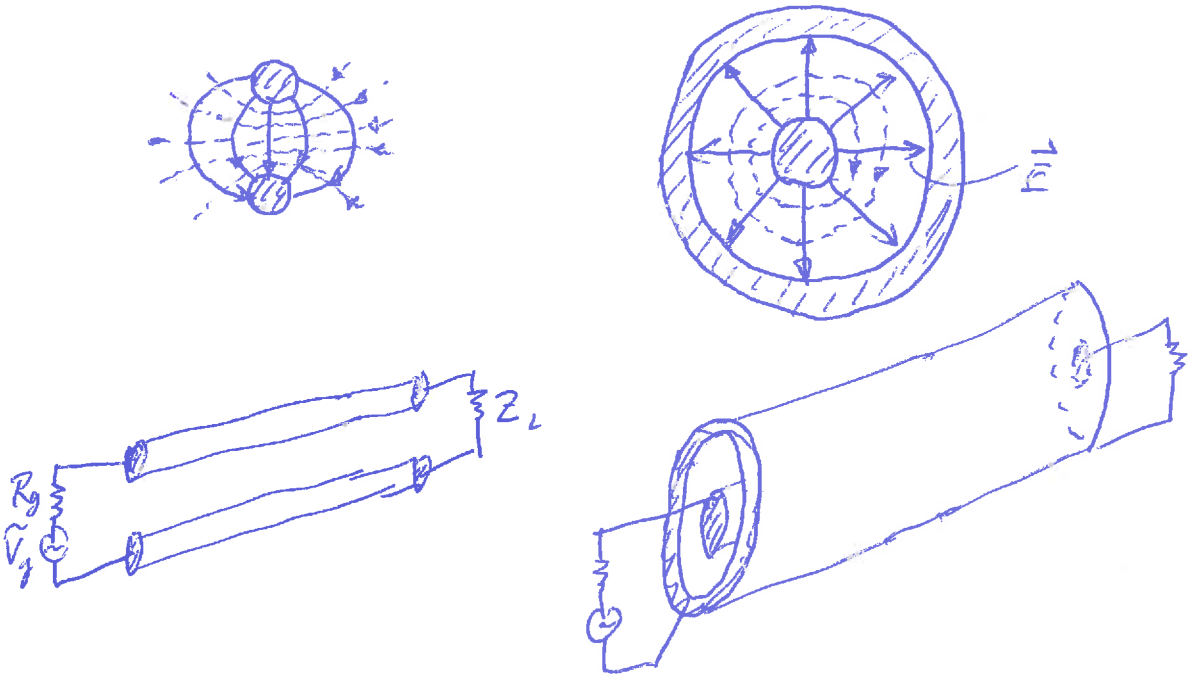
Now, we have finished our discussion on transmission lines.

The transmission line theory deals with a special type of waveguides, where you have two conductors.

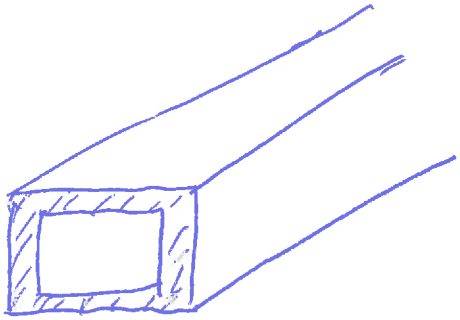


You can define local voltages $v(z, t)$ & currents $i(z, t)$ along the transmission line, and use the wave eq's of the v & i.

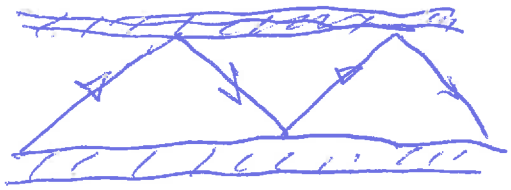
This is one step beyond the lumped component ckt theory.

But there are other waveguides.

You don't need two conductors to guide E.M. waves.



A metal tube is a wave guide



We can have a very coarse ray optics picture:
The metal walls are like mirrors.

The EM wave behaves like light rays.

Besides waveguides, you have antennas & other things, where you can not even define a "local" voltage or current, and therefore need a "true" EM field theory.

