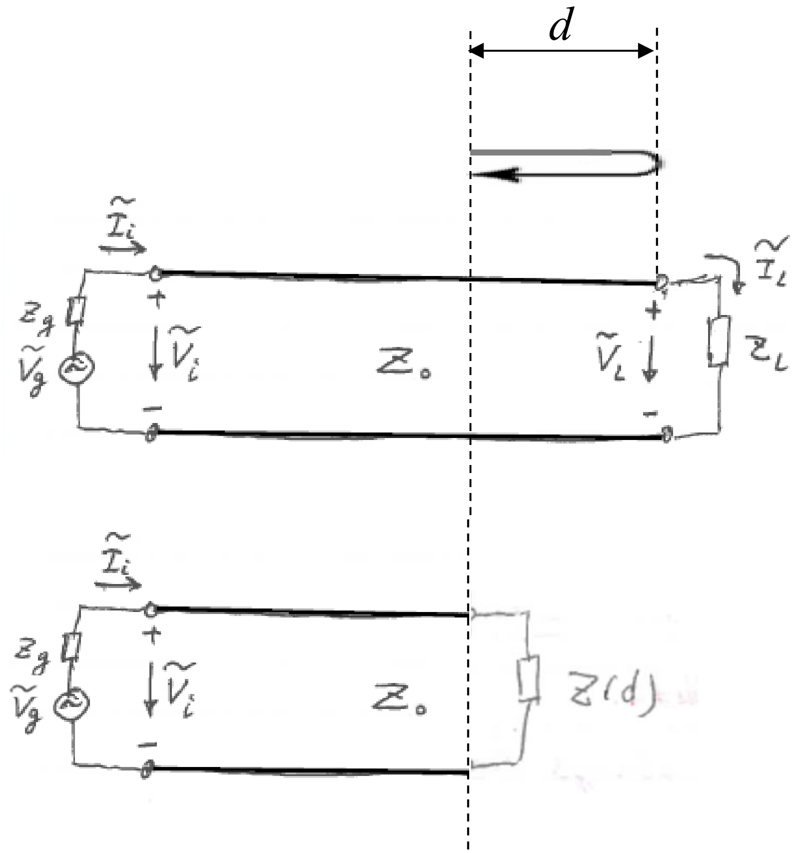
Power flow along transmission lines

At any distance *d*, we can use an equivalent impedance *Z*(*d*) to represent the stuff to the right of *d*.

Question: What is the impedance “felt” by the incident wave at *d*?



Let’s now have a quantitative look at the energy flowing along the transmission line.

Let’s first do this with the “real quantities” instead of phasors.

$$v(d,t) = Re\left(\tilde{V}(d) e^{j\omega t}\right)$$

We forgo the convenience for the moment, in order not to forget the real world quantities.

