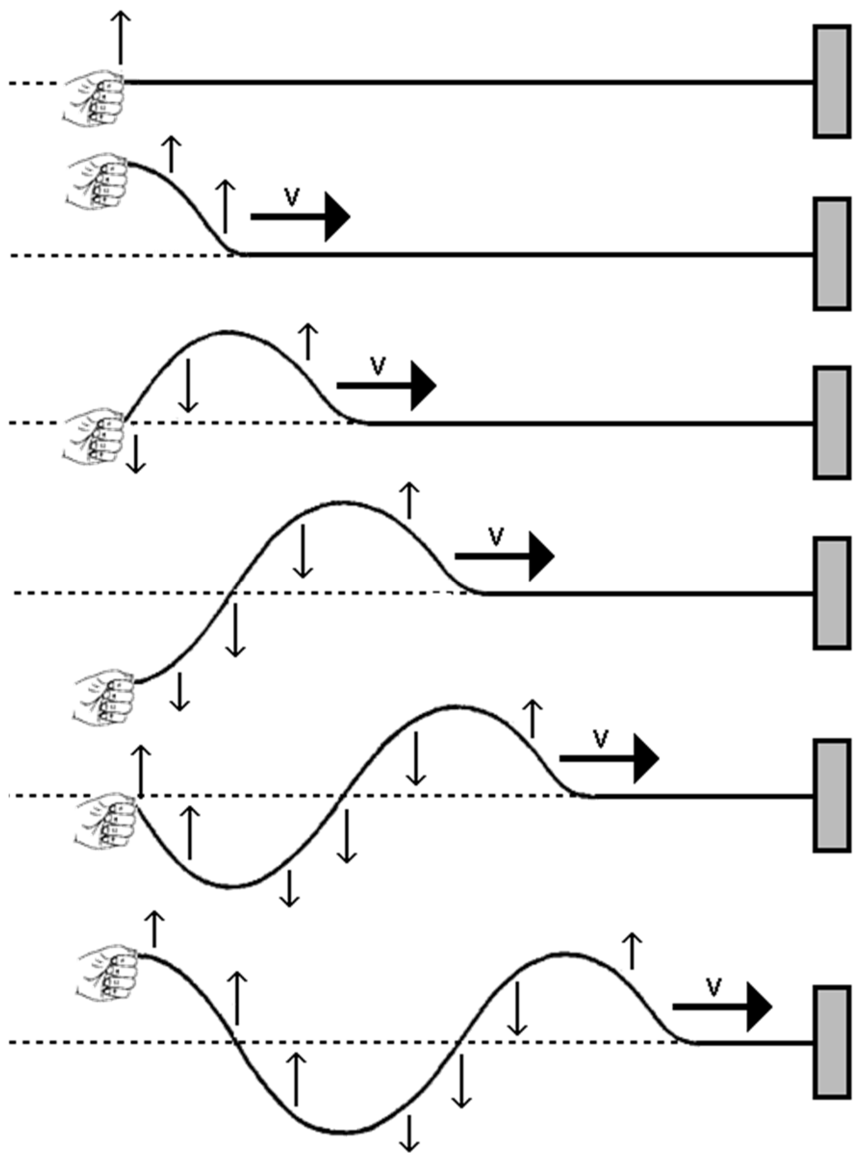
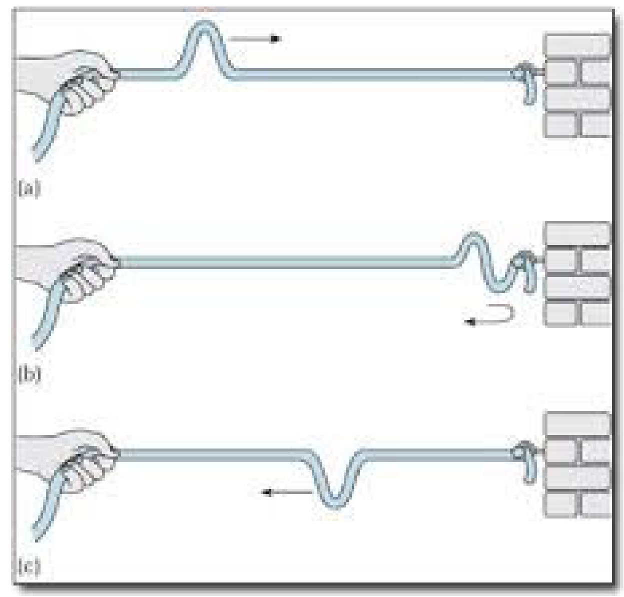
Traveling Waves

The one-dimensional (1D) case



The "perturbation" propagates on.

A traveling wave is the propagation of motion (disturbance) in a medium.