Keep in mind that the fields are vectors.
One reason the transmission line theory is simpler is that it deals w/ v & i, which are scalars.
To handle vectors, you need vector algebra & vector calculus.

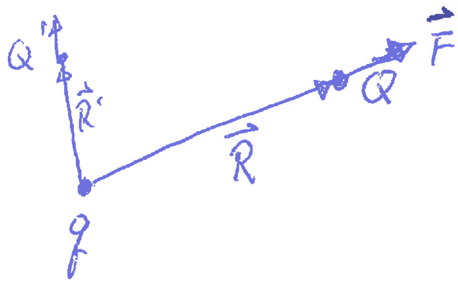
Chapter 3 of the book is all about the math

Read on your own. Individualized effort.

Coulomb's Law

Now, let's talk about fields.

First let's look @ Coulomb's law in vacuum.
(free space)



$$\vec{F} = \frac{qQ}{4\pi\varepsilon_0 R^2}\,\hat{R}$$

Notations of vectors.

Meanings of this eq.

A different way to write this is $\vec{F} = \frac{qQ}{4\pi\varepsilon_0 R^3}\,\vec{R}$

$\vec{R} = R\hat{R}$, $R = |\vec{R}|$

Hand-written

$$\vec{F} = \frac{qQ}{4\pi\varepsilon_0 R^2}\hat{R}$$

$$\vec{F} = \frac{qQ}{4\pi\varepsilon_0 R^3}\vec{R}$$

$$\vec{R} = R\hat{R}, \quad R = |\vec{R}|$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{R}$$

$$\vec{F} = Q\vec{E}$$

Printed

$$\mathbf{F} = \frac{qQ}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}}$$

Vectors: bold; italic or not
Scalars: not bold; italic
Numbers: not bold; not italic

$$\mathbf{F} = \frac{qQ}{4\pi\varepsilon_0 R^3}\mathbf{R}$$

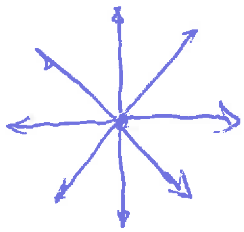
$$\mathbf{R} = R\hat{\mathbf{R}}$$

$$R = |\,\mathbf{R}\,|$$

$$\hat{\mathbf{R}} = \frac{\mathbf{R}}{R}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}}$$

$$\mathbf{F} = Q\mathbf{E}$$



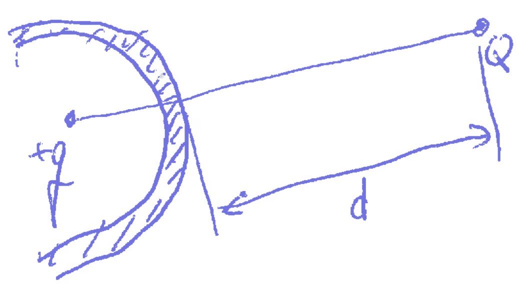
So. we can imagine something called the $\vec{E}$ field.

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{R}$$

when you put your testing charge Q any where, $\vec{F} = Q\vec{E}$

Long ago, people didn't know whether the field is just a mathematical convenience we created in our mind, or a real thing.

now, we know that. the field is real.



If you suddenly cover q w/ a metal lid. Q will feel the disappearance of the force after $t = d/c$.

In other words, any interaction cannot be instantaneous over distance.
The field is the medium for the interaction.

you need to handle vectors.

I encourage you to go thru it. (Chapter 3)

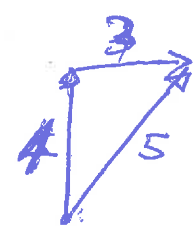
Math w/o a physics or engineering context is dry and not fun.

I won't go thru that in the class.

Instead, I will embed it in the physics

Anyway, when you go thru this math in the book, try to visualize it. Keep this one idea in your mind: a vector is just a quantity w/ a direction. That's it. With this, the concepts related to vectors are easy to understand.

For example, you walk 4 miles to the north, turn to your right, walk 3 miles to the east. You are only 5 miles away from the starting point, although you walked 7 miles.