The incident wave

$$v_{inc}(d,t) = Re\left(\underline{V_0^+} e^{j\beta d} e^{j\omega t}\right)$$

a complex number

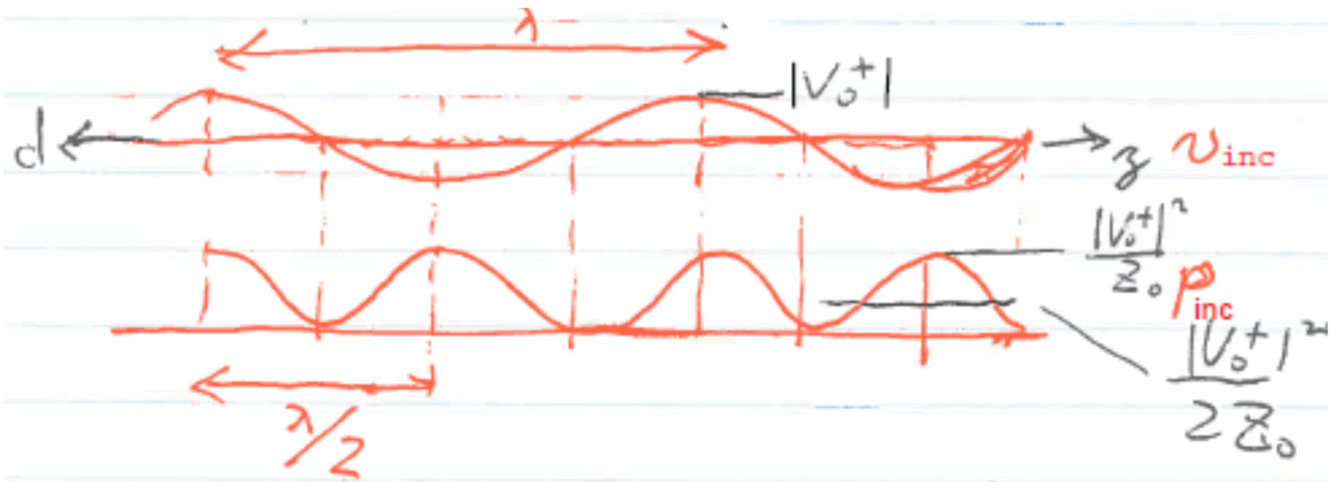
$$= Re\left(|V_0^+| e^{j(\omega t + \beta d + \phi^+)}\right)$$

$V_0^+ = |V_0^+| e^{j\phi^+}$

$$= |V_0^+| \cos(\omega t + \beta d + \phi^+)$$

Make sure you can do the conversion

The incident wave $v_{inc}(d,t) = |V_0^+| \cos(\omega t + \beta d + \phi^+)$



$$P_{inc} = \frac{v_{inc}^2}{Z_0} = \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t + \beta d + \phi^+)$$

talk about why.

Notice that we are talking about lossless transmission lines thus Z_0 is real. Also notice that P_{inc} is always positive.

The average flow:

$$\boxed{P_{inc,av} = \frac{1}{T}\int_0^T P_{inc}(d,t)\,dt = \frac{|V_0^+|^2}{2Z_0}}$$

Similarly, the reflected wave

$$v_{ref}(d,t) = \mathrm{Re}\left(\Gamma V_0^+ e^{-j\beta d} e^{j\omega t}\right)$$
$$= \mathrm{Re}\left[|\Gamma|\,|V_0^+|\, e^{j(\omega t - \beta d + \phi^+ + \theta_r)}\right]$$
$$= |\Gamma|\,|V_0^+| \cos(\omega t - \beta d + \phi^+ + \theta_r)$$

reflected

$$P_{ref} = -\frac{v_{ref}^2}{Z_0} = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t - \beta d - \phi^+ + \theta_r)$$

$$P_{ref,av} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}$$

The average total or net energy flow is

$$P_{av} = P_{inc,av} + P_{ref,av} = \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$$

You may do it the phasor way and just get the average values.

$$\boxed{P_{av} = \frac{1}{2} \operatorname{Re}\left(\tilde{V}\ \tilde{I}^*\right)} \qquad (1)$$

Apply the same to the incident, the reflected, and the total or net. You will get:

$$\boxed{P_{inc,av} = \frac{|V_0^+|^2}{2Z_0}} \qquad \boxed{P_{ref,av} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}}$$

$$\boxed{P_{av} = \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)} \qquad (2)$$

Now, look at them. Are they just intuitively obvious?

Do it on your own:

For the total or net power flow, derive Eq (2) from (1) on your own.

Think about a special case: $|\Gamma| = 0 \Rightarrow P_{av} = \dfrac{|V_0^+|^2}{2Z_0}$

$$P_{av} = P_{inc,av} = \frac{|V_0^+|^2}{2Z_0}$$

$$P_{ref,av} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = 0$$

Consider a class of special cases: $|\Gamma| = 1$

Review Textbook Section 2-9.
Finish HW3.