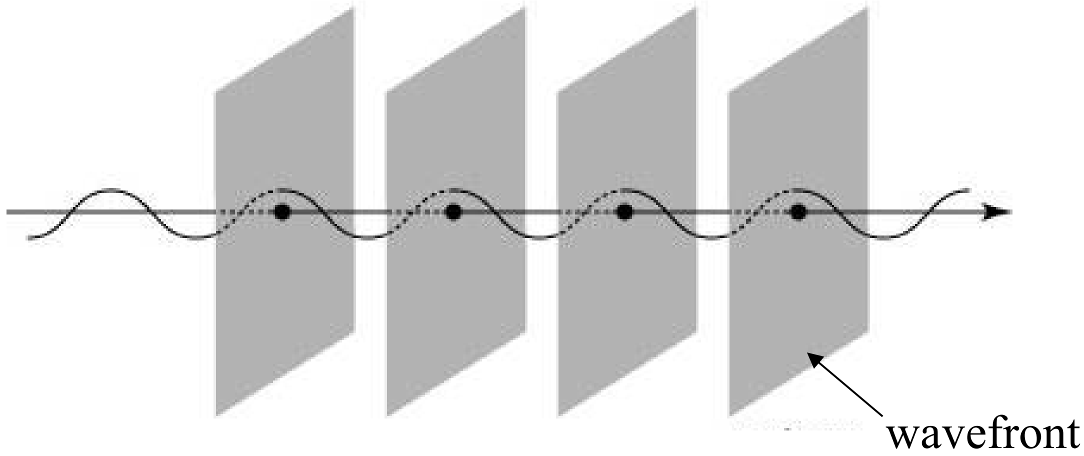


Reflection

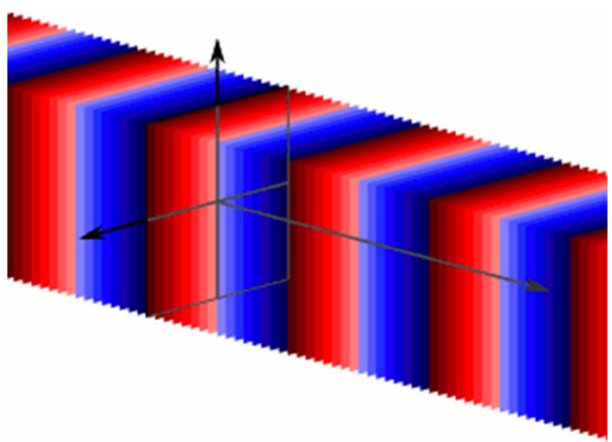
Why is there reflection?

Traveling Wave in Higher Dimensions

Plane waves in 3D



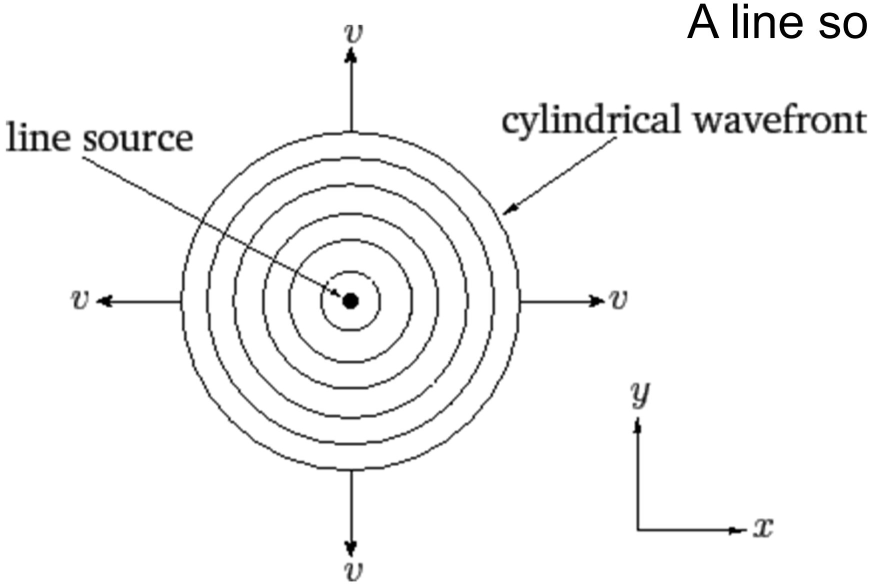
Example: sound waves



The plane wave is a 1D wave in 3D space:
No variation in the 2D plane of a wave front.
A wave front is a surface of equal phase.

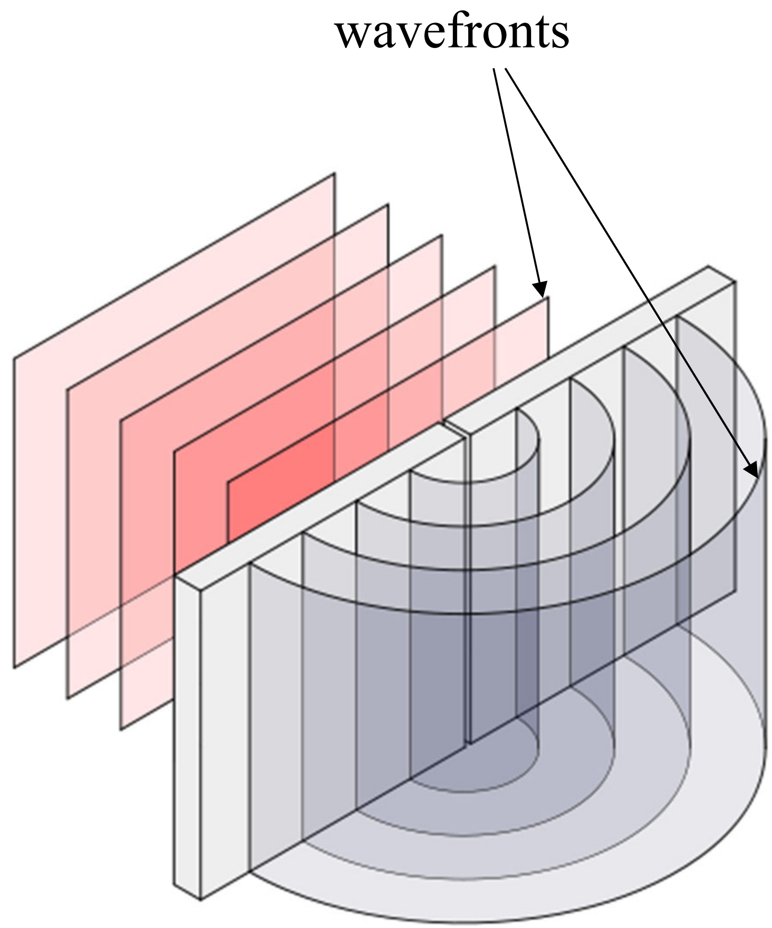
Watch animation: http://en.wikipedia.org/wiki/Plane_wave