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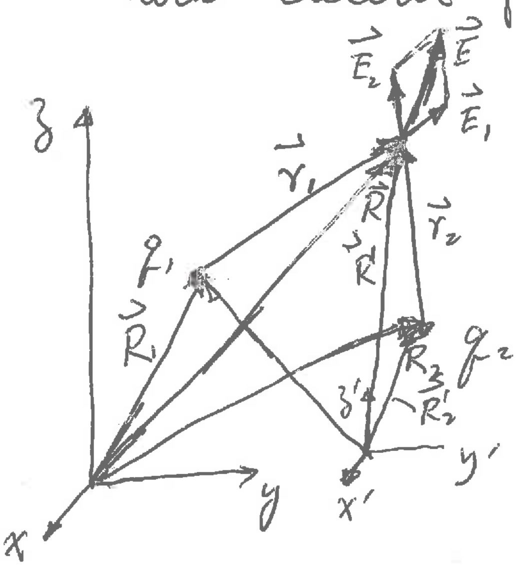
For our first example, let's calculate the total electric field due to two charges



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2\right)$$

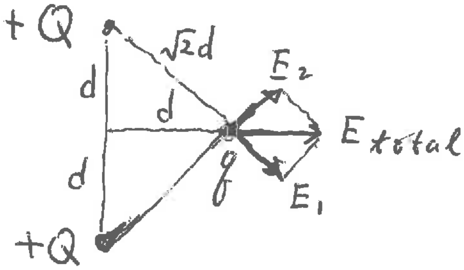
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{|\vec{R}-\vec{R}_1|^2}\,\frac{\vec{R}-\vec{R}_1}{|\vec{R}-\vec{R}_1|} + \frac{q_2}{|\vec{R}-\vec{R}_2|^2}\,\frac{\vec{R}-\vec{R}_2}{|R-\vec{R}_2|}\right)$$

$$\vec{r}_1 = \vec{R} - \vec{R}_1 = \vec{R}' - \vec{R}_1$$

$$\vec{r}_2 = \vec{R} - \vec{R}_2 = \vec{R}' - \vec{R}_2'$$

Notice that this applies to any choice of the origin.

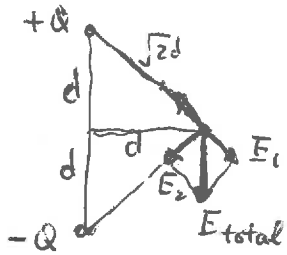
Example



$$E_1 = E_2 = E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(\sqrt{2}d)^2} = \frac{1}{8\pi\varepsilon}\frac{Q}{d^2}$$

$$E_{total} = \sqrt{2}\,E = \frac{\sqrt{2}}{8\pi\varepsilon}\frac{Q}{d^2}$$

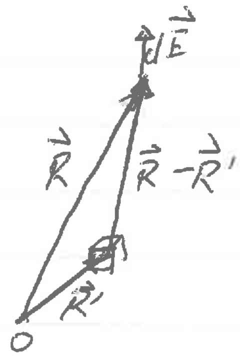
$$F = q\,E_{total}$$



If you have more than 2 charges. just add them all up.
Charge can be distributed continuously.
In this case, we use integral to do the summing.

~~Ato~~ In a small volume dV around position $\vec{R}'$, the charge density is ρ (or ρ_v).
At $\vec{R}$, the field is

$$d\vec{E} = \frac{\rho dV}{4\pi\varepsilon_0} \cdot \frac{1}{|\vec{R}-\vec{R}'|^2} \frac{\vec{R}-\vec{R}'}{|\vec{R}-\vec{R}'|}$$



The total field is

$$\vec{E} = \int d\vec{E}$$

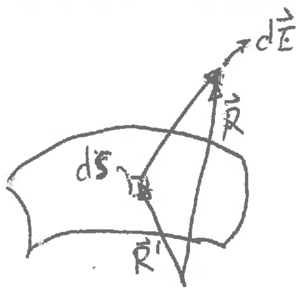
$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{|\vec{R}-\vec{R}'|^2} \frac{\vec{R}-\vec{R}'}{|\vec{R}-\vec{R}'|} dV$$

Of course, the total charge is

$$Q = \int \rho \, dV$$

(e.g. conductor surface)

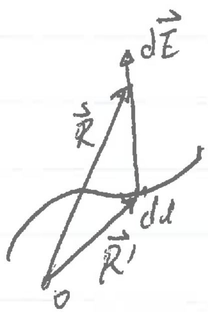
The charge could be confined in a surface
In this case, we define areal density ρ_s (or σ).



