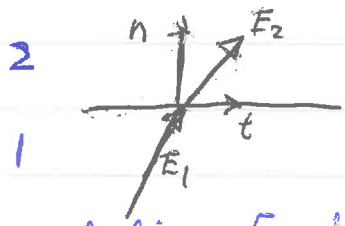
Electrostatics

Boundary conditions

You will often encounter boundries between two different media, either between two different dielectrics, or between a dielectric and a conductor.
We actually have already touched on this when we calculated the electric field due to a uniformly charged ball.
Remember the discontinuity in the $\vec{E}$ field:

Now lets look @ the so called boundary conditions both <u>in general</u> & <u>in detail</u>.



Let's draw a tiny loop

$$\oint \vec{E} \cdot d\vec{l} = 0$$

define E_t & E_n

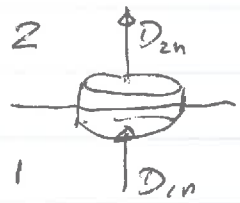
$$E_{2t} \Delta l - E_{1t} \Delta l = 0$$

$$\Rightarrow E_{2t} = E_{1t}$$

If medium 1 is an ideal conductor,

$$E_1 = 0 \Rightarrow E_{1t} = 0 = E_{2t} = 0$$

Now lets have a small pie @ the interface.



$$\oint \vec{D} \cdot d\vec{S} = Q$$

$$(D_{2n} - D_{1n})\Delta S = \rho_s \Delta S$$

$$D_{2n} - D_{1n} = \rho_s$$

$$\varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \rho_s$$

Special cases:

Does not include polarization charge

① Two dielectrics w/ $\rho_s = 0$

$$D_{2n} = D_{1n}, \iff \varepsilon_2 E_{2n} = \varepsilon_1 E_{1n}$$

This is the reason why we have the discontinuity at the surface of the charged ball.

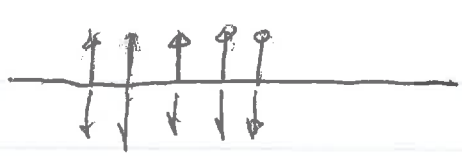
② Medium 1 is an ideal conductor.

$$E_1 = 0 \Rightarrow E_{1n} = 0$$

$$\Rightarrow \boxed{\varepsilon_2 E_{2n} = \rho_s}$$

Talk about dielectric surface. ⊕

Now let's consider an infinitely large sheet of charge, w/ a density ρ_s. It's a 2-D sheet, w/o a thickness.



By symmetry the $\vec{E}$ fields on the two sides must be equal, but in opposite directions.

And $\vec{E}$ is the same every where @ the sheet. And $\vec{E} \perp$ the sheet. coz...

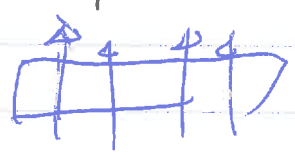
$$2\varepsilon E \, \Delta S = \rho_s \, \Delta S$$

$$\therefore \quad E = \frac{\rho_s}{2\varepsilon}$$

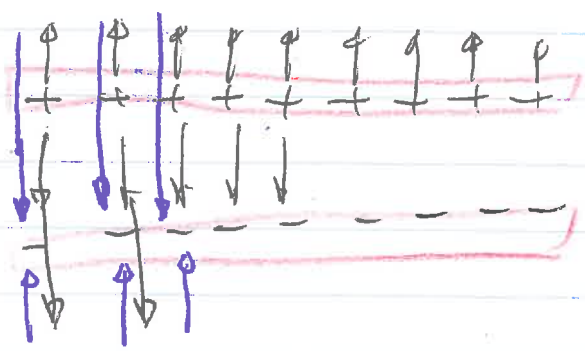
This is the same as the field at the center of the charged disk when we take $a \to \infty$.

What about elsewhere, i.e. far away from the sheet?

There's no charge.



Now, let's look @ two infinite sheets of opposite charge. $+\rho_s$ & $-\rho_s$.



Think:

Above the positive charge sheet.

$$E = \frac{\rho_s}{2\varepsilon} - \frac{\rho_s}{2\varepsilon} = 0$$

Between the two

$$E = \frac{-\rho_s}{2\varepsilon} + \frac{-\rho_s}{2\varepsilon} = -\frac{\rho_s}{\varepsilon}$$

Below the negative charge sheet.

$$E = -\frac{\rho_s}{2\varepsilon} + \frac{\rho_s}{2\varepsilon} = 0$$