Electrostatics

Boundary conditions

You will often encounter boundaries between two different media, either between two different dielectrics, or between a dielectric and a conductor. We actually have already touched on this when we calculated the electric field due to a uniformly charged ball. Remember the discontinuity in the $E$ field?

Now let's look at the so-called boundary conditions both in general & in detail.

Let's draw a tiny loop $\int \vec{E} \cdot d\vec{l} = 0$

Define $E_t$ & $E_n$

$E_{2t} \Delta l - E_{1t} \Delta l = 0$

$\Rightarrow E_{2t} = E_{1t}$

If medium 1 is an ideal conductor, $E_1 = 0 \Rightarrow E_{1t} = 0 = E_{2t} = 0$
Now let's have a small pile at the interface.

\[ \oint \mathbf{D} \cdot d\mathbf{S} = \mathcal{Q} \]

\[ (D_{2n} - D_{1n})dS = \rho_s dS \]

\[ D_{2n} - D_{1n} = \rho_s \]

\[ \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \rho_s \]

Special cases:

1. Two dielectrics w/ \( \rho_s = 0 \)
   \[ D_{2n} = D_{1n} \quad \Rightarrow \quad \varepsilon_2 E_{2n} = \varepsilon_1 E_{1n} \]
   This is the reason why we have the discontinuity at the surface of the charged shell.

2. Medium 1 is an ideal conductor.
   \[ E_1 = 0 \quad \Rightarrow \quad E_{1n} = 0 \]
   \[ \Rightarrow \varepsilon_2 E_{2n} = \rho_s \]

Now let's consider an infinitely large sheet of charge w/ a density \( \rho_s \). It's a 2-D sheet, w/o a thickness.

By symmetry, the \( E \) fields on the two sides must be equal, but in opposite directions.

And \( E \) is the same everywhere @ the sheet. And \( E @ the sheet. \)
\[ 2\varepsilon E \Delta S = P_s \Delta S \]

\[ E = \frac{P_s}{2\varepsilon} \]

This is the same as the field at the center of the charged disk when we take \( a \to \infty \).

What about elsewhere, i.e. far away from the sheet?

There's no charge.

Now, let's look at two infinite sheets of opposite charge, \( +P_s \) & \( -P_s \).

Above the positive charge sheet.

\[ E = \frac{P_s}{2\varepsilon} - \frac{P_s}{2\varepsilon} = 0 \]

Between the two.

\[ E = \frac{-P_s}{2\varepsilon} + \frac{-P_s}{2\varepsilon} = -\frac{P_s}{\varepsilon} \]

Below the negative charge sheet.

\[ E = -\frac{P_s}{2\varepsilon} + \frac{P_s}{2\varepsilon} = 0 \]