$$\vec{E} = \int_s \frac{1}{4\pi\varepsilon_0} \frac{\rho_s}{|\vec{R}-\vec{R}'|} \frac{\vec{R}-\vec{R}'}{|\vec{R}-\vec{R}'|} dS$$

$$Q = \int_s \rho_s \, dS$$

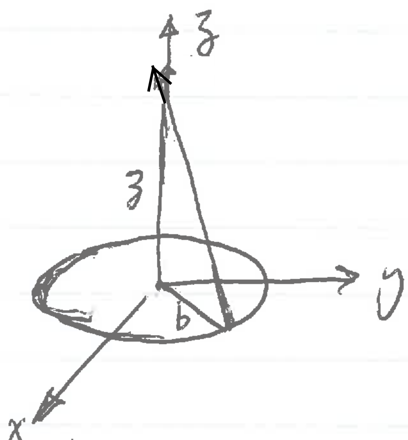
The charge could be confined along a line, in which case you can define the line density ρ_ℓ.



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_\ell}{|\vec{R}-\vec{R}'|^2}\,\frac{\vec{R}-\vec{R}'}{|\vec{R}-\vec{R}'|}\,d\ell$$

$$Q = \int \rho_\ell \, d\ell$$

These formulas are quite general, and therefore abstract. You may not get a lot of insight from them.
Lets look @ some examples now.



Let's say we have a ring of charge, w/ a density ρ_ℓ.
Radius b.
So the total charge

$$Q = \rho_\ell \cdot 2\pi b$$

Let's find the field at any point along the z axis.

only need to find dE_z due to symmetry

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{z^2+b^2}\cdot\rho_\ell\, b\, d\phi$$

$$dE_z = dE\cos\theta$$

In other words

$$\frac{dE_z}{dE} = \frac{z}{\sqrt{b^2+z^2}}$$

$$\therefore\ dE_z = dE\,\frac{z}{\sqrt{b^2+z^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{\rho_\ell\, b\, d\phi}{z^2+b^2}\cdot\frac{z}{\sqrt{z^2+b^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{z\rho_\ell b}{(z^2+b^2)^{\frac{3}{2}}}\,d\phi$$

$$E = E_z = \int_0^{2\pi}\frac{1}{4\pi\varepsilon_0}\,\frac{z\rho_\ell b}{(z^2+b^2)^{\frac{3}{2}}}\,d\phi$$

$$= \frac{2\pi}{4\pi\varepsilon_0}\,\frac{z\rho_\ell b}{(z^2+b^2)^{\frac{3}{2}}}, \quad Q = 2\pi b\rho_\ell$$

$$= \frac{z}{4\pi\varepsilon_0(b^2+z^2)^{\frac{3}{2}}}\,Q$$

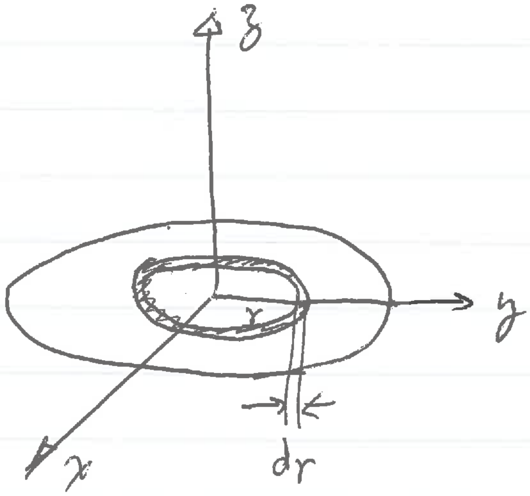
You can compare this method w/ the one in the text book. This is much simpler and faster. Why? ... symmetry.

Let's now do a sanity check.

$$E(z=0)=0$$

$$E(z\to\infty) = \frac{z}{4\pi\varepsilon_0 (z^2)^{\frac{3}{2}}} Q = \frac{Q}{4\pi\varepsilon_0 z^2}$$

Now we can use the result of this example to find the field along the axis of a charged disk.



We can ~~think~~ regard the disk as made of many many rings with a width of dr.

