Smith Chart for $z \leftrightarrow \Gamma$ Mapping:  
A Recap

• Each point within the $|\Gamma| = 1$ circle represents a $\Gamma$, which uniquely corresponds to a $z = r + jx$

• One family of circles, each the locus of all $\Gamma$ corresponding to a constant $r$

• One family of circles, each the locus of all $\Gamma$ corresponding to a constant $x$

• Moving along the transmission line away from the load (i.e. towards the generator) is equivalent to moving along a constant-SWR circle (i.e. constant-$|\Gamma|$ circle) clockwise

• The chart we use assumes a short at the load, corresponding to $\Gamma = -1 = 1 \angle 180^\circ$, but that doesn’t matter. Take the scales as relative.
The Complete Smith Chart
Black Magic Design
A quick quiz: $Z_0 = 50.2$, $Z_L = 75/2$.
Do you have a min or max $Z$ the load?

$Z_L > 1 \Rightarrow \max \text{ inc. & ref. in phase}$

$Z \rightarrow Y$ Transformation

From experience w/ circuit theory, you may know that sometimes it's more convenient to talk about impedances, while in other cases it's more convenient to talk about admittances.

For example, if two impedances are in parallel: $Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$

When $Z_1$ & $Z_2$ are complex, this is quite cumbersome.
But, if you use admittance:

$Y_1 = \frac{1}{Z_1}, \quad Y_2 = \frac{1}{Z_2}$

Then $Y = Y_1 + Y_2$

Transmission lines can be "branched"
Now, I'm gonna show you how we handle admittance with the Smith Chart.

\[ Y = \frac{1}{Z} \]
\[ Y_0 = \frac{1}{Z_0} \]

\[ \Rightarrow \quad y = \frac{-Y}{Y_0} = \frac{Z_0}{Z} = \frac{1}{3} \]

\[ y = g + jb \]
\[ Y = G + jB \quad \text{susceptance} \]
\[ g = \frac{G}{Y_0} = G \cdot Z_0 \quad b = \frac{B}{Y_0} = B \cdot Z_0 \]

OK, now recall that

\[ z = \frac{1+\Gamma}{1-\Gamma} \]

\[ y = \frac{1}{3} = \frac{1-\Gamma}{1+\Gamma} = \frac{1 + (-\Gamma)}{1 - (-\Gamma)} = \frac{1 + \Gamma e^{j\phi}}{1 - \Gamma e^{-j\phi}} \]

the "-" sign doesn't really matter.

Now, you know how to quickly find \( y \) from \( z \) using the Smith chart.
Get out your chart again.
Locate this point again: \( z = 2 - j \).

Extend this line diametrically and draw the circle (or use the alternative method).
Recall that it's called the constant SWR circle. The SWR is constant because \( |\Gamma| \) is constant.

Locate this point, which is diametrically opposite to \( z \).

The \( \Gamma \) of this point is, of course \( -\Gamma \).

\[ \Gamma = e^{-j\pi} \]

The "3" corresponding to \(-\Gamma\) is \( y \).

\[ y = 0.4 + 0.2j \]

Let's check how accurate we are.

\[ 3y = (2-j)(0.4+0.2j) \]
\[ = 0.8 - 0.4j + 0.4j + 0.2 = 1 \]

Cool!

So, the Smith chart is also a cool tool to find \( y \) from \( z \).
You can also think about this from a different angle.

Round trip $\frac{\lambda}{2}$

Still remember the $\frac{\lambda}{4}$ transformer? Let's look at it now from a different angle.

If $z_2$ is real, $x_2 = 0$.

$3 \left( d = \frac{\lambda}{4} \right)$ is real, too.

$z_{in} = \frac{1}{z_2}$

$3 \left( d = \frac{\lambda}{4} \right)$ is real, too.

$\frac{z_{in}}{z_0} \cdot \frac{z_2}{z_0} = 1$  $\Rightarrow$  $z_{in} \cdot z_2 = z_0^2$

$Z_0 = 50 \Omega$  $\Rightarrow$  $Z_2 = 75 \Omega$

$Z_{02} = \sqrt{50 \times 75} \Omega$
The ¼ transformer is one of the many ways for impedance matching.

In general, we can use a "matching network" to achieve impedance match.

If \( Z_L = R_L \), i.e. \( X_L = 0 \), the matching network is simply a ¼ transformer with \( Z_{02} = \sqrt{Z_{01} R_L} \).

If \( Z_L = R_L + jX_L, \ X_L \neq 0 \),

Recall that \( X(d_{max}) = 0, \ X(d_{min} = 0) \)

The "z chart"
Let's compare these two cases.
when $Z_2 = R_i$, $X_l = 0$, already
you only need to make $R_i$ match to $Z_0$.
So, you only need to "turn one knob,"
which is $Z_{o2}$.

In the more general case, $Z_2 = R_i + jX_l$, $X_l \neq 0$
you need to make it purely resistive. & the
resistive part to match $Z_0$.
So, you need "turn two knobs".
Here, one knob is the distance from the load
to the $\lambda/4$ transformer,
the 2nd knob is $Z_{o2}$ of the $\lambda/4$ transformer.
You can find two other knobs to turn.

Multiple reflection inside the matching network, as in the
anti-reflection coating.