A line source makes a cylindrical wave.



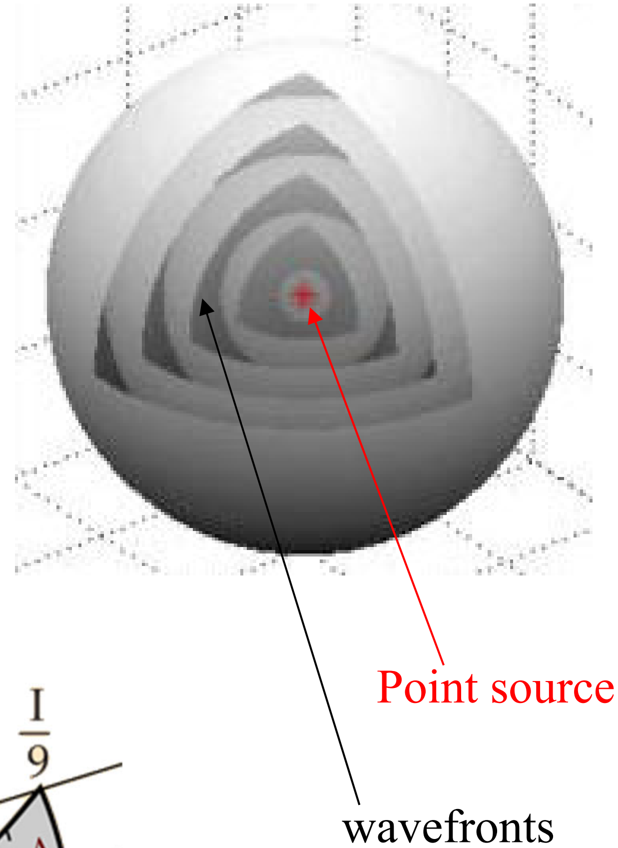
Cylindrical wave (3D; top view)

Water surface wave (2D)
(Circular wave)

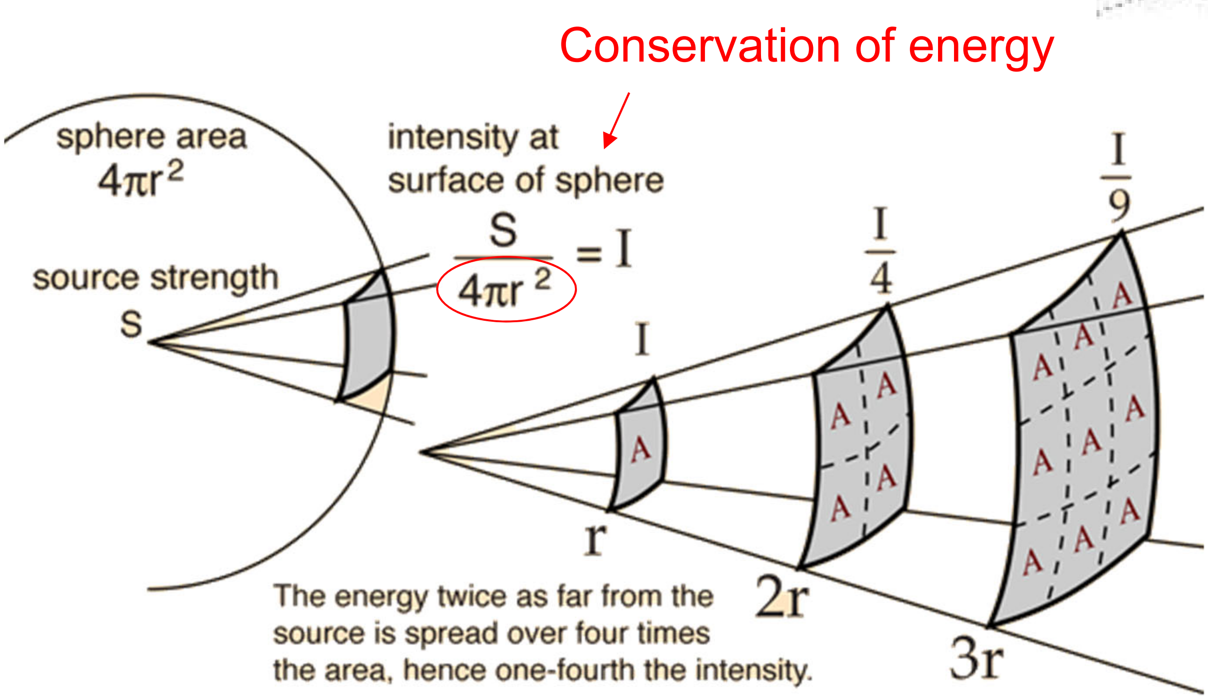


Make a cylindrical wave from a plane wave

A point source makes a spherical wave, the wave front of which are spherical surfaces.



