ECE 341 Notes

Chapter 2. Transmission Lines

Power Flows

We talked about power flow on a transmission line, when we discussed impedance match, reflection, etc. ....

Now let's look @ power flow quantitatively

We have used phasors for convenience.
But let's not forget the real world quantities,

$$v(d,t) = \mathrm{Re}\left(\tilde{V}(d)\, e^{j\omega t}\right)$$

$\tilde{V}(d)$: function of d

For the incident wave

$$v_{inc}(d,t) = \mathrm{Re}\left(V_0^+ e^{j\beta d} e^{j\omega t}\right)$$

V_0^+: a complex number

$$= \mathrm{Re}\left(|V_0^+| e^{j(\omega t + \beta d + \phi^+)}\right)$$

$$V_0^+ = |V_0^+| e^{j\phi^+}$$

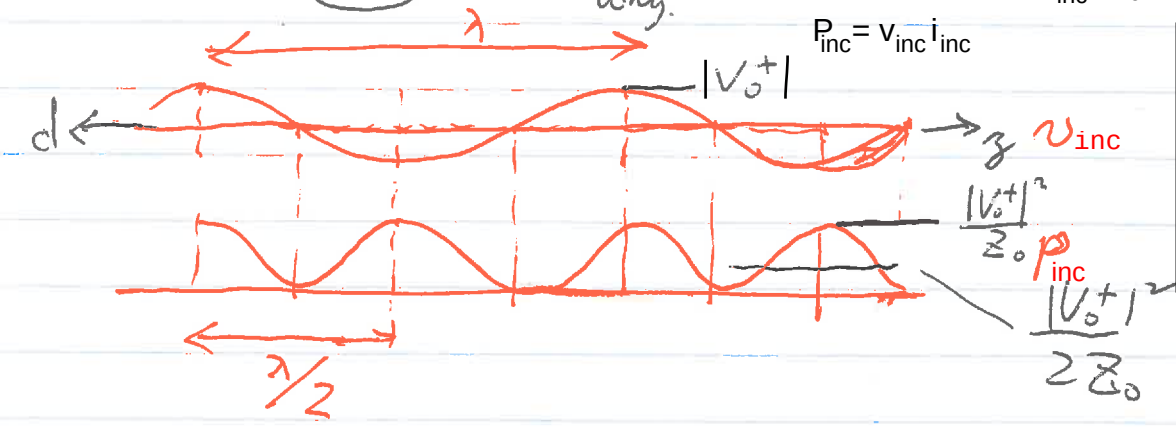
$$= |V_0^+| \cos(\omega t + \beta d + \phi^+)$$

$$P_{inc} = \frac{v_{inc}^2}{Z_0} = \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t + \beta d + \phi^+)$$

Z_0 — talk about why.

Incident wave only, $v_{inc}/i_{inc} = Z_0$

$P_{inc} = v_{inc}\, i_{inc}$



Talk about equivalent impedances seen by incident, reflected, and combined waves.

For the reflected wave.

$$v_{ref}(d,t) = \mathrm{Re}\left(\Gamma V_0^+ e^{-j\beta d} e^{j\omega t}\right)$$

$$= \mathrm{Re}\left[|\Gamma|\,|V_0^+|\, e^{j(\omega t - \beta d + \phi^+ + \theta_r)}\right]$$

$$= |\Gamma|\,|V_0^+| \cos(\omega t - \beta d + \phi^+ + \theta_r)$$

Reflected

$$P_{ref} = -\frac{V_{ref}^2}{Z_0} = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t - \beta d - \phi^+ + \theta_r)$$

Remember, we are talking about a lossless line. Z_0 is real.

Instantaneous $P_{inc}(d,t) > 0, \quad P_{ref}(d,t) < 0$

$$P_{inc,av} = \frac{1}{T}\int_0^T P_{inc}(d,t)\,dt = \frac{|V_0^+|^2}{2Z_0}$$

Talk about this factor of 2

$$P_{ref,av} = \ldots = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}$$

$$P_{av} = P_{inc,av} + P_{ref,av} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma|^2)$$

An easier way:

$$\boxed{P_{av} = \frac{1}{2}\,\mathrm{Re}(\tilde{V}\,\tilde{I}^*)}$$

Put $\tilde{V}$ & $\tilde{I}^*$ in, and do the algebra. you will get

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma|^2)$$

$|\Gamma|^2 = 1 \quad P_{av} = 0$! No power flow

standing wave. Reactive Load!

$$|\Gamma| = 0 \quad \Rightarrow \quad P_{av} = \frac{|V_0^+|^2}{2Z_0}$$

what does this mean?

Actually. $$P_{av} = P_{inc,av} = \frac{|V_0^+|^2}{2Z_0}$$

$$P_{ref} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = 0$$

actually these eq's are very intuitive. you don't need to go thru the painful math.

$|\Gamma| = 0$ means perfect impedance match.

— All traveling wave.