Power flow along transmission lines

At any distance $d$, we can use an equivalent impedance $Z(d)$ to represent the stuff to the right of $d$.

Question: What is the impedance “felt” by the incident wave at $d$?

Let’s now have a quantitative look at the energy flowing along the transmission line.

Let’s first do this with the “real quantities” instead of phasors.

$$v(d, t) = \text{Re} \left( \tilde{V}(d) e^{j\omega t} \right)$$

We forgo the convenience for the moment, in order not to forget the real world quantities. The incident wave

$$v_{\text{inc}}(d, t) = \text{Re} \left( \frac{V_0^+ e^{j\phi^+}}{a \text{ complex number}} e^{j\omega t} \right)$$

$$= \text{Re} \left( |V_0^+| e^{j(\omega t + \beta d + \phi^+)} \right)$$

$$= |V_0^+| \cos(\omega t + \beta d + \phi^+)$$

Make sure you can do the conversion
The incident wave \( v_{\text{inc}} (d, t) = |V_0^+| \cos (\omega t + \beta d + \phi^+) \)

Notice that we are talking about lossless transmission lines thus \( Z_0 \) is real. Also notice that \( P_{\text{inc}} \) is always positive.

The average flow:

\[
P_{\text{inc, av}} = \frac{1}{T} \int_0^T P_{\text{inc}} (d, t) \, dt = \frac{|V_0^+|^2}{2Z_0}
\]
Similarly, the reflected wave

\[ v_{\text{ref}}(x,t) = \text{Re} \left( \Gamma V_0^* e^{-j\beta_d t} e^{j\omega t} \right) \]
\[ = \text{Re} \left[ |\Gamma| |V_0^*| e^{j(\omega t - \beta_d x + \phi^* + \theta_r)} \right] \]
\[ = |\Gamma| |V_0^*| \cos (\omega t - \beta_d x + \phi^* + \theta_r) \]

reflected

\[ P_{\text{ref}} = -\frac{v_{\text{ref}}^2}{Z_0} = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0} \cos^2 (\omega t - \beta d - \phi^* + \theta_r) \]

\[ P_{\text{ref, av}} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} \]

The average total or net energy flow is

\[ P_{\text{av}} = P_{\text{Inc, av}} + P_{\text{ref, av}} = \frac{|V_0^+|^2}{Z_0} \left( 1 - |\Gamma|^2 \right) \]
You may do it the phasor way and just get the average values.

\[ P_{av} = \frac{1}{2} \text{Re} \left( \tilde{V} \tilde{I}^* \right) \]

Apply the same to the incident, the reflected, and the total or net. You will get:

\[ P_{inc, av} = \frac{1}{2} \frac{|V_0 + l|^2}{Z_0} \]
\[ P_{ref, av} = -|\Gamma|^2 \frac{|V_0 + l|^2}{2Z_0} \]
\[ P_{av} = \frac{1}{2} \frac{|V_0 + l|^2}{Z_0} \left(1 - |\Gamma|^2\right) \]

Now, look at them. Are they just intuitively obvious?

Think about a special case:

\[ |\Gamma| = 0 \quad \Rightarrow P_{av} = \frac{|V_0 + l|^2}{2Z_0} \]
\[ P_{av} = P_{inc, av} = \frac{|V_0 + l|^2}{2Z_0} \]
\[ P_{ref, av} = -|\Gamma|^2 \frac{|V_0 + l|^2}{2Z_0} = 0 \]