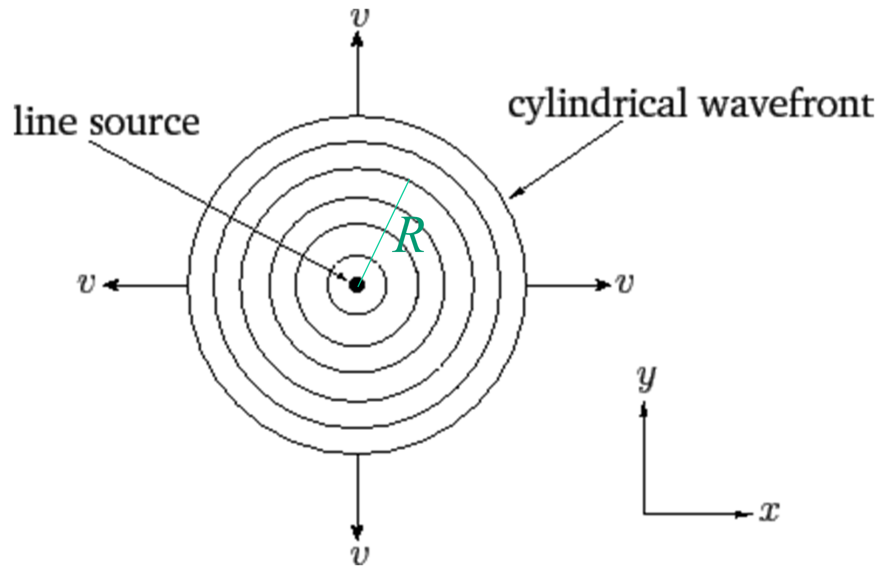
Intensity is energy carried per time per area, i.e., power delivered per area



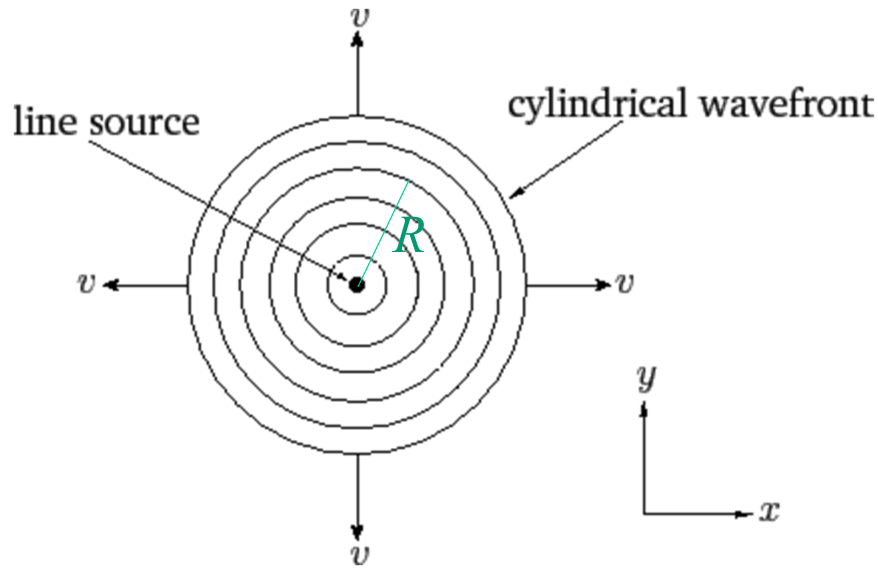
$$\text{Intensity} \propto 1/r^2$$
$$\text{Amplitude} \propto 1/r$$

How does the intensity & amplitude of a cylindrical wave depend on R?

$$R^2 = x^2 + y^2$$



How does the intensity & amplitude of a cylindrical wave depend on R?



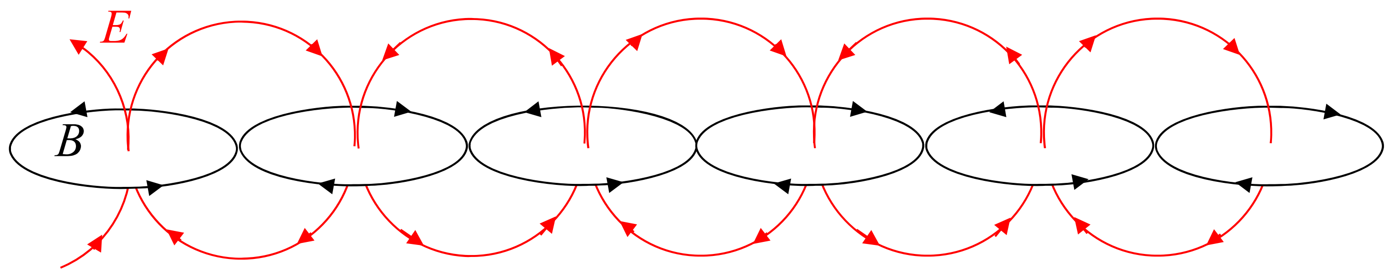
$$R^2 = x^2 + y^2$$

Circumference $\propto R$

Therefore,

$$\boxed{\begin{array}{l}\text{Intensity} \propto 1/R \\ \text{Amplitude} \propto 1/(R^{1/2})\end{array}}$$

