i^- = \Gamma L V_i^+$$

The reflection current

$$I_i^- = \Gamma L I_i^+$$

Here the superscript indicates the direction and the subscript 1 means the 1st roundtrip.

$$\Gamma L = \frac{R_L - Z_0}{R_L + Z_0}$$

Assuming $R_L \gg Z_0$.

In example drawn here.

Quiz: What is the voltage at time $T$ at the load?
At $t = 2T$, the front hits the source.

There's reflection by the source.

\[ \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} \]

\[ V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+ \]
\[ I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+ \]

Again, the superscripts indicate directions.

+ means from source to load, - ...

The subscript "2" means the 2nd round trip.

At $t = 3T$, hit the load again

\[ V_2^- = \Gamma_L V_2^+ \]
\[ I_2^- = -\Gamma_L I_2^+ \]

\[ V_2^+ + V_2^- = V_2^+ (1 + \Gamma_L) \]
\[ I_2^+ + I_2^- = I_2^- (1 - \Gamma_L) \]

Actually, for $ith$ round trip.
\[ V_i^+ + V_i^- = V_i^+ \quad (\pm \Gamma_e) \]

\[ I_i^+ + I_i^- = I_i^+ \quad (\mp \Gamma_e) \]

\[ \mathcal{U}(t = \infty) = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \ldots \]

\[ = \sum_{i=1}^{\infty} (V_i^+ + V_i^-) \]

\[ = V_1^+ (1 + \Gamma_e) + V_2^+ (1 + \Gamma_e) + \ldots = (1 + \Gamma_e) \left[ V_1^+ + V_2^+ + \ldots \right] \]

\[ = (1 + \Gamma_e) \sum_{i=1}^{\infty} V_i^+ \]

\[ V_2^+ = \Gamma_g \Gamma_e V_1^+ \quad \quad V_{i+1}^+ = \Gamma_g \Gamma_e V_i^+ \]

\[ \mathcal{U}(t = \infty) = V_1^+ (1 + \Gamma_e) \left[ 1 + \frac{\Gamma_g \Gamma_e}{1 + (\Gamma_g \Gamma_e)^2} + \ldots \right] \]

\[ = V_1^+ (1 + \Gamma_e) \sum_{i=0}^{\infty} (\Gamma_g \Gamma_e)^i \quad \chi = \Gamma_g \Gamma_e \]

\[ 1 + \chi + \chi^2 + \ldots = \sum_{i=0}^{\infty} \chi^i = \frac{1}{1 - \chi} \]

\[ \therefore \mathcal{U}(t = \infty) = V_1^+ (1 + \Gamma_e) \cdot \frac{1}{1 - \frac{\Gamma_g \Gamma_e}{1 + \Gamma_g \Gamma_e}} \]

\[ V_i^+ = \frac{V_g Z_o}{R_g + Z_o} \]

\[ \Gamma_g = \frac{R_g - Z_o}{R_0 + Z_o} \]

\[ \Gamma_e = \frac{1}{1 - \frac{\Gamma_g \Gamma_e}{1 + \Gamma_g \Gamma_e}} \]
\[ V(x = \infty) = \frac{V_0 R_L}{R_g + R_L} \]

- Exactly what we expected for dc voltage division.

Similarly,

\[ i(t = \infty) = I_*^+ (1 - \Gamma_L) \sum_{i=0}^{\infty} (\Gamma_L) \Gamma_g^{i} i \]

\[ = I_*^+ \frac{1 - \Gamma_L}{1 - \Gamma_g \Gamma_L} \]

\[ = \frac{V_0}{R_g + R_L} \]

It's pretty tedious to trace where the front is a certain time, and calculate the instantaneous values \( v(t) \) and \( i(t) \).

We now introduce a graphical tool to help us do that, tracking the bouncing back and forth of the front.

- It's called the Bounce Plot diagram.