Magnetostatics II

Exams (Test 2 & the Final) will not cover the vector potential.
This is an important topic, however, if you have further interest in microwave engineering, antennas, etc.

Read the following notes along with Section 5-4 of the text book on your own (running the extra mile):

Now, another analogy between the $\vec{E}$ & $\vec{B}$ fields.

For the $\vec{E}$ field (static). $\nabla \times \vec{E} = 0$ or $\oint \vec{E} \cdot d\vec{l} = 0$

$\nabla \times (\nabla V) = 0$ the curl of a gradient is 0.

therefore we can define $\nabla \underline{V} = -\vec{E}$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l}$$

$\underline{\vec{E} \text{ is conservative}}$ If its conservative, it must be something's gradient.

For the magnetic field

$$\nabla \cdot \vec{B} = 0$$

We say the $\vec{B}$ field is solenoidal.

Mathematically, we have

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$ The divergence of curl is 0.

The curl of any field has zero divergence.

This can help you remember ~~this~~ the math in a physical context.

$$\vec{J} = \nabla \times \vec{H}$$ Ampere's law

$$\nabla \cdot \vec{J} = 0$$ Kirchhoff's current law.

since $\nabla \cdot \vec{B} = 0$

We can always define some $\vec{A}$ such that $\vec{B} = \nabla \times \vec{A}$.

$\vec{A}$ is called the vector potential.

Notice that for a give $\vec{B}$, $\vec{A}$ is not unique.

for example, if $\vec{B} = \nabla \times \vec{A}$

the $B = \nabla \times (\vec{A} + \vec{A}_0)$ where $\vec{A}_0$ is a constant, since $\nabla \times \vec{A}_0 = 0$

Analogy. $V + V_0$.

(For your interest, read Section 5-4)

Why do we define $\vec{A}$?

$$\nabla \times \vec{H} = \vec{J} \qquad \vec{B} = \mu \vec{H}$$

$$\nabla \times \vec{B} = \mu \vec{J}$$

$$\nabla \times (\nabla \times \vec{A}) = \mu \vec{J}$$

Going thru the math, you'll get

$$\nabla^2 \vec{A} - \nabla(\nabla \cdot \vec{A}) = -\mu \vec{J}$$

I will define what $\nabla^2 \vec{A}$ means in a second.

Recall that the choice of $\vec{A}$ is not unique for $\vec{B} = \nabla \times \vec{A}$ so we can always choos $\vec{A}$ such that $\boxed{\nabla \cdot \vec{A} = 0}$. So,

$$\boxed{\nabla^2 \vec{A} = -\mu \vec{J}}$$

Define ∇^2.

$$\nabla^2 \vec{A} = \hat{x}\, \nabla^2 A_x + \hat{y}\, \nabla^2 A_y + \hat{z}\, \nabla^2 A_z$$

So, the eq. actually means three eqs.

$$\begin{cases} \nabla^2 A_x = -\mu J_x \\ \nabla^2 A_y = -\mu J_y \\ \nabla^2 A_z = -\mu J_z \end{cases}$$

Compare to the Poisson eq.

$$\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon}}$$

These are just formally 3 Poisson eq's

How are these used? You use them to find A_x, A_y & A_z so you have $\vec{A}$, and then $\vec{B} = \nabla \times \vec{A}$.

This is just the same as you use the Poisson eq to find the electric potential V and then $\vec{E} = -\nabla V$. The difference is that the mag field is more complicated you need to solve 3 eq's. That's why the $\vec{A}$ is called the vector potential & V is called the scalar potential.

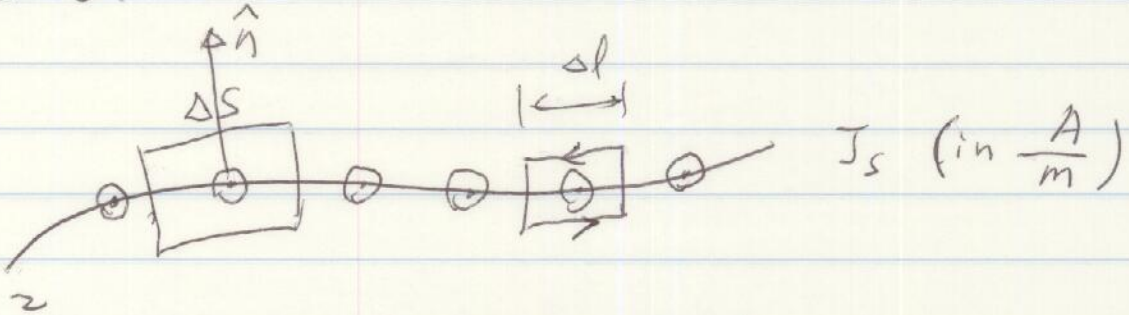
Now let's summarize the methods to find the E & B fields and compare them

Electric	Magnetic
① Coulomb's law $d\vec{E} = \frac{\rho dV}{4\pi\varepsilon R^2}\hat{R}$	① Biot-Savart law $d\vec{H} = \frac{I d\vec{l} \times \hat{R}}{4\pi R^2}$ general
Then take integral to find total $\vec{E}$ or $\vec{H}$	
② For hi-symmetry structures $\oint \vec{D} \cdot d\vec{S} = \int \rho dV = Q$	$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} = I$
You find the trick. Usually high school math is sufficient	
③. Poisson Eq $\nabla^2 V = -\frac{\rho}{\varepsilon}$ then $\vec{E} = -\nabla V$	$\nabla^2 \vec{A} = -\mu \vec{J}$ $\vec{B} = \nabla \times \vec{A}$

$\vec{A}$ is a very important concept in both physics & microwave engineering.
Here, I want you to have this big picture to know the 3 methods of finding the fields.
I also know that very few of you will actually become microwave engineers, or physists.
So, there won't be homework problems about $\vec{A}$, ~~neg~~ neither will it be tested in any exams.

I hope you take good notes and understand the concept.

Now, the last topic in Magnetostatics: the boundary conditions.



$$\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B}_1 \cdot \hat{n}\, \Delta S = \vec{B}_2 \cdot \hat{n}\, \Delta S \Rightarrow B_{1n} = B_{2n}$$

$$\oint \vec{H} \cdot d\vec{l} = I \Rightarrow -H_{1t} \Delta l + H_{2t} \Delta l = J_s \Delta L$$

$$\Rightarrow H_{2t} - H_{1t} = J_s$$

Think about the electrostatic analogy on your own.