Electromagnetic Wave



Somehow start with a changing electric field E, say $E \propto \sin\omega t$

The changing electric field induces a magnetic field, $B \propto \dfrac{\partial E}{\partial t} \propto \cos\omega t$

If the induced magnetic field is changing with time, it will in turn induce an electric field

$$E \propto -\frac{\partial B}{\partial t} \propto \sin\omega t$$

$\boxed{\text{Notice that } \dfrac{d}{dt}\cos\omega t = -\omega\sin\omega t}$

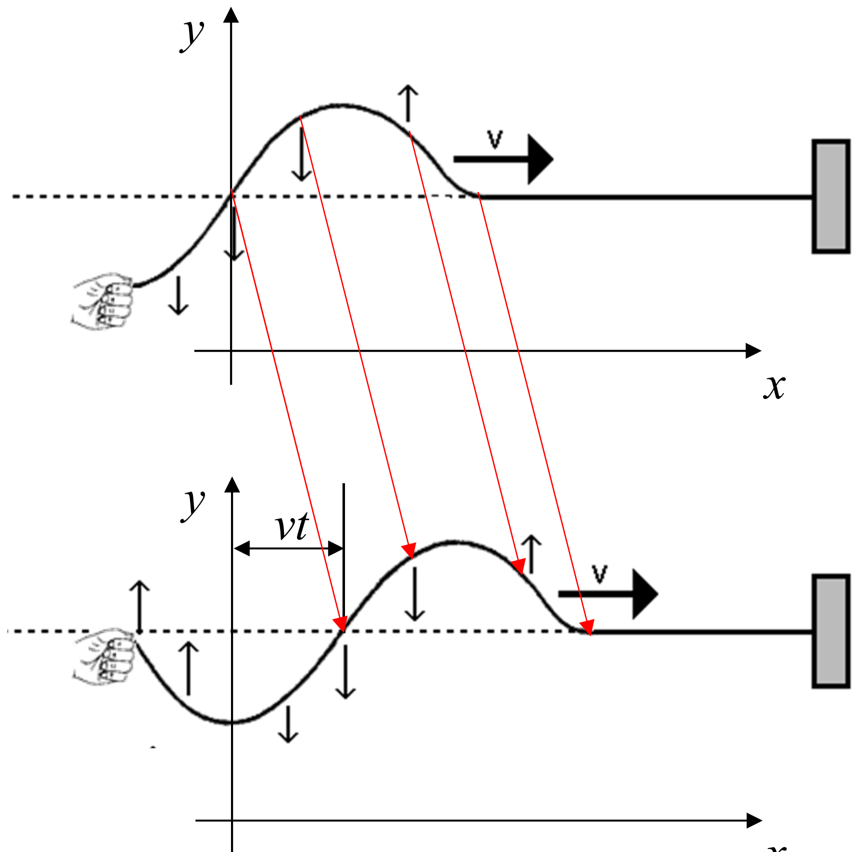
Negative signs cancel

And so on and so on....
Just as the mechanical wave on a string.

Note: The picture is NOT of a plane wave, but depicts a wave emitted by a source of limited size. Will revisit this picture later.

Mathematical Expression of the Traveling Wave

A traveling wave is the propagation of motion (disturbance) in a medium.



At time 0,
$y = f(x)$

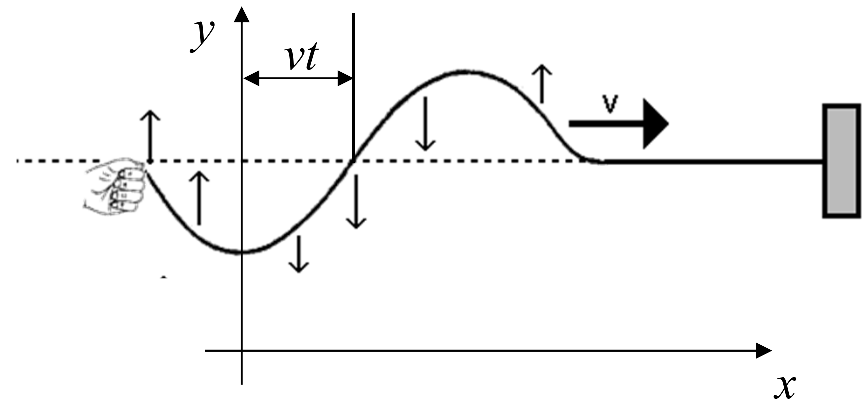
At time t,
$y = f(x-vt)$

The wave form shifts vt in time t.

This is the general expression of traveling waves.

Questions:
What kind of wave does $y = f(x+vt)$ stand for?
What about $y = f(vt-x)$?

What about $y = f(vt - x)$?



