Power flow along transmission lines

At any distance $d$, we can use an equivalent impedance $Z(d)$ to represent the stuff to the right of $d$.

Question: What is the impedance “felt” by the incident wave at $d$?

We stopped here on Tue 2/23/2021.

Let’s now have a quantitative look at the energy flowing along the transmission line.

Let’s first do this with the “real quantities” instead of phasors.

$$v(d, t) = \text{Re} \left( V(d) e^{j\omega t} \right)$$

We forgo the convenience for the moment, in order not to forget the real world quantities. The incident wave

$$v_{\text{inc}}(d, t) = \text{Re} \left( V_0 e^{j\omega t} \right)$$

$$= \text{Re} \left( |V_0| e^{j\omega t + \beta d + \phi^+} \right)$$

Make sure you can do the conversion
The incident wave

\[ v_{\text{inc}}(d, t) = |V_0^+| \cos(wt + \beta d + \phi^+) \]

Notice that we are talking about lossless transmission lines thus \( Z_0 \) is real. Also notice that \( P_{\text{inc}} \) is always positive.

The average flow:

\[ P_{\text{inc, av}} = \frac{1}{T} \int_0^T P_{\text{inc}}(d, t) \, dt = \frac{|V_0^+|^2}{2Z_0} \]
Similarly, the reflected wave

\[
\begin{align*}
\nu_{ref}(d,t) &= \text{Re} \left( \Gamma V_0^+ e^{-j\beta d} e^{j\omega t} \right) \\
&= \text{Re} \left[ |\Gamma| |V_0^+| e^{j(\omega t - \beta d + \phi^* + \theta_r)} \right] \\
&= |\Gamma| |V_0^+| \cos(\omega t - \beta d + \phi^* + \theta_r)
\end{align*}
\]

reflected

\[
P_{ref} = -\frac{\nu^2_{ref}}{Z_0} = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t - \beta d + \phi^* + \theta_r)
\]

\[
P_{ref,av} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}
\]

The average total or net energy flow is

\[
P_{av} = P_{inc,av} + P_{ref,av} = \frac{|V_0^+|^2}{2Z_0} \left( 1 - |\Gamma|^2 \right)
\]
You may do it the phasor way and just get the average values.

\[ P_{av} = \frac{1}{2} \Re(\bar{V} \bar{I}^*) \]  

(1)

Apply the same to the incident, the reflected, and the total or net. You will get:

\[ \begin{align*}
    P_{inc, av} &= \frac{1}{2} \frac{|V_0|^2}{2Z_0} \\
    P_{ref, av} &= -\left| \Gamma \right|^2 \frac{1}{2Z_0} \frac{|V_0|^2}{2Z_0} \\
    P_{av} &= \frac{1}{2} \frac{|V_0|^2}{2Z_0} \left( 1 - \left| \Gamma \right|^2 \right)
\end{align*} \]  

(2)

Now, look at them. Are they just intuitively obvious?

Do it on your own:

For the total or net power flow, derive Eq (2) from (1) on your own.
Think about a special case: \( |\Gamma| = 0 \Rightarrow P_{av} = \frac{|V_o|^2}{2Z_0} \)

Consider a class of special cases: \( |\Gamma| = 1 \)

Review Textbook Section 2-9.
Finish HW3.