

The Image Method

We can calculate the field due to a chunk of charge by

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{|\vec{R} - \vec{R}'|^2} \frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|} dV$$



What if we place an infinitely large plate of conductor under it?

The charged body will induce charge in the conductor plate.

The total field is the sum of the field due to the charged body and that of the induced charge.

But it's extremely difficult to determine the induced charge.

We could solve the Poisson's equation with the boundary conditions.

What equation?

What are the boundary conditions?

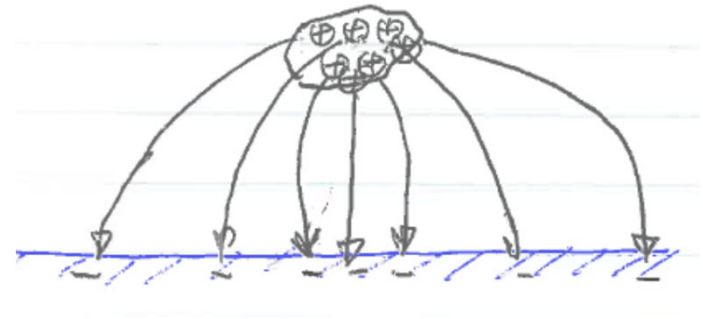
A charged body over an infinitely large conductor plate

The charged body will induce charge in the plate.

The total field is the sum of the field due to the charged body and that of the induced charge.

But it's extremely difficult to determine the induced charge.

We could solve the Poisson's equation with the boundary conditions.



$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon}$$

Boundary conditions:

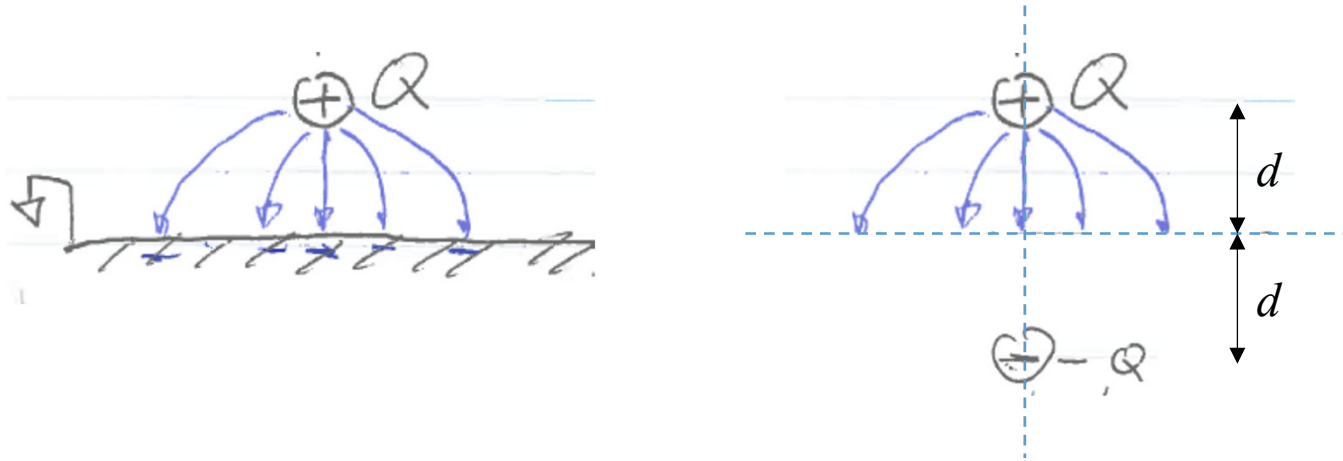
$$V(\infty) = 0$$

Let the plane of the upper surface of the plate be the x - y plane where $z = 0$.

$$V(x,y,0) = 0$$

But there is an easier way.

Let's first consider a point charge over a perfect conductor plate.



Intuitively, the field in the upper half of space is the same for the two situations.

This is correct. And, it's mathematically justified:

Same differential equation and same boundary conditions, then same solution.

“Uniqueness of solution.”

Boundary conditions: $V(\infty) = 0$, $V(x,y,0) = 0$

Now you just make sure you know how to find the field in the right figure.

If you have more than one point charge, or a continuous charge body, just use superposition...

Review textbook Section 4-11. Do Problem 7 of Homework 9.

Reminder: Finish reviewing Chapter 3 and finish Homework 8.

Parts 1 through 6 of the Project posted online.