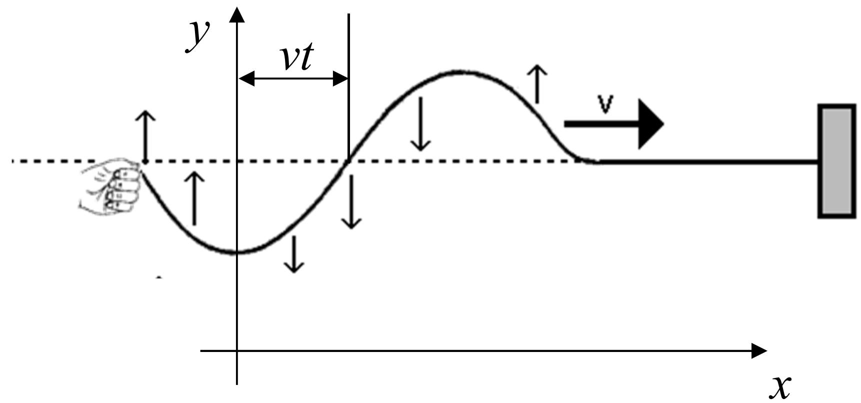
$$f(vt - x) = f[-(x - vt)]$$

Define $f(-x)$ your "new f", or $g(x) \equiv f(-x)$, so it's the same wave!

So, which way should I go? $f(x-vt)$ or $g(vt-x)$?
To your convenience!
No big deal. But this affects how we define our "sign conventions."
People in different disciplines use different conventions.

> If you are more concerned about seeing a waveform on an oscilloscope, you like $g(vt-x)$ better.
> If you are more concerned about the spatial distribution of things, you like $f(x-vt)$ better.
> We will talk later about how this choice affects the ways to write the "same" (but apparently different) equations in different disciplines (EE vs. physics).

What about $y = f(vt - x)$?



$f(vt-x)$
$=f[v(t-x/v)]$

At any x, you have a time-delayed version of $f(vt)$.
The time delay is simply the time for the wave to travel a distance x, i.e., x/v.

For a single-wavelength, sinusoidal wave, this is always true.
Because the wave travels at just one speed, v.

But, a general wave has components of different wavelengths/frequencies.
The speeds of the different components may be different.

I am already talking about an important concept.

Then, a distance x later, the "waveform" in time (as you see with an oscilloscope) will change.
This is called "dispersion"

Let's now look at the special case of the sinusoidal wave

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

ω: angular frequency

β: propagation constant

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right]$$

Phase as a function of x

$$= A\cos\left[\beta(v t - x) + \phi_0\right]$$

$$= f(x - vt) = g(vt - x)$$

You can write this function, or group the terms, in so many ways.
It's just about how you view them

Watch Wikipedia animation:
https://en.wikipedia.org/wiki/Wave#/media/File:Simple_harmonic_motion_animation.gif

Next, we try to understand β

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right]$$

$$= A\cos[\beta(vt - x) + \phi_0]$$

$$= f(x - vt) = g(vt - x)$$

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

$$= \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T}$$

One wavelength traveled in one period.

$\boxed{\lambda \text{ is the "spatial period"; } \frac{1}{\lambda} \text{ is the "spatial frequency".}}$

And, β is the spatial equivalent of ω.
Call it the wave vector or propagation constant.

For the free space (i.e. vacuum), $v = c = \frac{\omega}{\beta}$, or $\omega = c\beta$.

No dispersion.
(c is a constant)

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right]$$

$$= A\cos\left[\beta(vt - x) + \phi_0\right]$$

$$= f(x - vt) = g(vt - x)$$

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

$$= \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T}$$

One wavelength traveled in one period.

For the free space (i.e. vacuum), $v = c = \frac{\omega}{\beta}$, or $\omega = c\beta$.

No dispersion (c is a constant)

The relation between β and ω for a wave traveling in a medium is a material property of the medium. We call it the "dispersion relation" or just "dispersion."

Recall that we used the term to describe a phenomenon. Related.

In general the dispersion relation is not perfectly linear.

$\frac{\omega}{\beta} \equiv v$ is not a constant. We call it the phase velocity, v_p.

Thus the dispersion!

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right]$$

$$= A\cos\left[\beta(v t - x) + \phi_0\right]$$

ϕ_0 is the reference phase (the wave's phase with time and space set to zero)

$$y(x,t) = A\cos\left[\omega\left(t + \frac{\phi_0}{\omega}\right) - \beta x\right]$$ ϕ_0 viewed as shift in time

$$= A\cos\left[\omega t - \beta\left(x - \frac{\phi_0}{\beta}\right)\right]$$ ϕ_0 viewed as shift in position

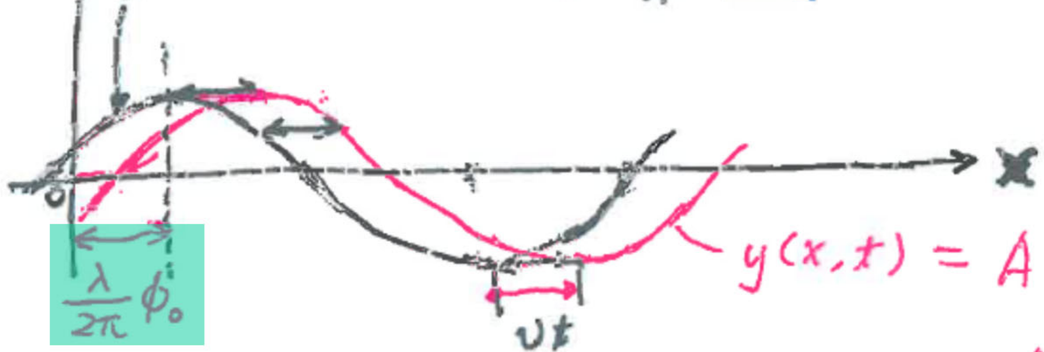
Two ways to look at this.
Two ways to group the terms.

