

Dimensions & Units

Dimension: The type of a physical quantity, e.g. length, time, temperature.

We use the SI system, in which there are 6 fundamental dimensions.

Kind of arbitrarily defined.

Time and current are among them.

Therefore, charge = current × time is “derived”.

If time and charge had been defined fundamental, then current = charge / time would be derived.

A number and a unit together make the value of a physical quantity.

In physics and engineering, we should always check units (although this sanity check does not guarantee an equation’s correctness).

$$E = \frac{q}{4\pi\epsilon_r\epsilon_0 R^2}$$

$$\frac{C}{\frac{C}{Vm} \cdot m^2} = \frac{V}{m}$$

$$q - C; \quad \epsilon_0 - \frac{F}{m} = \frac{C}{Vm}; \quad R - m$$

coulomb
meter
volt

Field is the gradient of potential, therefore V/m.

$$E = \frac{q}{4\pi\epsilon_r\epsilon_0 R^2}$$

$$q \text{ --- coulomb} ; \epsilon_0 \text{ --- } \frac{\text{farad}}{\text{meter}} = \frac{\text{C}}{\text{Vm}} ; R \text{ --- m}$$

$$\frac{\text{C}}{\frac{\text{C}}{\text{Vm}} \cdot \text{m}^2} = \frac{\text{V}}{\text{m}}$$

Field is the gradient of potential, therefore V/m.

On the other hand, if we place a probe charge q' in this field, q' will feel a force

$$F = q'E \Rightarrow E = \frac{F}{q'}$$

$$\frac{\text{N}}{\text{C}} \leftarrow \begin{array}{l} \text{newton} \\ \text{coulomb} \end{array}$$

Can you prove that $\text{N/C} = \text{V/m}$?

Recall that $W = F \cdot l$

$$J = \text{Nm} \leftarrow \text{joule}$$

If the force is an electrostatic one moving a charge Q , and the potential difference over l is V , then

$$W = Q \cdot V$$

$$J = \text{C} \cdot \text{V} \leftarrow \begin{array}{l} \text{coulomb} \\ \text{volt} \end{array}$$

You do the rest.

Review textbook Section 1-2.