So, the charge on such a ring is

$$dQ = 2\pi r \cdot dr \cdot \rho_s$$

The field due to such a ring is

$$dE = \frac{z}{4\pi\varepsilon_0 (r^2+z^2)^{\frac{3}{2}}} 2\pi\rho_s r\,dr$$

~~So~~ Now, you add up all the rings

$$E = \frac{z}{\underset{2}{\cancel{4\pi}}\varepsilon_0} \cancel{2\pi}\rho_s \int_0^a \frac{r\,dr}{(r^2+z^2)^{\frac{3}{2}}}$$

Now let's have a bit of math exercise

$$\int_0^a \frac{r\,dr}{(r^2+z^2)^{\frac{3}{2}}} = \frac{1}{2}\int_0^a \frac{d(r^2)}{(r^2+z^2)^{\frac{3}{2}}}$$

$$\int \frac{dx}{(x+x_0)^{\frac{3}{2}}} = \int \frac{d(x+x_0)}{(x+x_0)^{\frac{3}{2}}}$$

Recall that

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Here $n = -\frac{3}{2}$

$$\cdots = \frac{1}{-\frac{3}{2}+1}(x+x_0)^{-\frac{3}{2}+1}$$

$$= -2\,(x+x_0)^{-\frac{1}{2}} = -2\frac{1}{\sqrt{x+x_0}}$$

$$\therefore \; = \frac{1}{2}(-2)\frac{1}{\sqrt{r^2+z^2}}\Bigg|_0^a$$

$$= \frac{1}{\sqrt{r^2+z^2}}\Bigg|_a^0$$

$$= \frac{1}{z} - \frac{1}{\sqrt{a^2+z^2}}$$

$$\therefore \; E = \frac{z\rho_s}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{a^2+z^2}}\right) = \frac{\rho_s}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{a^2+z^2}}\right)$$

for $z > 0$.

OK. now let's do some sanity check again.

$$\boxed{\rho_s = \frac{Q}{\pi a^2} \qquad E = \frac{1}{2\varepsilon_0}\,\frac{Q}{\pi a^2}\left(1-\frac{z}{\sqrt{a^2+z^2}}\right)}$$

$$\cancel{= \frac{Q}{2\varepsilon\pi a^2}\left(1-\frac{z/a}{\sqrt{1+(z/a)^2}}\right)}$$

$$\cancel{E(z\to 0) = \frac{Q}{2\pi\varepsilon a^2}\left[1-\frac{z}{a}\left(1-\frac{1}{2}\left(\frac{z}{a}\right)^2\right)\right]}$$

$$= \frac{1}{2\varepsilon_0}\,\frac{Q}{\pi a^2}\left[1-\frac{1}{\sqrt{1+\left(\frac{a}{z}\right)^2}}\right]$$

$$E(z\to\infty) = \frac{1}{2\varepsilon}\,\frac{Q}{\pi a^2}\left[1-\left(1-\frac{1}{2}\left(\frac{a}{z}\right)^2\right)\right]$$

$$= \frac{1}{2\varepsilon_0}\cdot\frac{Q}{\pi \cancel{a^2}}\cdot\frac{1}{2}\,\frac{\cancel{a^2}}{z^2}$$

$$= \frac{Q}{4\pi\varepsilon_0 z^2}$$ yes!

$$E(z\to 0) = \frac{1}{2\varepsilon}\cdot\frac{Q}{\pi a^2} \neq 0 \ !$$

anything wrong ??! Donut vs pie???!

actually. $z\to 0 \iff a\to\infty \quad \left(\frac{z}{a}\right)\to 0.$

$$E\left(\frac{z}{a} \to 0\right) = \frac{\rho_s}{2\varepsilon}$$

The field of an infinite sheet of charge.

We will get back to this after we discuss the Gauss's law.

Okay. now you can calculate the field of a charged cylinder.

~~How~~?

You can use the same method to find the field of a charged sphere at a point outside the sphere. Have the fun yourself.

Since you already got the point, I don't want to bore you w/ more math.

Next, we talk about Gauss's law.

Recall that I said I would embed the math in physics, coz I don't want to talk about the math w/o a context, which is dry and boring.

So, when you study after the class you should read the relevant sections in both Ch. 4 & Ch. 3.