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0) = A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \phi_0\right)$$

"Spatial period"

$$y(x, t=0) = A\cos\left(-\frac{2\pi}{\lambda}x + \phi_0\right)$$

Take snapshots at different times



Convert phase to distance

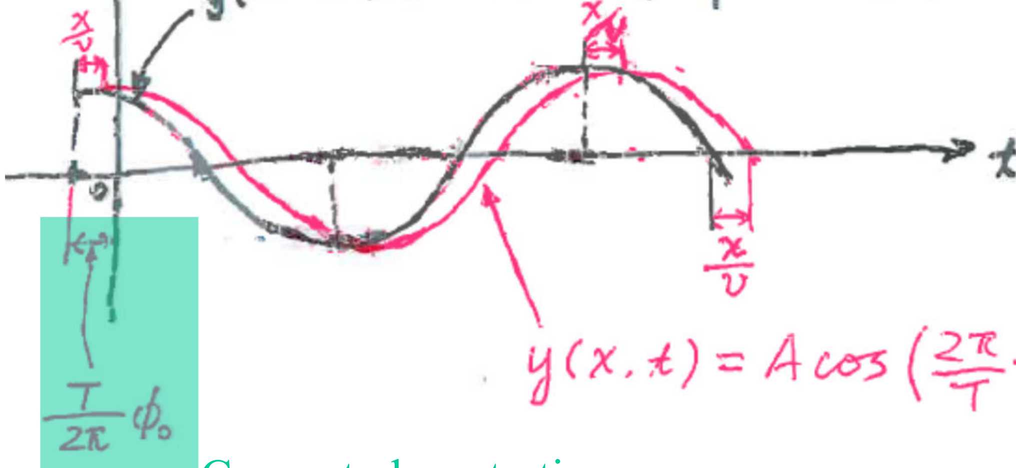
$$y(x,t) = A\cos\left[\left(-\frac{2\pi}{\lambda}x + \phi_0\right) + \frac{2\pi}{T}t\right]$$

$$= A\cos\left[-\frac{2\pi}{\lambda}x + \left(\phi_0 + \frac{2\pi}{T}t\right)\right]$$

For the same wave,

$$y(x=0, t) = A\cos\left(\frac{2\pi}{T}t + \phi_0\right)$$

Measure waveforms at different locations



$$y(x,t) = A\cos\left(\frac{2\pi}{T}t + \phi_0 - \frac{2\pi}{\lambda}x\right) = A\cos\left[\frac{2\pi}{T}t + \left(\phi_0 - \frac{2\pi}{\lambda}x\right)\right]$$

Convert phase to time

Now, you can work on HW1, P1 – P7.

Both the Homework & Answer sheet are online.
Review class notes and read the textbook, **then** do the homework, **then** check answers.

Waves carry information.

How much information does a sinusoidal wave carry?

Why do we study sinusoidal waves?

We want the wave to carry the "undistorted" information after it travels a distance x to reach us.

We want its snapshots in space to be the same as at $t = 0$.
We want its waveform in time to be the same as at $x = 0$.

Recall that we talked about dispersion.

In most cases, the dispersion is not too bad.
The $\omega(\beta)$ is only slightly nonlinear.

The "signal" or "wave packet" or "envelope" travels at a different speed than v_p, which is different for different frequencies anyway.
That speed is the "group velocity" v_g.

Run the extra mile:
Find out the expression for v_g, given the dispersion $\omega(\beta)$. Derive it.
You'll have a deep understanding about wave propagation.

Attenuation

In some cases, the amplitude A decreases as the wave propagates.

For many types of waves, the power density $\propto A^2$

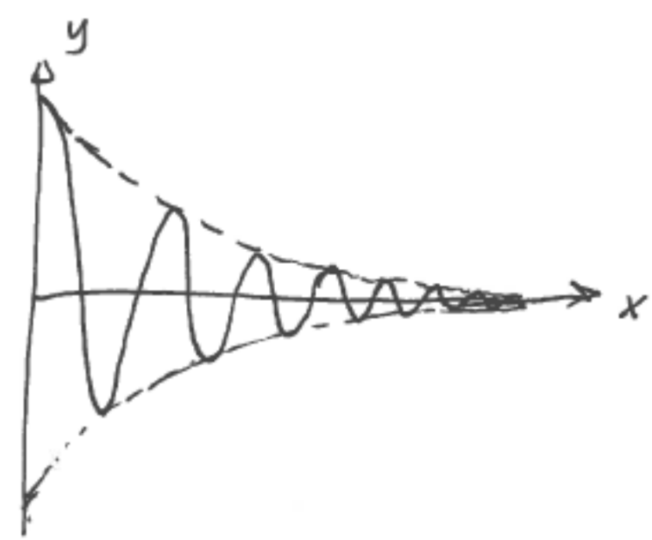
For unit distance traveled, a certain fraction of power density is lost:

