Propagation of Pulses on Transmission Lines

So far we have been talking about single-frequency (harmonic) “signals”, which carry no information.

To transmit information, we must modulate the signal.

For example, we modulate a microwave carrier to send a bit down the line.

This envelope travels along the line at speed \( v_g \), the “group velocity,” which is usually a little different from \( v_p \), due to dispersion.
For simplicity, we ignore dispersion and assume \( v_g = v_p \).

If \( Z_L = Z_0 \), this pulse is totally absorbed upon arrival at the load. This is what we want.

For a lossless line, \( Z_0 \) is real.

If \( Z_L \) is purely resistive, this match is (assumed to be) frequency-independent.
If $Z_L \neq Z_0 \neq Z_g$, things become complicated.

We first look at the case, $\tau < l/v_p$:

Lots of echoes. Echoes die off.
First case, $\tau < l/v_p$:

Lots of echoes. Echoes die off.
May corrupt other bits

Second case, $\tau > l/v_p$,
i.e., $l < v_p\tau$:

The bit is distorted and broadened.
Recall that we always have multiple reflections inside any matching network.

Does impedance matching really help us? Why?

\[ y(d) = 1 + 1.58j \]

\[ d = 0.063\lambda \]

\[ z_L = 0.5 - j \]

\[ y_L = 0.4 + 0.8j \]

\[ l = 0.09\lambda \]

Single stub matching example
Recall that we always have multiple reflections inside any matching network.

**Does impedance matching really help us? Why?**

Notice that we can always choose to have $d < \lambda/2$ and $l < \lambda/2$.

The time to travel $\lambda/2$ is $1/(2f)$.

Thus the bit is broadened only by several $1/(2f)$, at most.

Without matching, we have echoes.

The modulated case is quite complicated. We now look into a simple case quantitatively.
First, let’s list the basic assumptions to be used:

1. Lossless line. $Z_0$ is purely real.
3. Therefore, $\Gamma$ is frequency-independent.
   
   (If $R_L = Z_0$, impedance matched for all frequencies)
4. Dispersionless: $v_g = v_p$ for all frequencies.

Know the simplifying assumptions. Know the limitations.
Propagation of a voltage step on a transmission line

For $0 < t < T = l/v_p$,

The “turn-on” event has not reached the load yet. It does not know about $R_L$.

The transmission line feels like infinitely long. In other words, no reflection yet.

What is the equivalent input impedance at $z = 0$?
Propagation of a voltage step on a transmission line

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The “turn-on” event has not reached the load yet. It does not know about $R_L$. The transmission line feels like infinitely long. In other words, no reflection yet.

The equivalent input impedance at $z = 0$ is $Z_0$.

Subscript “1” means the first round trip. Superscript “+” means the incident direction.
Propagation of a voltage step on a transmission line

The equivalent input impedance at $z = 0$ is $Z_0$.

Snapshots at $t = T/2$

The leading edge reaches the load at $t = T$. Reflection.

Edge/front moving at $v_p$ (actually $v_g$)

$$V_i^+ = \frac{V_g Z_0}{R_g + Z_0}$$

$$I_i^+ = \frac{V_g}{R_g + Z_0}$$

$$i(z, T/2)$$

$$V_i^- = \Gamma_l V_i^+$$

$$I_i^- = \frac{-\Gamma_l I_i^+}{R_L + Z_0}$$

What is the voltage at the load at $t = T$?
What is the voltage at the load at $t = T$?

Snapshots at $t = 3T/2$

Snapshots at $t = 5T/2$

At $t = 2T$, the front hits the source. Reflection.

$$V_2^+ = \Gamma_g \cdot V_1^- = \Gamma_g \cdot \Gamma_L \cdot V_1^+$$

$$I_2^+ = -\Gamma_g \cdot I_1^- = \Gamma_g \cdot \Gamma_L \cdot I_1^+$$

Assuming

$$\Gamma_g = \frac{R_g - Z_o}{R_g + Z_o}$$

$$\Gamma_L > 0$$

$$\Gamma_g > 0$$
At \( t = 3T \), the front hits the load again.

\[
V_2^- + V_2^+ = V_2^+ (1 + \Gamma_L)
\]
\[
I_2^- + I_2^+ = I_2^- (1 - \Gamma_L)
\]

Again, notice the sign.

Again, notice that reflection happens instantaneously.

It goes on and on. For the \( i \)th round trip,

\[
V_i^+ + V_i^- = V_i^+ (1 + \Gamma_L)
\]
\[
I_i^+ + I_i^- = I_i^+ (1 - \Gamma_L)
\]

\[
V(\tau = \infty) = V_1^+ + V_1^- + V_2^+ + V_2^- + \ldots
\]
\[
= \sum_{i=1}^{\infty} (V_i^+ + V_i^-)
\]
\[
= V_1^+ (1 + \Gamma_L) + V_2^+ (1 + \Gamma_L) + \ldots = (1 + \Gamma_L) \left[ V_1^+ + V_2^+ + \ldots \right]
\]
\[
= (1 + \Gamma_L) \sum_{i=1}^{\infty} V_i^+
\]

Note: At the steady state, \( v \) is the same at all \( z \), therefore we do not specify \( z \).
We get:

\[ u(t = \infty) = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \ldots \]

\[ = \sum_{i=1}^{\infty} (V_i^+ + V_i^-) \]

\[ = V_1^+ (1 + \Gamma_L) + V_2^+ (1 + \Gamma_L) + \ldots = (1 + \Gamma_L) \left[ V_1^+ + V_2^+ + \ldots \right] \]

\[ = (1 + \Gamma_L) \sum_{i=1}^{\infty} V_i^+ \]

\[ V_2^+ = \Gamma_g \Gamma_L V_1^+ \]

\[ V_{i+1}^+ = \Gamma_g \Gamma_L V_i^+ \]

\[ u(t = \infty) = V_1^+ (1 + \Gamma_L) \left[ 1 + \frac{\Gamma_g \Gamma_L}{\gamma g g L} + \frac{(\Gamma_g \Gamma_L)^2}{(1 + \Gamma_L)} + \ldots \right] \]

\[ = V_1^+ (1 + \Gamma_L) \sum_{i=0}^{\infty} \left( \frac{\Gamma_g \Gamma_L}{\gamma g g L} \right)^i \]

Use

\[ 1 + x + x^2 + \ldots = \sum_{i=0}^{\infty} x^i = \frac{1}{1 - x}, \]

with \( \chi = \Gamma_g \Gamma_L \)

We get:

\[ u(t = \infty) = V_1^+ (1 + \Gamma_L) \cdot \frac{1}{1 - \Gamma_g \Gamma_L} \]
Surprising?

Similarly,

\[
\begin{align*}
\dot{v}(t = \infty) &= V_i^+ (1 - \Gamma_L) \sum_{i=0}^{\infty} (\Gamma_g \Gamma_L)^i \\
&= I_i^+ \left( \frac{l - \Gamma_L}{l - \Gamma_g \Gamma_L} \right) \\
&= \frac{V_g}{R_g + R_L}
\end{align*}
\]

We have traced \( v(t) \) and \( i(t) \) all the way to \( t = \infty \). That’s quite tedious.

We have a graphical tool to trace this bouncing back and forth.

It’s called the bounce diagram.

Review textbook Section 2-12 overview, Section 2-12.1