$$\frac{d A^2(x)}{A^2(x)} = -2\alpha \, dx$$

$$\frac{d A^2(x)}{dx} = -2\alpha A^2(x)$$

$$A^2(x) = A^2(0) e^{-2\alpha x}$$

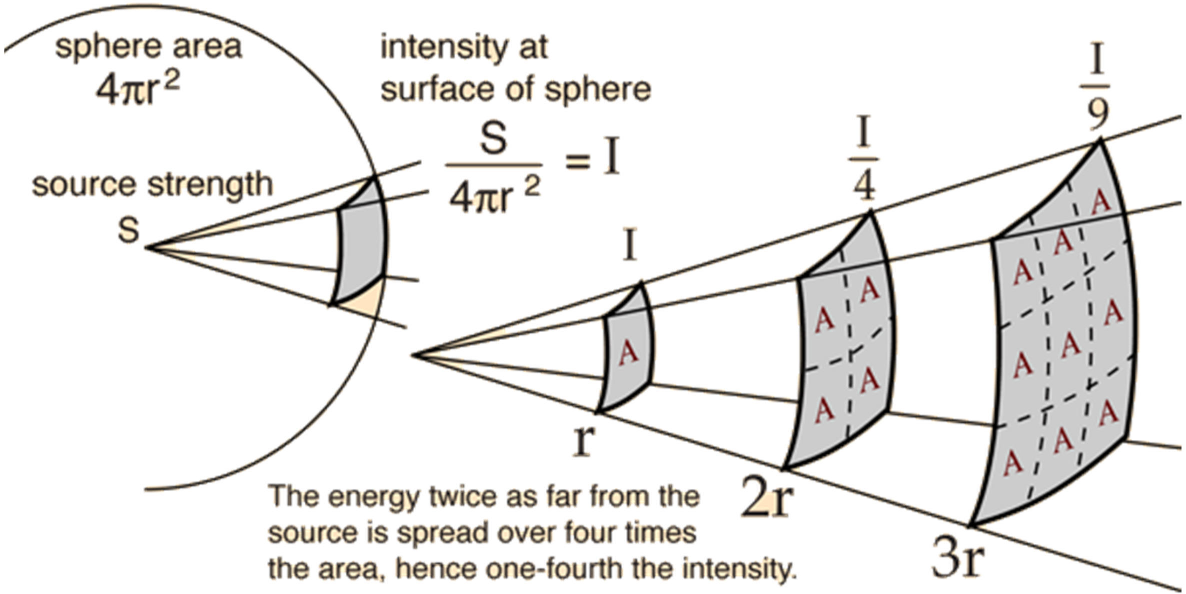
$$A(x) = A(0) e^{-\alpha x}$$



$$y(x,t) = A_0 e^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$$

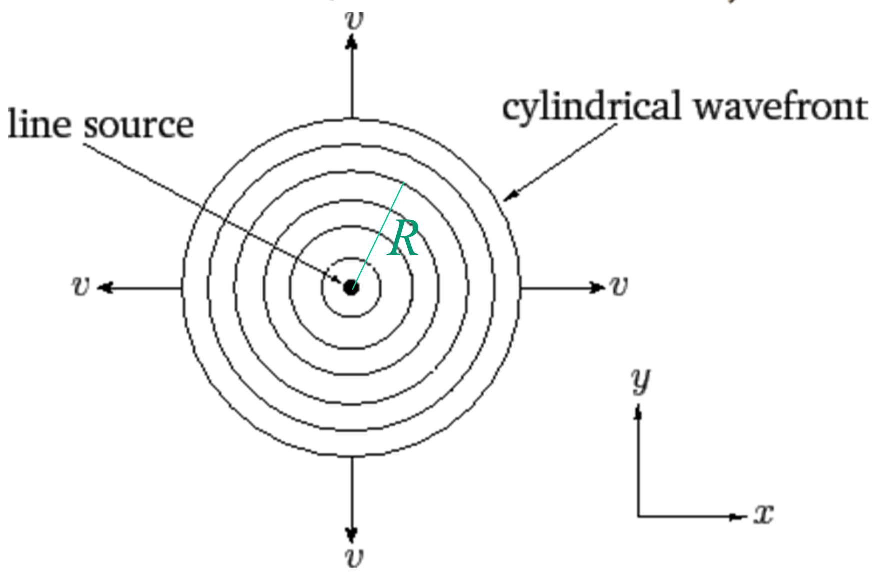
$$y(x,t) = A_0 e^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$$

The exponential attenuation of the plane wave is due to loss, not to be confused with the amplitude reduction due to “spreading”.



$$y(r,t) = A_0(1/r)\cos(\omega t + \beta r + \phi_0)$$

Intensity $\propto 1/r^2$
Amplitude $\propto 1/r$



$$y(r,t) = A_0(1/R^{1/2})\cos(\omega t + \beta R + \phi_0)$$

Intensity $\propto 1/R$
Amplitude $\propto 1/(R^{1/2})$

Just to mention another mechanism that weakens a propagating wave: scattering

Finish HW1:P1 through P9.

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Review class notes and read the textbook, **then** do the homework, **then** check answers.